Homework 5
Fundamental Algorithms, Fall 2005, Professor Yap

Due: Wed Dec. 14, in class.

INSTRUCTIONS:

- Please try to give clear but brief answers. Usually, the length of your answer will go down in proportion to the amount of thought given to the problem.

Question 1 (10 Points)

Our "Counter Example" analysis can yield the exact cost to increment from any initial counter value to any final counter value. What is the *exact* number of work units to increment a counter from 71 to 254?

Question 2 (20 Points)

Consider the insertion of the following sequence of keys into an initially empty tree: $-1, 1, -2, 2, -3, 3, \ldots, -n, n$.
Let T_n be the final splay tree.
(a) Show T_n for $n = 1, 2, 3$.
(b) Make a conjecture about the shape of T_n, and prove it.

Question 3 (40 Points)

(a) Carry out the following operations on a Fibonacci heap, starting from an initially empty structure. Show the heap after each operation. Please indicate any marked nodes with an "X".
Insert(a,10) i.e., insert item a with key 10
Insert(b,20)
Insert(c,30)
Insert(d,40)
Insert(e,15)
Insert(f,50)
Insert(g,35)
Insert(h,45)
deleteMin()
decreaseKey(c,14) i.e., decrease the key of c to 14
deleteMin().

(b) We want to do a variation of Fibonacci heaps. The idea is to combine the root list H and the array A used in the consolidation process. In this way, we avoid the consolidation process altogether. First, all sibling lists must be ordered by degree, and furthermore, there is at most one child of each degree. Second, the rootlist H is treated like a sibling list (of some imaginary superroot). This implies that the rootlist and sibling lists have length at most $D(n) = O(\lg n)$. Third, each sibling list (including rootlist) has a potential that is equal to the number of nodes in the sibling list, plus the maximum degree among the siblings. The potential of the data structure $\Phi(H)$ is the sum of the potentials of all sibling lists, including the rootlist, plus twice the number of marked nodes. Marking is as before.

Show that we can perform insert, union and decreaseKey in amortized constant time.

Question 4 (30 Points)

Consider the LCS problem where we are constructing the matrix L corresponding to input strings X, Y.

(a) Suppose $L[i, j] = 89$. What are all the possible values of $L[i-1, j-1]$? What are all the possible values of $L[i, j+1]$? You must prove your claims.

(b) Consider the string $X_n = 010^2 10^3 1 \cdots 0^{n-1} 10^n 1$ and $Y_n = 101^2 01^3 0 \cdots 1^{n-1} 01^n 0$. Compute $L(X_4, Y_4)$. Next, prove that $L(X_n, Y_n) = 2n - 1$.

(c) You can represent $LCS(X, Y)$ by a suitable digraph whose size is $O(mn)$ where $|X| = m, |Y| = n$. This graph can be constructed from the matrix L used to solve the LCS problem. Please describe an algorithm to construct this graph.

(d) Instead of $O(mn)$, give a sharper bound on the space used by this graph. HINT: express the bound in terms of $L(X, Y) = k$.

(e) Given a string Z, how fast can you check whether $Z \in LCS(X, Y)$ using this graph G?