Homework 2
Fundamental Algorithms, Fall 2004, Professor Yap

Due: Thu Oct 7, in class

INSTRUCTIONS:

- Please read questions carefully. When in doubt, please ask.

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1. (20 Points)
   Give the Θ-order for the following sums. If the sum is of polynomial-type or exponential-type sum, you must prove this fact before you invoke the general theorem for such sums. If it is neither polynomial-type nor exponential-type sum, you may be able to reduce to such a sum. [cf. Hw2 from Fall 2001]
   (a) $S_n = \sum_{i=1}^{n} \binom{i}{3}$.
   (b) $S_n = \sum_{i=1}^{n} (\lg^i n)$.
   (c) $S_n = \sum_{i=1}^{n} \frac{i!}{\lg^i n}$.
   (d) $S_n = \sum_{i=2}^{n} i^{1/\lg i}$.

2. (15 Points)
   Use the Master Theorem to solve the following. You must justify why a given recurrence falls under any of the 3 possible cases.
   (a) $T(n) = 18T(n/3) + n^3$.
   (b) $T(n) = 27T(n/3) + n^3$.
   (c) $T(n) = 18T(n/3) + n^2 \lg n$.

3. (15 Points)
   Suppose algorithm A_1 has running time satisfying the recurrence

   $$T_1(n) = aT(n/2) + n$$

   and algorithm A_2 has running time satisfying the recurrence

   $$T_2(n) = 2aT(n/4) + n.$$

   Here, $a > 0$ is some constant. Compare these two running times for various values of a.

4. (30 Points)
   Use domain and range transformations to solve the following two recurrences:
   (a) $T(n) = 4T(n/2) + n^2/\log n$.
   (b) $T(n) = 4T(n/2) + n^2 \log n$.
   Do not use real induction to solve this. You might wish to refer to a similar question in Hw2, Spring 2003.

5. (20 Points)
   Using real induction, give good upper and lower bounds for

   $$T(n) = T(n - \lg n) + n.$$

   HINT: first try to find an upper bound by expanding the recurrence and simplifying the intermediate expressions.