Lecture 10

1 Separation Results: Introduction

Let K, K' be classes. A "separation result" for (K, K') is one showing the existence of a language L in $K - K'$. We say L "separates" K from K'.

(1) Complete languages in K are not necessarily good choices for separators!

(2) Translation results:

$$K_1 \in K_2 \Rightarrow K_1' \subseteq K_2'.$$

Thus a separation for (K_1', K_2') implies a separation for (K_1, K_2). This is useful for nondeterministic classes.

(3) Infer lower bounds on specific languages in K.

2 Basic Separation

These are for deterministic classes. "Basic" because most other separation results ultimately depend on them. Also known as hierarchy theorems.

Theorem 1: (Det.Space)

Let $s_2(n) = \omega(s_2(n))$ and s_2 is space constructible. Then $DSPACE(s_2) - DSPACE(s_1)$ is non-empty.

Theorem 2: (Det.Time)

Let $t_2(n) = \omega(t_1(n) \log t_1(n))$ and t_2 be time constructible. Then $DTIME(t_2) - DTIME(t_1)$ is non-empty.

3 Translational Lemma

If we try to obtain a separation result for $(NTIME(t_2), NTIME(t_1))$ using the above kind of arguments, t_2 would have to be at least exponential in t_1 (as far as we know).

Therefore another approach is needed.

Lemma (Translation for Space): Let $s_1(n) \geq n, s_2(n) \geq n, f(n) \geq n$ be complexity functions with $f(n)$ space constructible. Then $NSPACE(s_1) \subseteq NSPACE(s_2)$ implies $NSPACE(s_1(f(n))) \subseteq NSPACE(s_2(f(n)))$.

The proof is not too hard, and uses "padding".

This can be used to show: For all $a > b \geq 1$,

$$NSPACE(n^a) - NSPACE(n^b)$$

is non-empty. A much harder proof due to Cook (omitted for this class) is

$$NTIME(n^a) - NTIME(n^b).$$

READ UP: Another translational lemma from SAVAGE can be used to show that

$$DLOG = NLOG ==> LBA = DLBA.$$

4 APPLICATION TO LOWER BOUNDS

READ UP:

We can infer complexity lower Bounds from separation results: if there is a separation result for (K, K'), then we know that (usually) a K-complete language L_0 cannot be in K'. Let $\leq$ be the reducibility used in this definition of completeness.

Suppose $K' = NTIME(t)$. Then we can infer that ANY TM that accepts L_0 must take more than time $F(t(n))$ INFINITELY OFTEN.

Here, $F()$ is a functional that depends on the efficiency of the reducibility $\leq$.

OMITTED FOR THIS CLASS: Sometimes, we can sharpen the INFINITELY OFTEN result into an EVENTUALLY lower bound.