Lecture 6: Reducibilities

1 Introduction

Outline of the theory of NP-completeness. See Chapter 3 of Handout notes for general introduction to this theory.

Today we begin the study of reducibilities covered in Chapter 4.

2 Basic Inclusion Lemma

Motivated by the success of the NP-completeness theory, we want to apply the same techniques to a variety of other important inclusion questions:

$$\text{Is } K \subseteq K'?$$

for various classes K, K'.

Definition of reducibility $\leq$: this is a reflexive binary relation on languages, written with infix notation, $L \leq L'$. Note that $\leq$ need not be transitive.

Lemma 1 (Basic Inclusion Lemma) *f K' is $\leq$-closed and L is K-complete, then $K \subseteq K'$ iff $L \in K'$.*

How to use this lemma to study the (K, K') inclusion question: we need to choose the reducibilities with the appropriate strength for K, K'!

3 Many-One Reducibility

Basic properties.

Result: log-space transformations are closed under functional composition.

4 Turing Reducibility

Define oracle machines.

Space-bounded oracle computations have unusual properties!