Lecture 14

1 Finish Up from Last Lecture

We had a bug in the proof that the pebbling time for a bounded in-degree dag is $O(m/logm)$. Will assign hw exercise to fix this.

2 Stochastic Choices

New topic today: stochastic Choices is exemplified by probabilistic machines, but we can also make stochastic choices by using basic functions of the form $\oplus$ (probabilistic-or)and $\otimes$ (probabilistic-and).

- Forms of Computational Error: If the least fixed point value $Val_M(x) = [a, b]$, we can define 4 kinds of errors.

  We have (a) acceptance versus rejection errors, and (b) pessimistic versus optimistic errors.

  Pessimistic errors: we are interested in the bounded or unbounded case.

  Optimistic errors: we are interested in zero-error or not.

  IMPORTANT COMBINATION: bounded-error with zero-error rejection.

- Example of primality testing. These concepts are illustrated there.

- Discussion: Nondeterminism versus probabilistic machine. SHOW: $NTIME(t) \subseteq PrTIME(t + 1)$.

- Discussion: Zero-error. What it means when we do not have negation.

- Discussion: One-sided error versus nondeterminism. SHOW: $PrTIME_1(t) \subseteq NTIME(t)$.

- ERROR NOTATIONS: subscripts of $0, 1, b$ in addition to r. Thus

  $$PrTIME_0(t) \subseteq PrTIME_1(t) \subseteq PrTIME_b(t) \subseteq PrTIME_r(t) \subseteq PrTIME(t)$$

  Also the polynomial analogue of these classes are denoted:

  $$ZPP \subseteq RP \subseteq BPP \subseteq PP$$

  DISCUSS class inclusion diagram (FIGURE 8.1 of lecture notes)

- THREE TECHNIQUES TO AMPLIFY ERROR GAPS:

  (1) Zero-error rejection machines: by repeating a computation t times, and answer YES iff at least one YES shows up.

  (2) Bounded-error machines: do majority voting.

  (3) Stochastic machines: do special A(x) operator.

- What about AVERAGE complexity? Pathological without bounded-error!