Lecture 7: Ladner's Theorem

1 Review

2 Efficient Universal Machines

A characteristic class is a pair (K, χ) where K is a class and χ is some complexity characteristic. E.g., $\chi = (nondet, poly - time)$.

A universal acceptor for (K, χ) is a universal acceptor U that accepts K and such that each U_i has characteristic χ.

E.g., $(PSPACE, (nondet, poly - space))$ and $(PSPACE, (det, poly - space))$.

3 Strengths of Reducibilities

We know that $\leq_T^P$ is as strong as $\leq_m^P$. But is it strictly stronger?

If L' is a non-trivial language, then $L' \leq_T^P$ *co*-L'. Hence it suffices to construct a nontrivial L' such that $L' \leq_m^P$ *co*-L' is false.

We have an infinite sequence of conditions $C_0, C_1, \ldots$ that needs to be satisfied by L':

$$C_i : (\exists y_i) y_i \in L' \leftrightarrow T_i(y_i) \in L'.$$

Here T_i is the ith polytime transducer.

We can use a simple diagonalization argument to show this.

4 Polynomial Analogue of Post's Problem

Suppose $P \neq NP$. Does there exist a language $L' \in NP \setminus P$ that is not NP-complete under $\leq_T^P$?

The answer is yes, and is called Ladner's Theorem. This is the polynomial analogue of a question of Post.

The proof is presented in class.