SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

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Maps
Maps
Maps

Simplicial Complex Augmentation Framework for Bijective Maps

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Maps

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Maps

Simplicial Complex Augmentation Framework for Bijective Maps

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Better Maps
Better Maps
Better Maps
Better Maps

Distortion

Simplicial Complex Augmentation Framework for Bijective Maps
Better Maps

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Better Maps
Better Maps

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Better Maps

Simplicial Complex Augmentation Framework for Bijective Maps

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Overlapping
Overlapping
Overlapping
Overlapping

Local Flip
Overlapping

Local Flip

Simplicial Complex Augmentation Framework for Bijective Maps
Overlapping

Local Flip
Overlapping

Local Flip
Overlapping

Local Flip
Overlapping

Local Flip

Global Overlap
Simplicial Complex Augmentation Framework for Bijective Maps

Locally Injective Maps

Results

Bijective Maps

Comparisons

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Locally Injective Maps

Results

Bijective Maps

Comparisons
Bijective Condition: Local

\[ \text{det}(\Delta) > 0 \quad \forall \Delta \in \mathcal{M} \]
Symmetric Dirichlet Energy
Symmetric Dirichlet Energy
Symmetric Dirichlet Energy
Symmetric Dirichlet Energy
Symmetric Dirichlet Energy
Symmetric Dirichlet Energy
Symmetric Dirichlet Energy

Reference Triangle

$J$
Symmetric Dirichlet Energy

Reference Triangle

\[ J \]

\[ \infty \quad 2 \quad 4 \]

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Symmetric Dirichlet Energy

\[ E = \|J\|_F^2 + \|J^{-1}\|_F^2 \]
Symmetric Dirichlet Energy

\[ E = \|J\|_F^2 + \|J^{-1}\|_F^2 \]

\[ = \sigma_1^2 + \sigma_2^2 + \sigma_1^{-2} + \sigma_2^{-2} \]
Locally Injective Mappings

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[CM: Shtengel et al 2017]

[SLIM: Rabinovich et al 2017]

[LIM: Schueller et al 2013]

[AKVP: Claci et al 2016]

[AQP: Kovalsky et al 2016]
Scalable Locally Injective Mappings

\[ \mathcal{B}[X_{k-1}](x) \]

Rotation Invariance Energies

[Rabinovich et al 2017]
Scalable LocallyInjective Mappings

\[ \mathcal{P}(x_{k-1})(x) \]

Rotation Invariance Energies

[Rabinovich et al 2017]
Scalable Locally Injective Mappings

Rotation Invariance Energies

[Rabinovich et al 2017]

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Scalable Locally Injective Mappings

\[ X_k = \operatorname{argmin}_x \mathcal{Y}[X_{k-1}](x) \]

Rotation Invariance Energies

[Rabinovich et al 2017]
Locally Injective Maps

Bijective Maps

Results

Comparisons
Bijective Condition: Global

\[ \Delta_1 \cap \Delta_2 = \emptyset \]
\[ \forall (\Delta_1, \Delta_2) \in \mathcal{M} \times \mathcal{M} \]
Bijective Maps: Smith and Schaefer 2015

- Collision Detection on Boundary
- Local Support Energy
Scaffold: Natural Collision Detector
Scaffold: Natural Collision Detector
Scaffold Pipeline

[Zhang et al 2005]
Scaffold Pipeline

[Zhang et al 2005]

[Tutte 1963]
Scaffold Pipeline

- Build Scaffold

[Zhang et al 2005]

[Tutte 1963]
Scaffold Pipeline

- Tutte Embedding
- Build Scaffold
- Valid Interior Deformation
Scaffold Restricts
Scaffold Restricts
Scaffold Restricts
Scaffold Restricts
Scaffold Restricts
Scaffold Restricts
Scaffold Restricts
Scaffold Restricts
More Progressive [Müller et al 2015]
More Progressive [Müller et al 2015]

- Start from Identity
More Progressive [Müller et al 2015]

- Start from Identity
- Deform Arbitrarily
More Progressive [Müller et al 2015]

- Start from Identity
  - Deform Arbitrarily
  - Bounce Back
More Progressive [Müller et al 2015]

- Start from Identity
- Deform Arbitrarily
- Bounce Back
More Progressive [Müller et al 2015]

• Start from Identity
  • Deform Arbitrarily
  • Bounce Back
Scaffold

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Scaffold Helps

global
Scaffold Helps
SCAF Helps
SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

• Previous
  • Local Knowledge
  • Collision Detection
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- Scaffold Optimization
  - Global Knowledge
  - Collision Resolution
SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

• Scaffold Optimization
  • Global Knowledge
  • Collision Resolution
SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

• Scaffold Optimization
  • Global Knowledge
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SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

- Scaffold Optimization
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SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

- Scaffold Optimization
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SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

- Scaffold Optimization
  - Global Knowledge
  - Collision Resolution
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- Scaffold Optimization
  - Global Knowledge
  - Collision Resolution
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- Scaffold Optimization
  - Global Knowledge
  - Collision Resolution

\[ E = E_M + E_S \]
SCAF: Simplicial Complex Augmentation Framework for Bijective Maps

- Scaffold Optimization
  - Global Knowledge
  - Collision Resolution

\[ E = E_M + E_S \]
Formulation

\[ E_M(X) \]

coordinates
Formulation

\[ E_M(X) \]

coordinates
Formulation

\[ E_M(X) + E_{S^*}(X) \]

coordinates
Formulation

\[ X^* = \arg\min_X E_M(X) + E_{S^*}(X) \]
Formulation

$$\min_{X_k} E(X_k) = E_M(X_k) + E_{S_{k-1}}(X_k) \text{ from } X_{k-1}$$

$$E = E_M + E_S$$
Formulation

\[ \min_{X_k} E(X_k) = E_M(X_k) + E_{S_{k-1}}(X_k) \text{ from } X_{k-1} \]

\[ E = E_M + E_S \]
Formulation

\[
\min_{X_k} E(X_k) = E_M(X_k) + E_{S_{k-1}}(X_k) \text{ from } X_{k-1}
\]

\[
E = E_M + E_S
\]
Formulation

\[ \min_{X_k} E(X_k) = E_M(X_k) + E_{S_{k-1}}(X_k) \] from \( X_{k-1} \)

\[ E = E_M + E_S \]
Formulation

\[
\min_{X_k} E(X_k) = E_M(X_k) + E_{S_{k-1}}(X_k) \text{ from } X_{k-1}
\]

\[
S_k = \text{Update}(X_k) \text{ from } S_{k-1}
\]

\[
E = E_M + E_S
\]
Triangulation

- Local Update
- Edge Flip [Zhang et al. 2005, Müller et al. 2015]

$$S_k = \text{Update}(X_k) \quad \text{from} \quad S_{k-1}$$
Triangulation

- Local Update
- Edge Flip

$S_k = \text{Update}(X_k)$ from $S_{k-1}$

[Zhang et al. 2005, Müllner et al. 2015]
Triangulation

- **Local Update**
- **Edge Flip**

\[ S_k = \text{Update}(X_k) \text{ from } S_{k-1} \]
Triangulation

• Direct reconstruct
• Coarse
Triangulation

• Direct reconstruct
• Coarse
Triangulation

• Direct reconstruct
• Coarse
Triangulation

- Direct reconstruct
- Coarse
Triangulation

- Direct reconstruct
- Coarse
Triangulation

- Direct reconstruct
- Coarse
Triangulation

• Direct reconstruct
• Coarse
Triangulation

- Direct reconstruct
- Coarse
Triangulation

- Direct reconstruct
- Coarse
Locally Injective Maps

Bijective Maps

Results

Comparisons
Result
Robustness

• **Global Parameterization Dataset** [Myles et al. 2014]

• **119 meshes cut by** [Bommes et al. 2009] [Myles et al. 2014]
Robustness

• Global Parameterization Dataset [Myles et al. 2014]
• 119 meshes cut by [Bommes et al. 2009] [Myles et al. 2014]
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Simplicial Complex Augmentation Framework for Bijective Maps
Simplicial Complex Augmentation Framework for Bijective Maps
Volume

- Same Formulation
- Different Tessellation
  - TetGen [Si 2015]
  - Local Operations [Klinger 2009]
• Same Formulation
• Different Tessellation
  • TetGen [Si 2015]
  • Local Operations [Klinger 2009]
Volume

- Same Formulation
- Different Tessellation
  - TetGen [Si 2015]
  - Local Operations [Klinger 2009]
Locally Injective Maps

Bijective Maps

Results

Comparisons
Quality

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Quality

9K faces
Quality

9K faces
Quality

9K faces

[Smith and Schaefer 2015]: 2 hour 20 min
Quality

[Smith and Schaefer 2015]: 2 hour 20 min

< 2 min

9K faces
Efficiency

80K faces
Efficiency

80K faces
Efficiency

< 40 min
Efficiency

Over 5 days!

< 40 min
Efficiency

Over 5 days!

< 40 min
Scalability

- Inherited from SLIM [Rabinovich et al. 2017]
Limitations

- Local Minimum
- Valid Initialization
  - Easy in 2D
  - Hard in 3D
- Implementation in 3D
  - Less Efficient
  - More Involved
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Efficient

Robust

Scalable

Simplicial Complex Augmentation Framework for Bijective Maps

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https://github.com/jiangzhongshi/scaffold-map