Lecture 11

1 Finishing Up Separability

We show a simple technique of Book to show "weak separability" of two classes K and J. That is, we show $K \neq J$ without resolving the question whether it is $K \setminus J$ or $J \setminus K$ that is non-empty.

The proof for $PLOG \neq NP$ goes as follows: ...

We can extract the underlying properties that makes this proof work: we need that K has a complete language (under $\leq$) and J is "stratified" and each strata J_i is closed under $\leq$-reducibility.

2 Alternating Choices: Introduction

– two main examples: probabilistic machines, non-deterministic machines.

– initial example: assume finite computation trees

– generalize non-determinism to alternation

– More general: introduce functions on $[0,1]$. E.g. Probabilistic-Alternation.

– What properties do we need? Theory of "Valuation"

– Many other models falls under our valuation machine model: interactive proof systems (Goldwasser, Micali and Rackoff), arthur-merlin games (Babar), probabilistic-game automata (Condon-Ladner), logical type machines (Hong), etc.

3 Interval Algebra

MOTIVATION for intervals.

INT= all closed intervals in [0,1]

$\perp$ is the special value $[0,1] \in INT$.

Six functions: $\neg, \wedge, \vee, \textcircled{\top}, \otimes, \oplus$.

For negation, we have $\neg[u,v] = [1-v, 1-u]$.

The remaining 5 are best understood as extensions of their real counterpart. We then extend them to their interval analog using

$$[x,y] \circ [u,v] = [x \circ u, y \circ v].$$

LEMMA:

(1) The 5 binary functions are commutative.

(2) With the exception of $\textcircled{\top}$, they are also associative.

(3) De Morgan's Law holds for each of the pairs $(\wedge, \vee)$, $(\textcircled{\top}, \textcircled{\top})$ and $(\otimes, \oplus)$. E.g.

$$\neg(I \otimes J) = (\neg I) \oplus (\neg J).$$

TWO partial orders:

(A) $[x,y] \leq [u,v]$ iff $x \leq u$ and $y \leq u$.

(B) (information order) $[x,y] \sqsubseteq [u,v]$ iff $[x,y] \supseteq [u,v]$.

E.g., strong 3-valued algebra of Kleene with operations $\neg, \vee, \wedge$ over $\{0, 1, \perp\}$.

DEFINITION: $f : INT^n \to INT$.

(1) f is **monotonic** if $J_i \sqsubseteq J_i'$ implies

$$f(J_1, \ldots, J_n) \sqsubseteq f(J_1', \ldots, J_n').$$

(2) f is **continuous** if it is monotonic and for all

$$I_i^{(1)} \sqsubseteq I_i^{(2)} \sqsubseteq \cdots$$

we have

$$f(\lim_j \{I_1^{(j)}\}, \ldots, \lim_j \{I_n^{(j)}\}) = \lim_j f(I_1^{(j)}, \ldots, I_n^{(j)}). \tag{1}$$

LEMMA:

(1) The 6 functions are continuous.

(2) Function composition preserves monotonicity and continuity.

4 Theory of Valuations

WHAT IS NEW:

(a) We refine "accept/reject" into "accept/reject/undecided"

(b) GLOBAL versus LOCAL ANSWERS. GLOBAL = "accept/reject/undecided". LOCAL = "yes/no/yo".

BASIS: Let B be a set of functions from INT^n to INT (for various n depending on the function). We call B a **basis** if B contains the constants $0, 1$ and identity id, and each function in B is continuous.

Let δ be a Turing transition table. A **B-machine** (or **choice machine**) is a pair $M = (\delta, \gamma)$ where γ assigns to each state of δ a basis function in B. If state q is assigned an n-ary function $\gamma(q)$, then any configuration C with state q has exactly n successors $C_1, \ldots, C_n$.

OPERATION OF CHOICE MACHINES.

If the machines are halting, then the interpretation is clear! We may define the language $L(M)$ as usual.

Otherwise, you need the theory of valuation that tells you that there is a unique least solution.

5 Palindrom Example

This is an exercise in understanding how alternating machines operate. We show that palindromes can be accepted by an alternating machine in linear time and logarithmic space. It is known that if a nondet. TM accepts palindromes in time-space (t, s) then $t(n)s(n) = \Omega(n^2)$. This proves that alternation is more powerful than nondeterminism.