

Homework 4  
Fundamental Algorithms, Fall 2001, Professor Yap

- DUE: Mon Nov 12, in class.
- 1. [ **20 POINTS** ] What is the smallest size of an AVL tree  $T$  such that the deletion of a node in  $T$  will cause two rotation events? Here a “rotation event” is either a single rotation or a double rotation. Give this number  $n$ , and draw such an AVL tree with  $n$  nodes; be sure to indicate the special node  $u$  such that if we delete  $u$ , the rebalancing will cause two rotation events.
- 2. Exercise 15.2-1, page 338. [ **15 POINTS** ] Matrix multiplication problem. You must display your working by filling the entries of a suitable matrix.
- 3. Exercise 15.4-1, page 355. LCS problem. [ **15 POINTS** ]
- 4. Exercise 15.4-5, page 356. Longest monotone subsequence. [ **25 POINTS** ] HINT: Let  $X$  be the input sequence of numbers, and  $X_i$  be the prefix of  $X$  of length  $i$ . Suppose we maintain a list  $L_i = (a_1 \leq a_2 \leq \dots \leq a_{j(i)})$  of numbers where  $a_k$  (for  $k = 1, \dots, j(i)$ ) is the smallest number of  $X_i$  such that there exists a monotone subsequence of  $X_i$  of length  $k$  ends in  $a_k$ . The length of  $L_i$  is  $j(i)$  means that there are no monotone subsequence of  $X_i$  of length  $j(i) + 1$ . Note that  $j(i) \leq i$ . How do you construct  $L_{i+1}$  from  $L_i$ ?

The following exercises are NOT to be handed in, but we encourage you to try to solve them.

1. Exercise 15-3, page 364. Edit distance of two strings. Do part (a) only. HINT: if the input strings are  $X$  and  $Y$ , let  $X_i$  and  $Y_j$  denote their respective prefixes of lengths  $i$  and  $j$ . Let  $D(X_i, Y_j)$  be the edit distance between  $X_i$  and  $Y_j$ . Derive a recursive formula for  $D(X_i, Y_j)$ .