Energy-Based Models: Structured Learning Beyond Likelihoods

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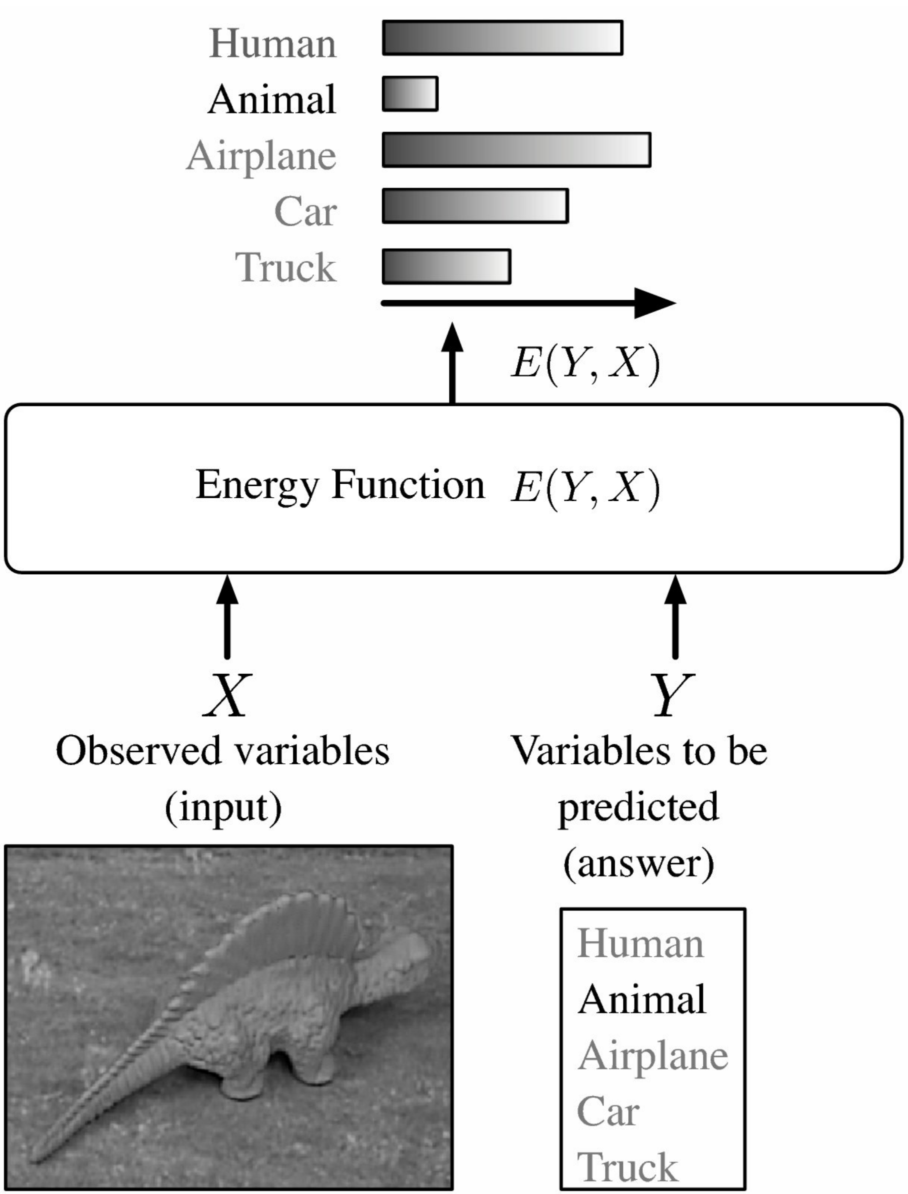
Two Big Problems in Machine Learning

- **1. The "Deep Learning Problem"**
  - "Deep" architectures are necessary to solve the invariance problem in vision (and perception in general)
- **2. The "Partition Function Problem"**
  - Give high probability (or low energy) to good answers
  - Give low probability (or high energy) to bad answers
  - There are too many bad answers!
- **This tutorial discusses problem #2**
  - The partition function problem arises with probabilistic approaches
  - Non-probabilistic approaches may allow us to get around it.
- **Energy-Based Learning provides a framework in which to describe probabilistic and non-probabilistic approaches to learning**
- **Paper: LeCun et al. : "A tutorial on energy-based learning"**
  - http://yann.lecun.com/exdb/publis
  - http://www.cs.nyu.edu/~yann/research/ebm

Plan of the Tutorial

- **Introduction to Energy-Based Models**
  - Energy-Based inference
  - Examples of architectures and applications, structured outputs
- **Training Energy-Based Models**
  - Designing a loss function. Examples of loss functions
  - Which loss functions work, and which ones don't work
  - Getting around the partition function problem with EB learning
- **2. Architectures for Structured Outputs**
  - Energy-Based Graphical Models (non-probabilistic factor graphs)
  - Latent variable models
  - Linear factors: Conditional Random Fields and Maximum Margin Markov Nets
  - Gradient-based learning with non-linear factors
- **Applications: supervised and unsupervised learning**
  - Integrated segmentation/recognition in vision, speech, and OCR.
  - Invariant feature learning, manifold learning

Energy-Based Model for Decision-Making

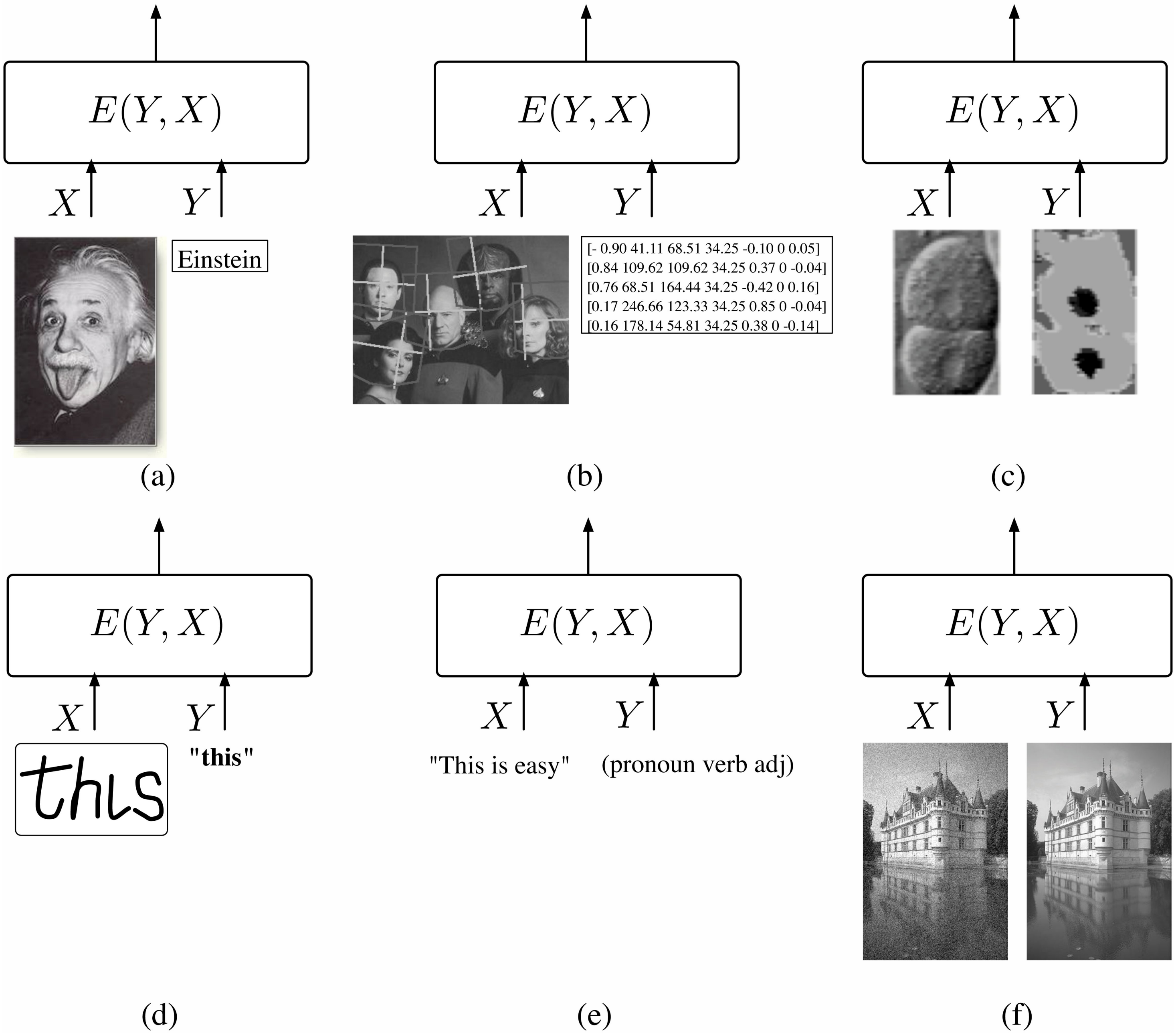


- **Model:** **Measures the compatibility between an observed variable X and a variable to be predicted Y through an energy function E(Y,X).**

$$Y^* = \operatorname{argmin}_{Y \in \mathcal{Y}} E(Y, X).$$

- **Inference:** **Search for the Y that minimizes the energy within a set** $\mathcal{Y}$
- **If the set has low cardinality, we can use exhaustive search.**

Complex Tasks: Inference is non-trivial



- **When the cardinality or dimension of Y is large, exhaustive search is impractical.**
- **We need to use a “smart” inference procedure: min-sum, Viterbi, ...**

What Questions Can a Model Answer?

1. Classification & Decision Making:

- "which value of Y is most compatible with X?"
- Applications: Robot navigation,.....
- Training: give the lowest energy to the correct answer

2. Ranking:

- "Is Y1 or Y2 more compatible with X?"
- Applications: Data-mining....
- Training: produce energies that rank the answers correctly

3. Detection:

- "Is this value of Y compatible with X"?
- Application: face detection....
- Training: energies that increase as the image looks less like a face.

4. Conditional Density Estimation:

- "What is the conditional distribution P(Y|X)?"
- Application: feeding a decision-making system
- Training: differences of energies must be just so.

Decision-Making versus Probabilistic Modeling

Energies are uncalibrated

- The energies of two separately-trained systems cannot be combined
- The energies are uncalibrated (measured in arbitrary untis)

How do we calibrate energies?

- We turn them into probabilities (positive numbers that sum to 1).
- Simplest way: Gibbs distribution
- Other ways can be reduced to Gibbs by a suitable redefinition of the energy.

$$P(Y|X) = \frac{e^{-\beta E(Y,X)}}{\int_{y \in \mathcal{Y}} e^{-\beta E(y,X)}},$$

Partition function

Inverse temperature

Architecture and Loss Function

- **Family of energy functions** $\mathcal{E} = \{E(W, Y, X) : \quad W \in \mathcal{W}\}.$
- **Training set** $\mathcal{S} = \{(X^i, Y^i) : i = 1 \ldots P\}$
- **Loss functional / Loss function** $\mathcal{L}(E, \mathcal{S}) \qquad \mathcal{L}(W, \mathcal{S})$
  - Measures the quality of an energy function
- **Training** $$W^* = \min_{W \in \mathcal{W}} \mathcal{L}(W, \mathcal{S}).$$
- **Form of the loss functional**
  - invariant under permutations and repetitions of the samples

$$\mathcal{L}(E, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} L(Y^i, E(W, \mathcal{Y}, X^i)) + R(W).$$

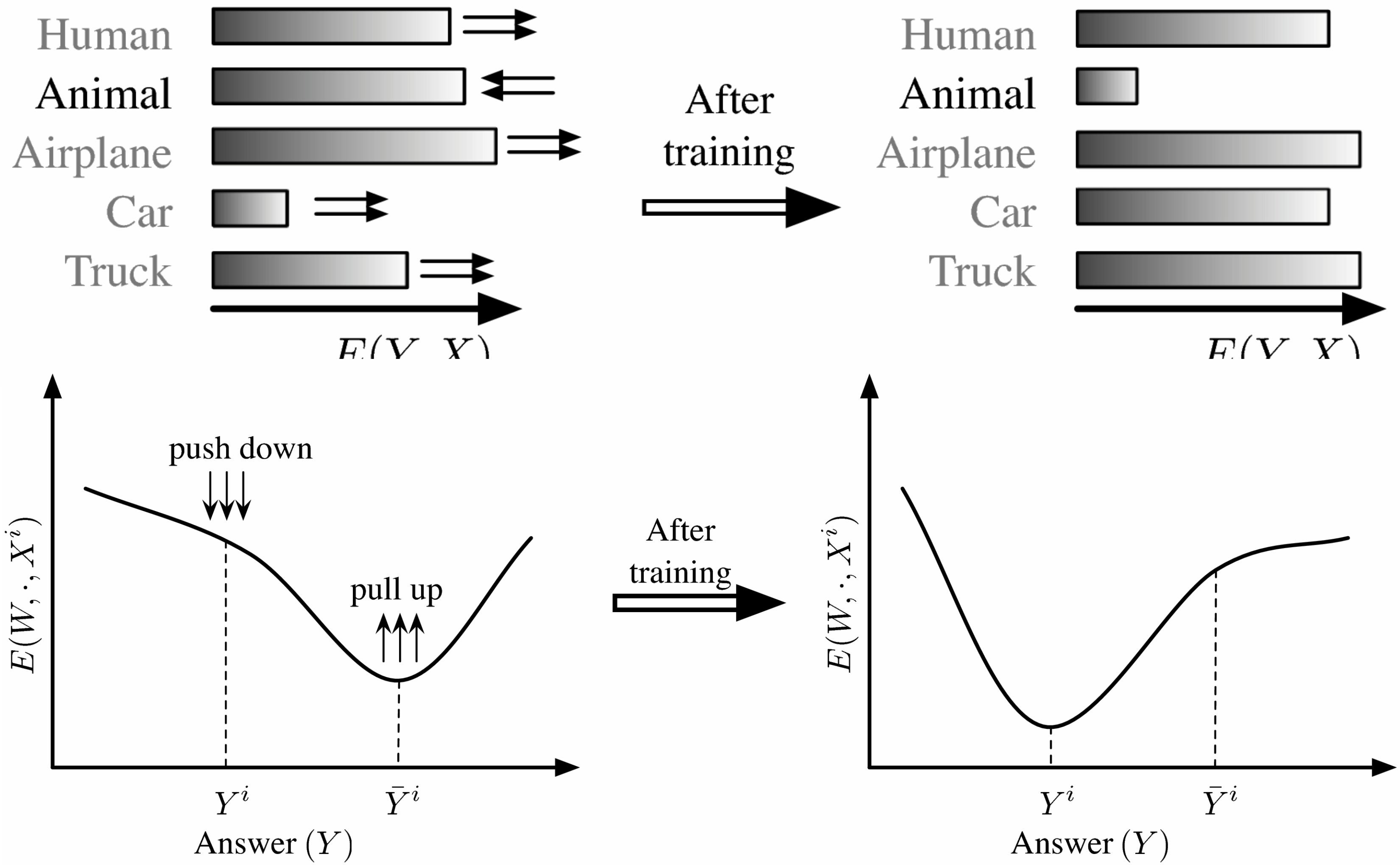
Per-sample loss

Desired answer

Energy surface for a given Xi as Y varies

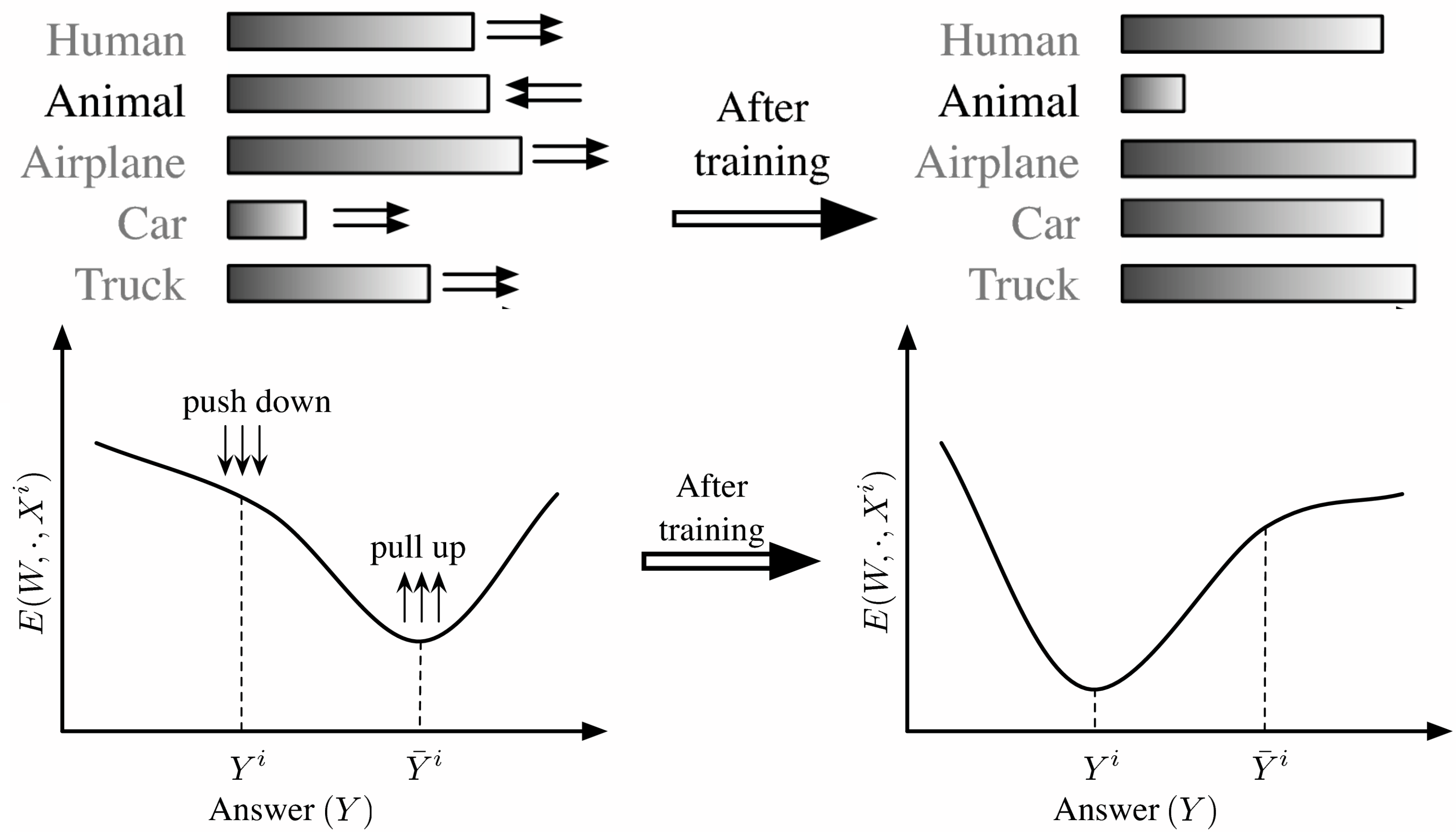
Regularizer

Designing a Loss Functional



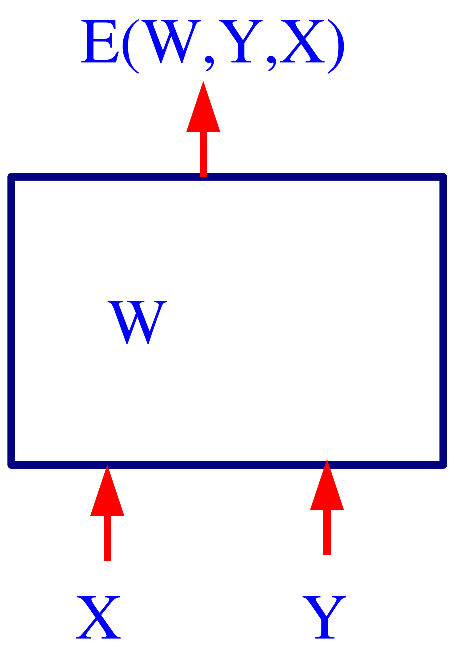
- **Correct answer has the lowest energy -> LOW LOSS**
- **Lowest energy is not for the correct answer -> HIGH LOSS**

Designing a Loss Functional



- **Push down on the energy of the correct answer**
- **Pull up on the energies of the incorrect answers, particularly if they are smaller than the correct one**

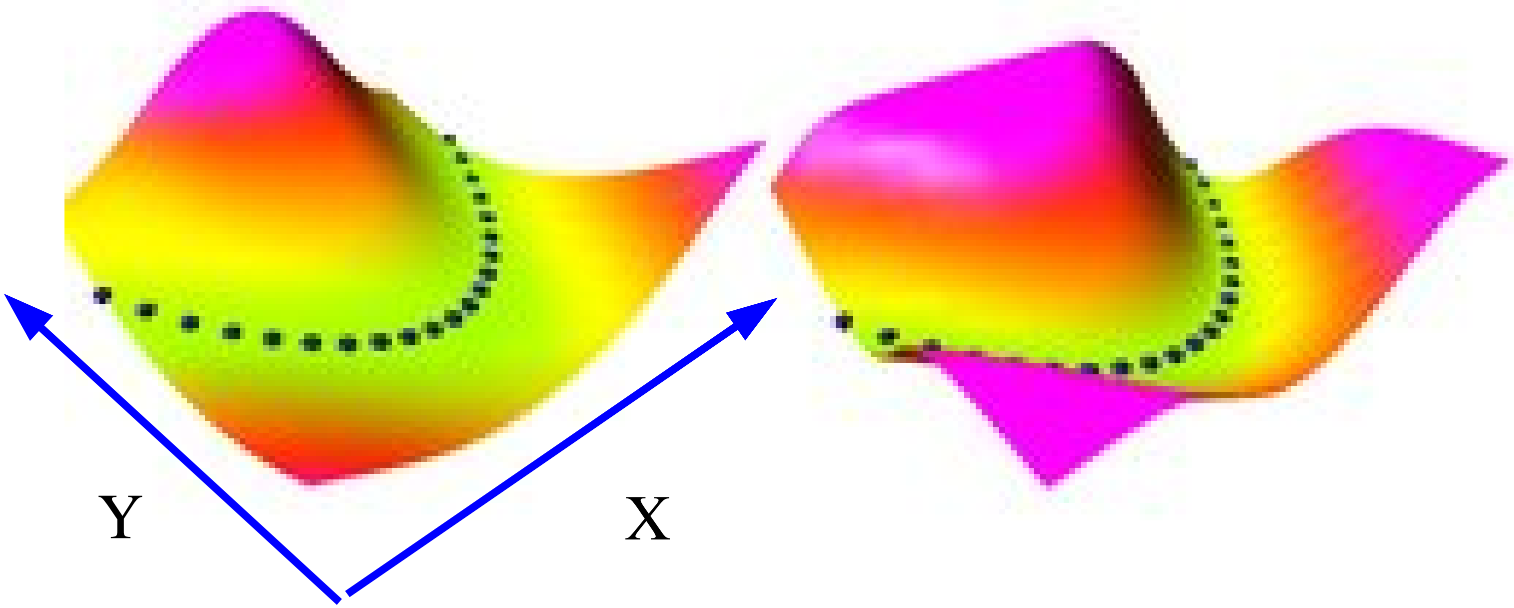
Architecture + Inference Algo + Loss Function = Model



- **1. Design an architecture:** a particular form for E(W,Y,X).
- **2. Pick an inference algorithm for Y:** MAP or conditional distribution, belief prop, min cut, variational methods, gradient descent, MCMC, HMC.....
- **3. Pick a loss function:** in such a way that minimizing it with respect to W over a training set will make the inference algorithm find the correct Y for a given X.
- **4. Pick an optimization method.**

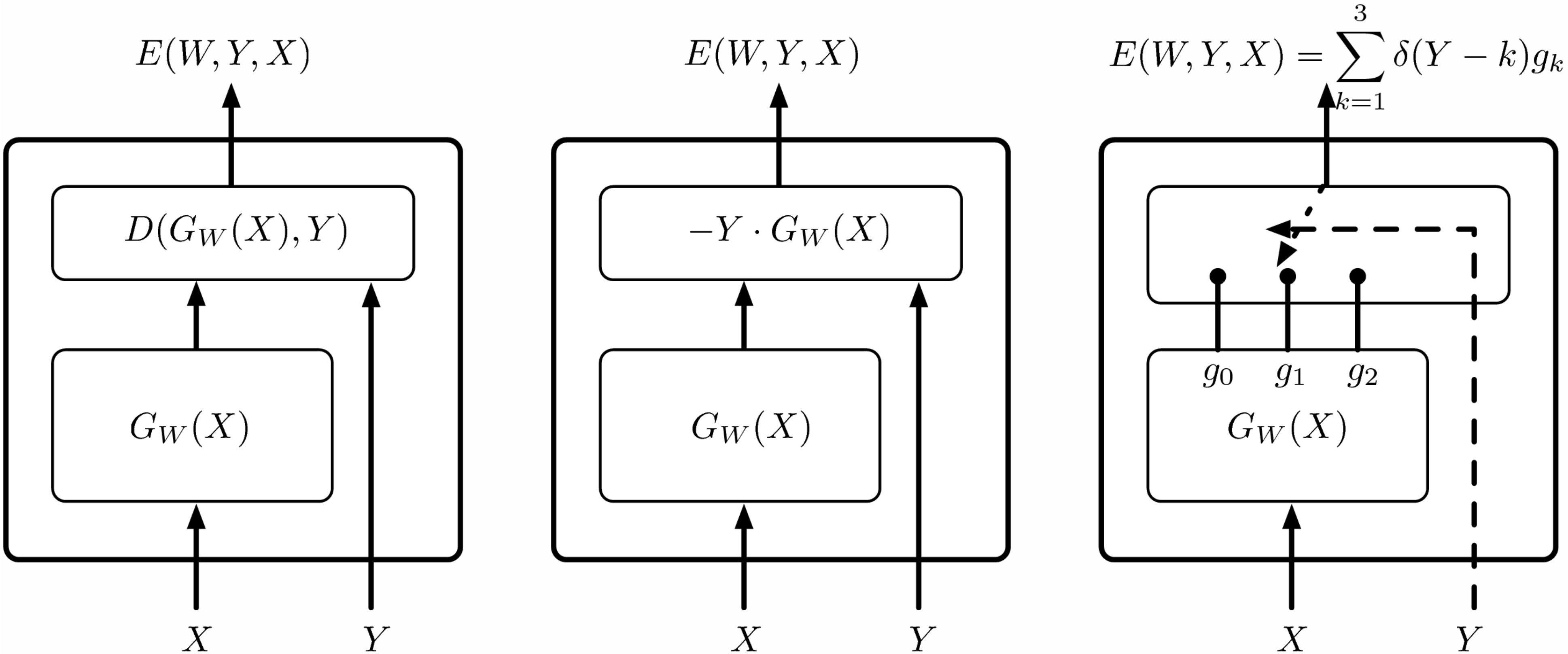
- **PROBLEM: What loss functions will make the machine approach the desired behavior?**

Several Energy Surfaces can give the same answers



- Both surfaces compute Y=X^2
- MINy E(Y,X) = X^2
- Minimum-energy inference gives us the same answer

Simple Architectures



- **Regression**

$$E(W,Y,X) = \frac{1}{2}||G_W(X) - Y||^2.$$

- **Binary Classification**

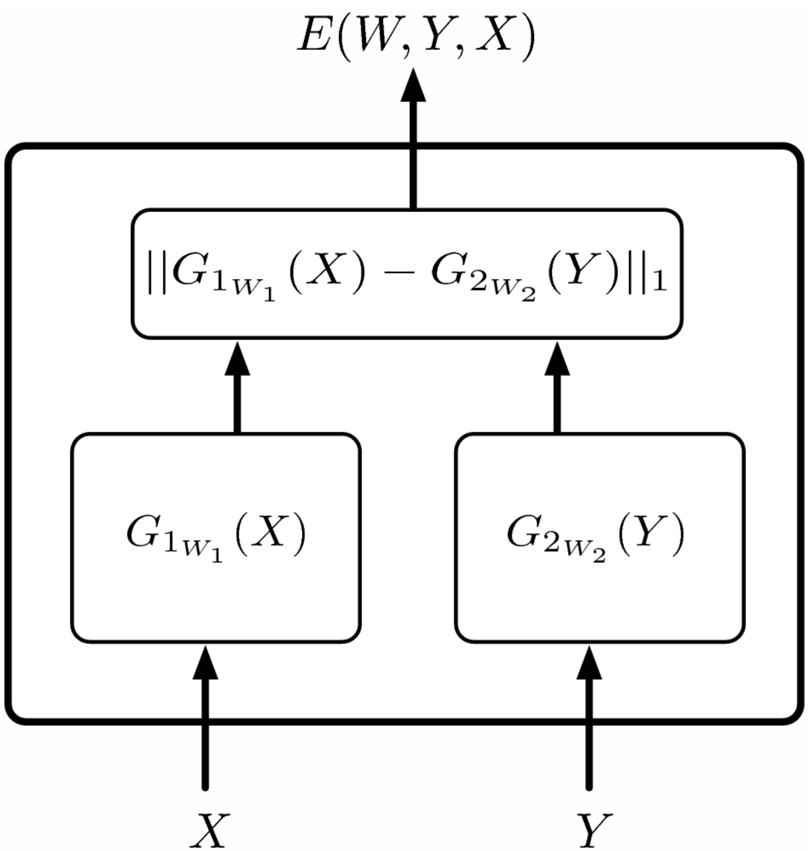
$$E(W,Y,X) = -YG_W(X),$$

- **Multi-class Classification**

Simple Architecture: Implicit Regression

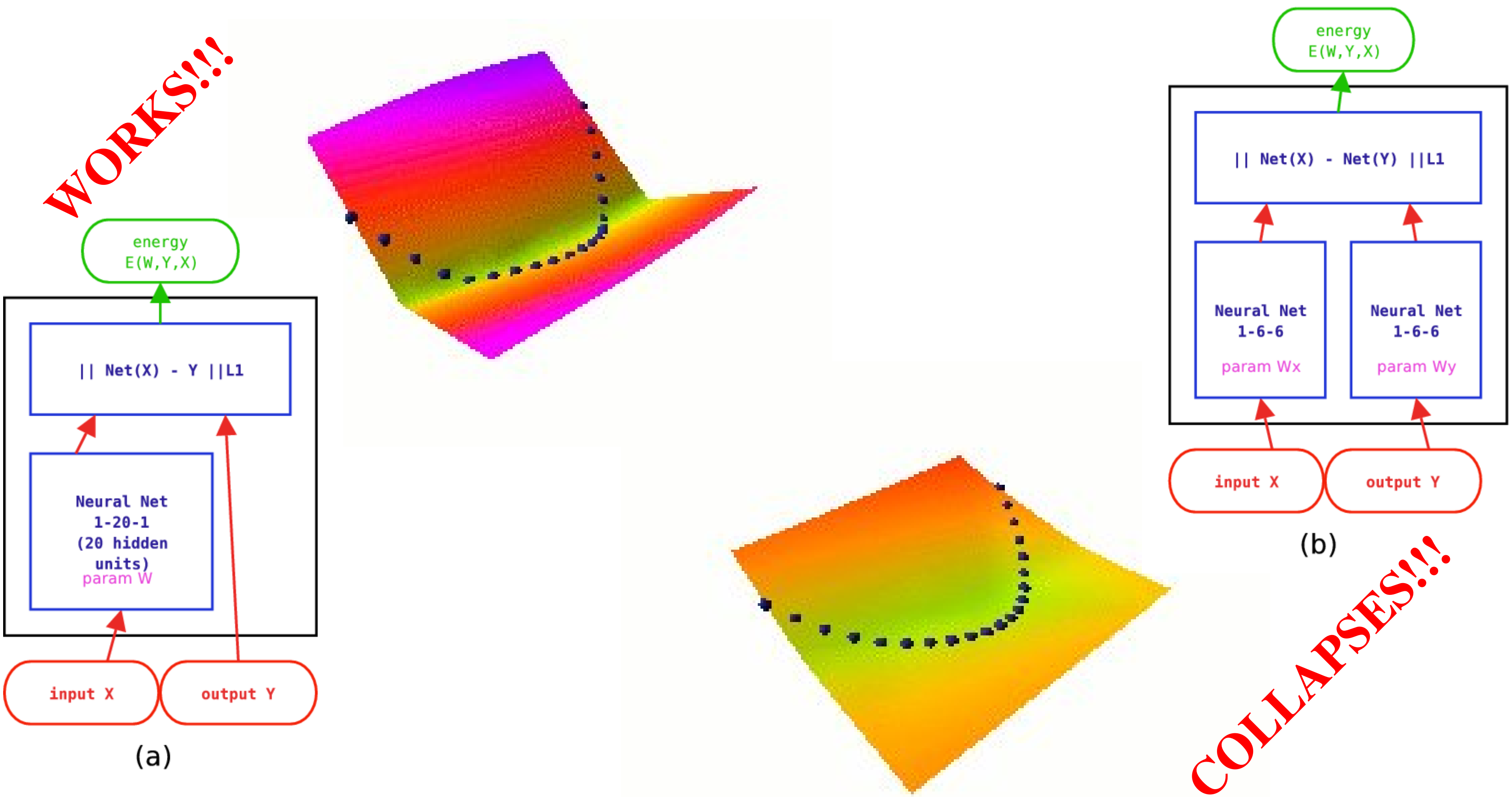
$$E(W, X, Y) = ||G_{1_{W_1}}(X) - G_{2_{W_2}}(Y)||_1,$$

- **The Implicit Regression architecture**
  - allows multiple answers to have low energy.
  - Encodes a constraint between X and Y rather than an explicit functional relationship
  - This is useful for many applications
  - Example: sentence completion: "The cat ate the {mouse,bird,homework,...}"
  - [Bengio et al. 2003]
  - But, inference may be difficult.



Examples of Loss Functions: Energy Loss

- **Energy Loss** $L_{energy}(Y^i, E(W, \mathcal{Y}, X^i)) = E(W, Y^i, X^i).$
  - Simply pushes down on the energy of the correct answer



Examples of Loss Functions:Perceptron Loss

$$L_{perceptron}(Y^i, E(W, \mathcal{Y}, X^i)) = E(W, Y^i, X^i) - \min_{Y \in \mathcal{Y}} E(W, Y, X^i).$$

- **Perceptron Loss [LeCun et al. 1998], [Collins 2002]**
  - Pushes down on the energy of the correct answer
  - Pulls up on the energy of the machine's answer
  - Always positive. Zero when answer is correct
  - No “margin”: technically does not prevent the energy surface from being almost flat.
  - Works pretty well in practice, particularly if the energy parameterization does not allow flat surfaces.

Perceptron Loss for Binary Classification

$$L_{perceptron}(Y^i, E(W, \mathcal{Y}, X^i)) = E(W, Y^i, X^i) - \min_{Y \in \mathcal{Y}} E(W, Y, X^i).$$

- **Energy:** $E(W, Y, X) = -Y G_W(X),$
- **Inference:** $Y^* = \text{argmin}_{Y \in \{-1,1\}} - Y G_W(X) = \text{sign}(G_W(X)).$
- **Loss:** $\mathcal{L}_{\text{perceptron}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \left(\text{sign}(G_W(X^i)) - Y^i\right) G_W(X^i).$
- **Learning Rule:** $W \leftarrow W + \eta \left(Y^i - \text{sign}(G_W(X^i))\right) \frac{\partial G_W(X^i)}{\partial W},$
- **If Gw(X) is linear in W:** $E(W, Y, X) = -Y W^T \Phi(X)$

$$W \leftarrow W + \eta \left(Y^i - \text{sign}(W^T \Phi(X^i))\right) \Phi(X^i)$$

Examples of Loss Functions: Generalized Margin Losses

- **First, we need to define the Most Offending Incorrect Answer**

- **Most Offending Incorrect Answer: discrete case**

Definition 1 *Let Y be a discrete variable. Then for a training sample (X^i, Y^i), the **most offending incorrect answer** $\bar{Y}^i$ is the answer that has the lowest energy among all answers that are incorrect:*

$$\bar{Y}^i = \text{argmin}_{Y \in \mathcal{Y} and Y \neq Y^i} E(W, Y, X^i). \tag{8}$$

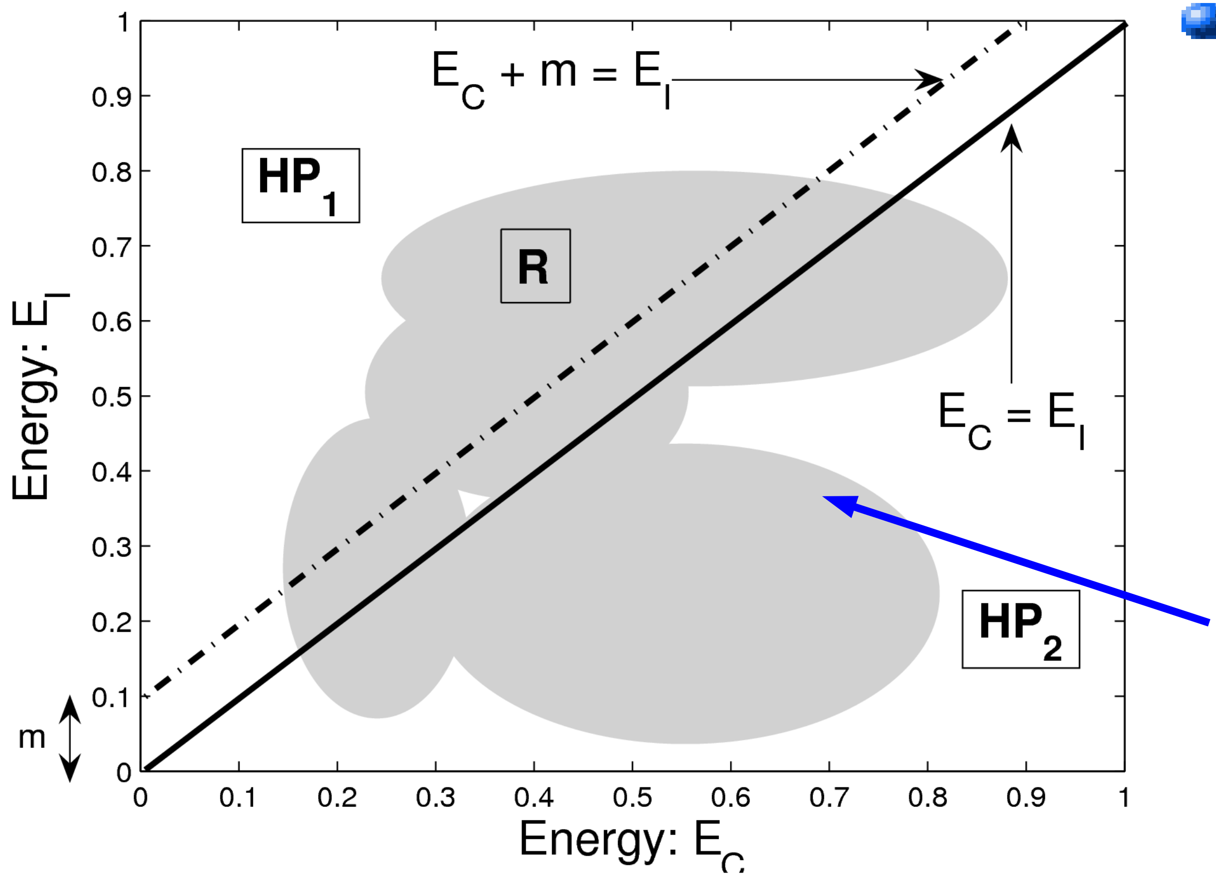
- **Most Offending Incorrect Answer: continuous case**

Definition 2 *Let Y be a continuous variable. Then for a training sample (X^i, Y^i), the **most offending incorrect answer** $\bar{Y}^i$ is the answer that has the lowest energy among all answers that are at least ϵ away from the correct answer:*

$$\bar{Y}^i = \text{argmin}_{Y \in \mathcal{Y}, \|Y - Y^i\| > \epsilon} E(W, Y, X^i). \tag{9}$$

Examples of Loss Functions: Generalized Margin Losses

$$L_{\text{margin}}(W, Y^i, X^i) = Q_m\left(E(W, Y^i, X^i), E(W, \bar{Y}^i, X^i)\right).$$



Generalized Margin Loss

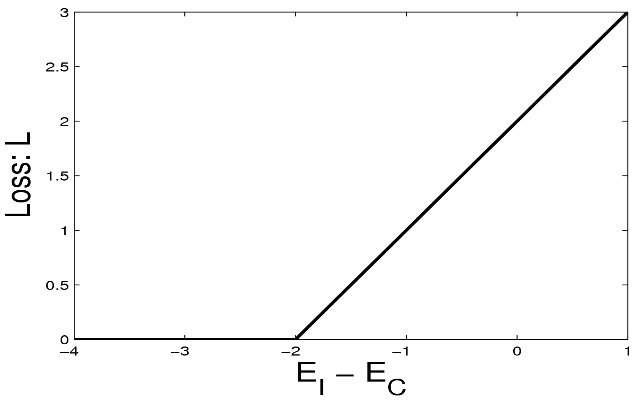
- Qm increases with the energy of the correct answer
- Qm decreases with the energy of the most offending incorrect answer
- whenever it is less than the energy of the correct answer plus a margin m.

Examples of Generalized Margin Losses

$$L_{\text{hinge}}(W, Y^i, X^i) = \max\left(0, m + E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right),$$

Hinge Loss

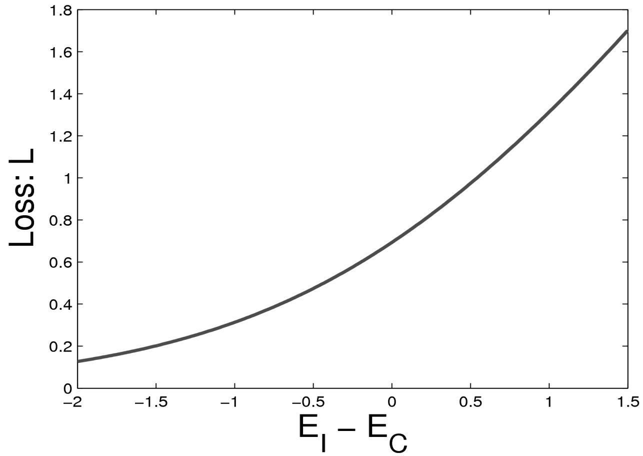
- [Altun et al. 2003], [Taskar et al. 2003]
- With the linearly-parameterized binary classifier architecture, we get linear SVMs



$$L_{\log}(W, Y^i, X^i) = \log\left(1 + e^{E(W,Y^i,X^i) - E(W,\bar{Y}^i,X^i)}\right).$$

Log Loss

- “soft hinge” loss
- With the linearly-parameterized binary classifier architecture, we get linear Logistic Regression

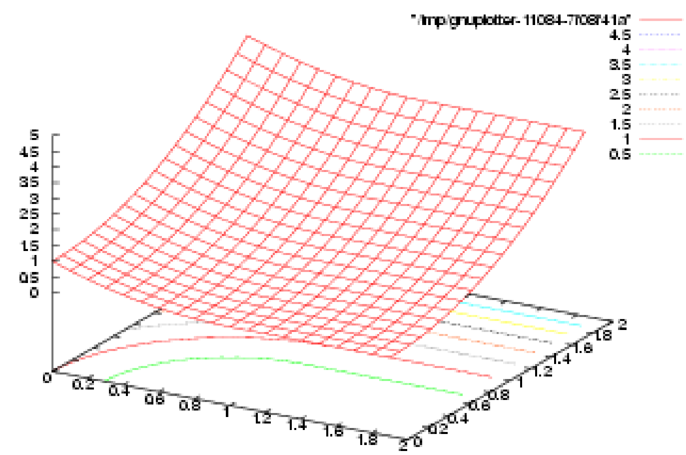


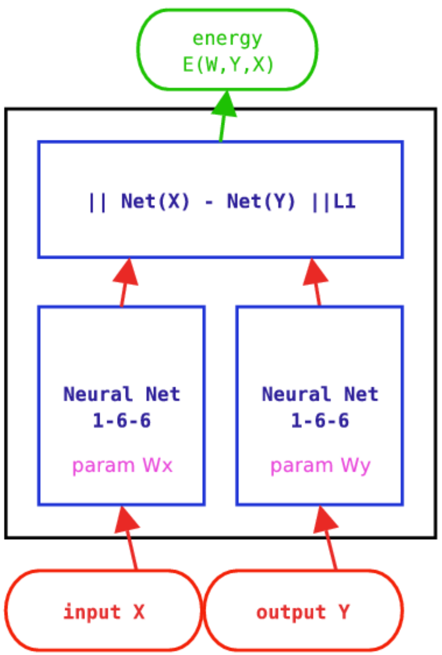
Examples of Margin Losses: Square-Square Loss

$$L_{\mathrm{sq-sq}}(W, Y^i, X^i) = E(W, Y^i, X^i)^2 + \left(\max(0, m - E(W, \bar{Y}^i, X^i))\right)^2.$$

Square-Square Loss

- [LeCun-Huang 2005]
- Appropriate for positive energy functions





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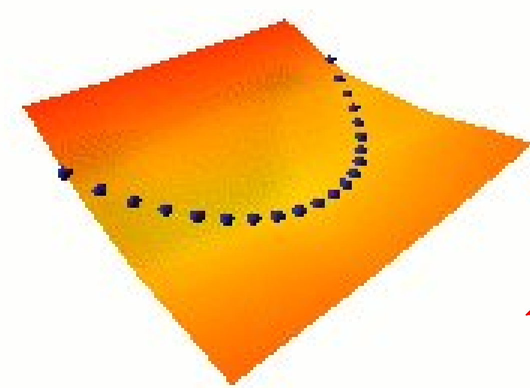
Learning Y = X^2



NO COLLAPSE!!!

Other Margin-Like Losses

- **LVQ2 Loss** [Kohonen, Oja], Driancourt-Bottou 1991]

$$L_{\text{lvq2}}(W, Y^i, X^i) = \min\left(1, \max\left(0, \frac{E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)}{\delta E(W, \bar{Y}^i, X^i)}\right)\right),$$

- **Minimum Classification Error Loss** [Juang, Chou, Lee 1997]

$$L_{\text{mce}}(W, Y^i, X^i) = \sigma\left(E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right),$$

$$\sigma(x) = (1 + e^{-x})^{-1}$$

- **Square-Exponential Loss** [Osadchy, Miller, LeCun 2004]

$$L_{\text{sq-exp}}(W, Y^i, X^i) = E(W, Y^i, X^i)^2 + \gamma e^{-E(W, \bar{Y}^i, X^i)},$$

Negative Log-Likelihood Loss

- **Conditional probability of the samples (assuming independence)**

$$P(Y^1,\dots,Y^P|X^1,\dots,X^P,W) = \prod_{i=1}^{P} P(Y^i|X^i,W).$$

$$-\log \prod_{i=1}^{P} P(Y^i|X^i,W) = \sum_{i=1}^{P} -\log P(Y^i|X^i,W).$$

- **Gibbs distribution:** $$P(Y|X^i,W) = \frac{e^{-\beta E(W,Y,X^i)}}{\int_{y\in\mathcal{Y}} e^{-\beta E(W,y,X^i)}}.$$

$$-\log \prod_{i=1}^{P} P(Y^i|X^i,W) = \sum_{i=1}^{P} \beta E(W,Y^i,X^i) + \log \int_{y\in\mathcal{Y}} e^{-\beta E(W,y,X^i)}.$$

- **We get the NLL loss by dividing by P and Beta:**

$$\mathcal{L}_{\text{nll}}(W,\mathcal{S}) = \frac{1}{P}\sum_{i=1}^{P}\left(E(W,Y^i,X^i) + \frac{1}{\beta}\log \int_{y\in\mathcal{Y}} e^{-\beta E(W,y,X^i)}\right).$$

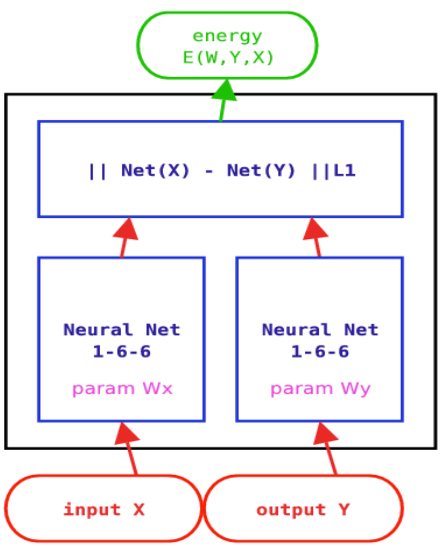
- **Reduces to the perceptron loss when Beta->infinity**

Negative Log-Likelihood Loss

- **Pushes down on the energy of the correct answer**
- **Pulls up on the energies of all answers in proportion to their probability**

$$\mathcal{L}_{\rm nll}(W,\mathcal{S}) = \frac{1}{P}\sum_{i=1}^{P}\left(E(W,Y^i,X^i) + \frac{1}{\beta}\log\int_{y\in\mathcal{Y}} e^{-\beta E(W,y,X^i)}\right).$$

$$\frac{\partial L_{\rm nll}(W,Y^i,X^i)}{\partial W} = \frac{\partial E(W,Y^i,X^i)}{\partial W} - \int_{Y\in\mathcal{Y}} \frac{\partial E(W,Y,X^i)}{\partial W} P(Y|X^i,W),$$



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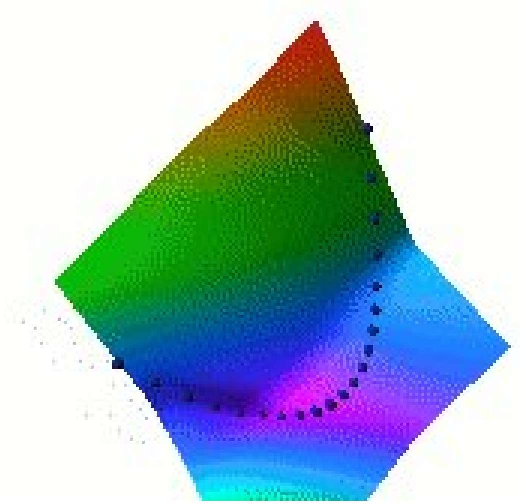


Negative Log-Likelihood Loss: Binary Classification

- **Binary Classifier Architecture:**

$$\mathcal{L}_{\text{nll}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \left[-Y^i G_W(X^i) + \log \left(e^{Y^i G_W(X^i)} + e^{-Y^i G_W(X^i)} \right) \right].$$

$$\mathcal{L}_{\text{nll}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \log \left(1 + e^{-2Y^i G_W(X^i)} \right),$$

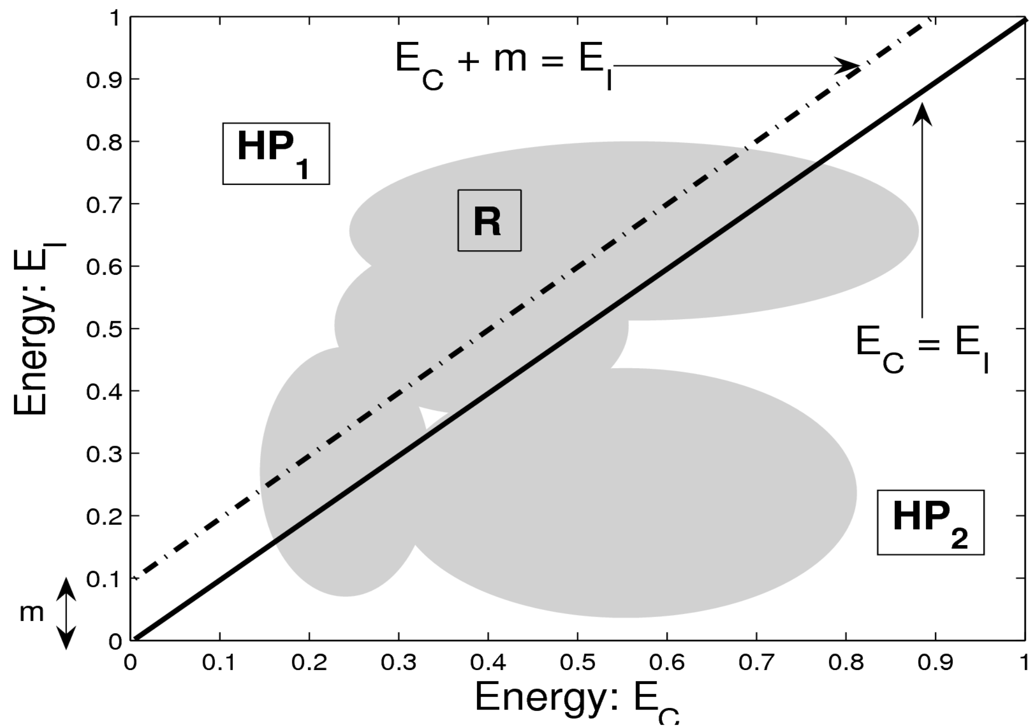
- **Linear Binary Classifier Architecture:**

$$\mathcal{L}_{\text{nll}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \log \left(1 + e^{-2Y^i W^T \Phi(X^i)} \right).$$

- **Learning Rule: logistic regression**

What Makes a "Good" Loss Function

- **Good loss functions make the machine produce the correct answer**
  - Avoid collapses and flat energy surfaces



Sufficient Condition on the Loss

Let (X^i, Y^i) be the i^{th} training example and m be a positive margin. Minimizing the loss function L will cause the machine to satisfy $E(W, Y^i, X^i) < E(W, Y, X^i) - m$ for all $Y \neq Y^i$, if there exists at least one point (e_1, e_2) with $e_1 + m < e_2$ such that for all points (e'_1, e'_2) with $e'_1 + m \geq e'_2$, we have

$$Q_{[E_y]}(e_1, e_2) < Q_{[E_y]}(e'_1, e'_2),$$

where $Q_{[E_y]}$ is given by

$$L(W, Y^i, X^i) = Q_{[E_y]}(E(W, Y^i, X^i), E(W, \bar{Y}^i, X^i)).$$

What Make a "Good" Loss Function

Good and bad loss functions

Loss (equation #)	Formula	Margin
energy loss	$E(W, Y^i, X^i)$	none
perceptron	$E(W, Y^i, X^i) - \min_{Y \in \mathcal{Y}} E(W, Y, X^i)$	0
hinge	$\max\left(0, m + E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right)$	m
log	$\log\left(1 + e^{E(W,Y^i,X^i) - E(W,\bar{Y}^i,X^i)}\right)$	> 0
LVQ2	$\min\left(M, \max(0, E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right)$	0
MCE	$\left(1 + e^{-\left(E(W,Y^i,X^i) - E(W,\bar{Y}^i,X^i)\right)}\right)^{-1}$	> 0
square-square	$E(W, Y^i, X^i)^2 - \left(\max(0, m - E(W, \bar{Y}^i, X^i))\right)^2$	m
square-exp	$E(W, Y^i, X^i)^2 + \beta e^{-E(W,\bar{Y}^i,X^i)}$	> 0
NLL/MMI	$E(W, Y^i, X^i) + \frac{1}{\beta} \log \int_{y \in \mathcal{Y}} e^{-\beta E(W,y,X^i)}$	> 0
MEE	$1 - e^{-\beta E(W,Y^i,X^i)} / \int_{y \in \mathcal{Y}} e^{-\beta E(W,y,X^i)}$	> 0

Advantages/Disadvantages of various losses

- **Loss functions differ in how they pick the point(s) whose energy is pulled up, and how much they pull them up**
- **Losses with a log partition function in the contrastive term pull up all the bad answers simultaneously.**
  - This may be good if the gradient of the contrastive term can be computed efficiently
  - This may be bad if it cannot, in which case we might as well use a loss with a single point in the contrastive term
- **Variational methods pull up many points, but not as many as with the full log partition function.**
- **Efficiency of a loss/architecture: how many energies are pulled up for a given amount of computation?**
  - The theory for this is does not exist. It needs to be developed

Latent Variable Models

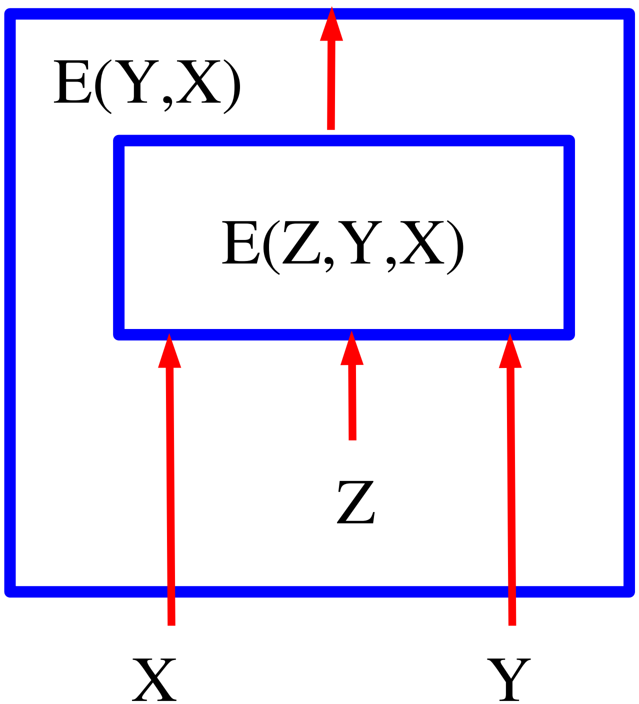
- **The energy includes "hidden" variables Z whose value is never given to us**
  - We can minimize the energy over those latent variables
  - We can also "marginalize" the energy over the latent variables

Minimization over latent variables:

$$E(Y, X) = \min_{Z \in \mathcal{Z}} E(Z, Y, X).$$

Marginalization over latent variables:

$$E(X, Y) = -\frac{1}{\beta} \log \int_{z \in \mathcal{Z}} e^{-\beta E(z, Y, X)}$$



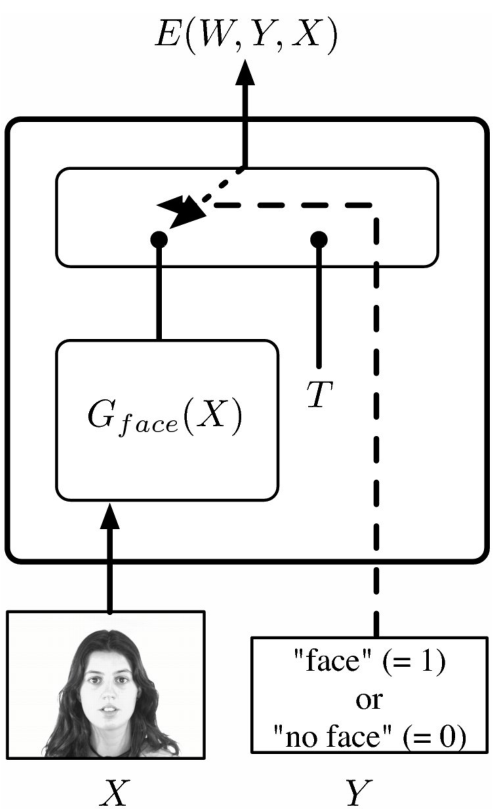
Estimation this integral may require some approximations (sampling, variational methods,....)

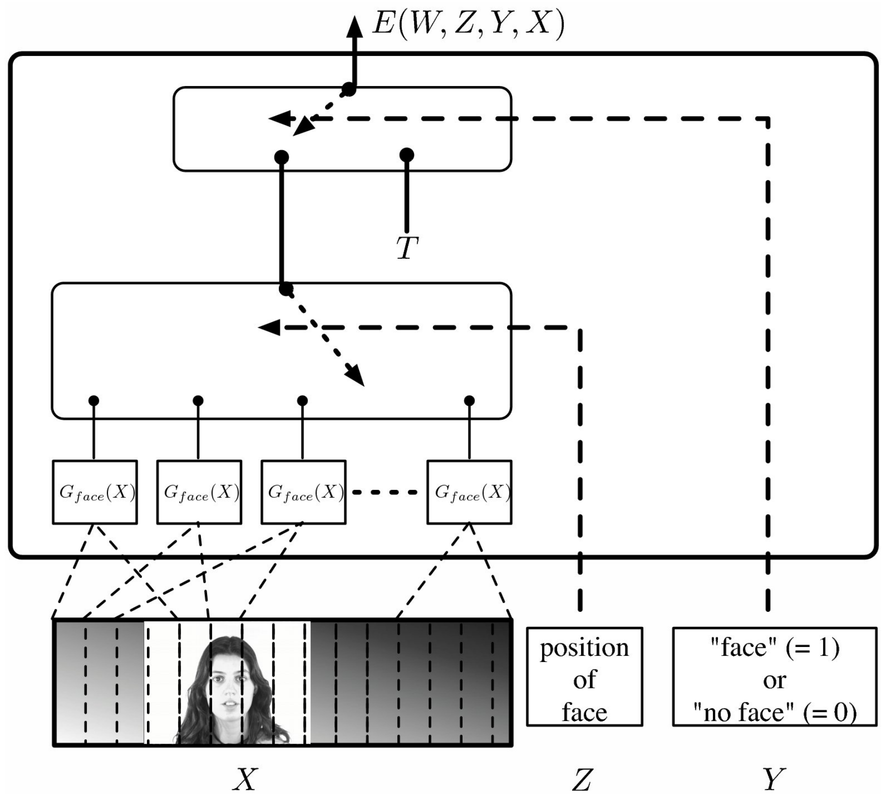
Latent Variable Models

- **The energy includes "hidden" variables Z whose value is never given to us**

$$E(Y, X) = \min_{Z \in \mathcal{Z}} E(Z, Y, X).$$

$$Y^* = \text{argmin}_{Y \in \mathcal{Y}, Z \in \mathcal{Z}} E(Z, Y, X).$$





What can the latent variables represent?

- **Variables that would make the task easier if they were known:**
  - **Face recognition**: the gender of the person, the orientation of the face.
  - **Object recognition**: the pose parameters of the object (location, orientation, scale), the lighting conditions.
  - **Parts of Speech Tagging**: the segmentation of the sentence into syntactic units, the parse tree.
  - **Speech Recognition**: the segmentation of the sentence into phonemes or phones.
  - **Handwriting Recognition**: the segmentation of the line into characters.
- **In general, we will search for the value of the latent variable that allows us to get an answer (Y) of smallest energy.**

Probabilistic Latent Variable Models

- **Marginalizing over latent variables instead of minimizing.**

$$P(Z,Y|X) = \frac{e^{-\beta E(Z,Y,X)}}{\int_{y\in\mathcal{Y},\ z\in\mathcal{Z}} e^{-\beta E(y,z,X)}}.$$

$$P(Y|X) = \frac{\int_{z\in\mathcal{Z}} e^{-\beta E(Z,Y,X)}}{\int_{y\in\mathcal{Y},\ z\in\mathcal{Z}} e^{-\beta E(y,z,X)}}.$$

- **Equivalent to traditional energy-based inference with a redefined energy function:**

$$Y^* = \text{argmin}_{Y\in\mathcal{Y}} - \frac{1}{\beta}\log\int_{z\in\mathcal{Z}} e^{-\beta E(z,Y,X)}.$$

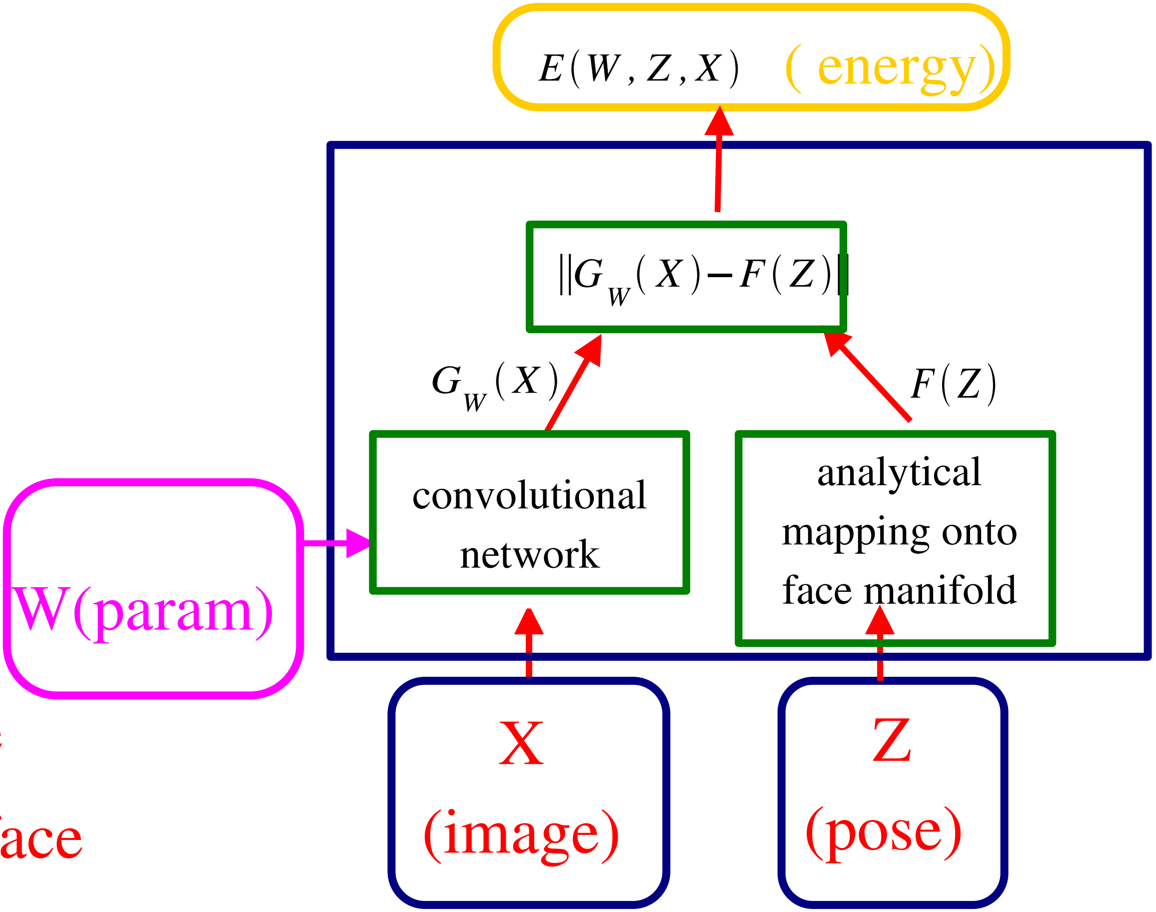
- **Reduces to minimization when Beta->infinity**

Face Detection and Pose Estimation with a Convolutional EBM

- **Training:** 52,850, 32x32 grey-level images of faces, 52,850 selected non-faces.
- Each training image was used 5 times with random variation in scale, in-plane rotation, brightness and contrast.
- **2nd phase:** half of the initial negative set was replaced by false positives of the initial version of the detector .

$$E^*(W,X) = \min_Z ||G_W(X) - F(Z)||$$

$$Z^* = \mathrm{argmin}_Z ||G_W(X) - F(Z)||$$

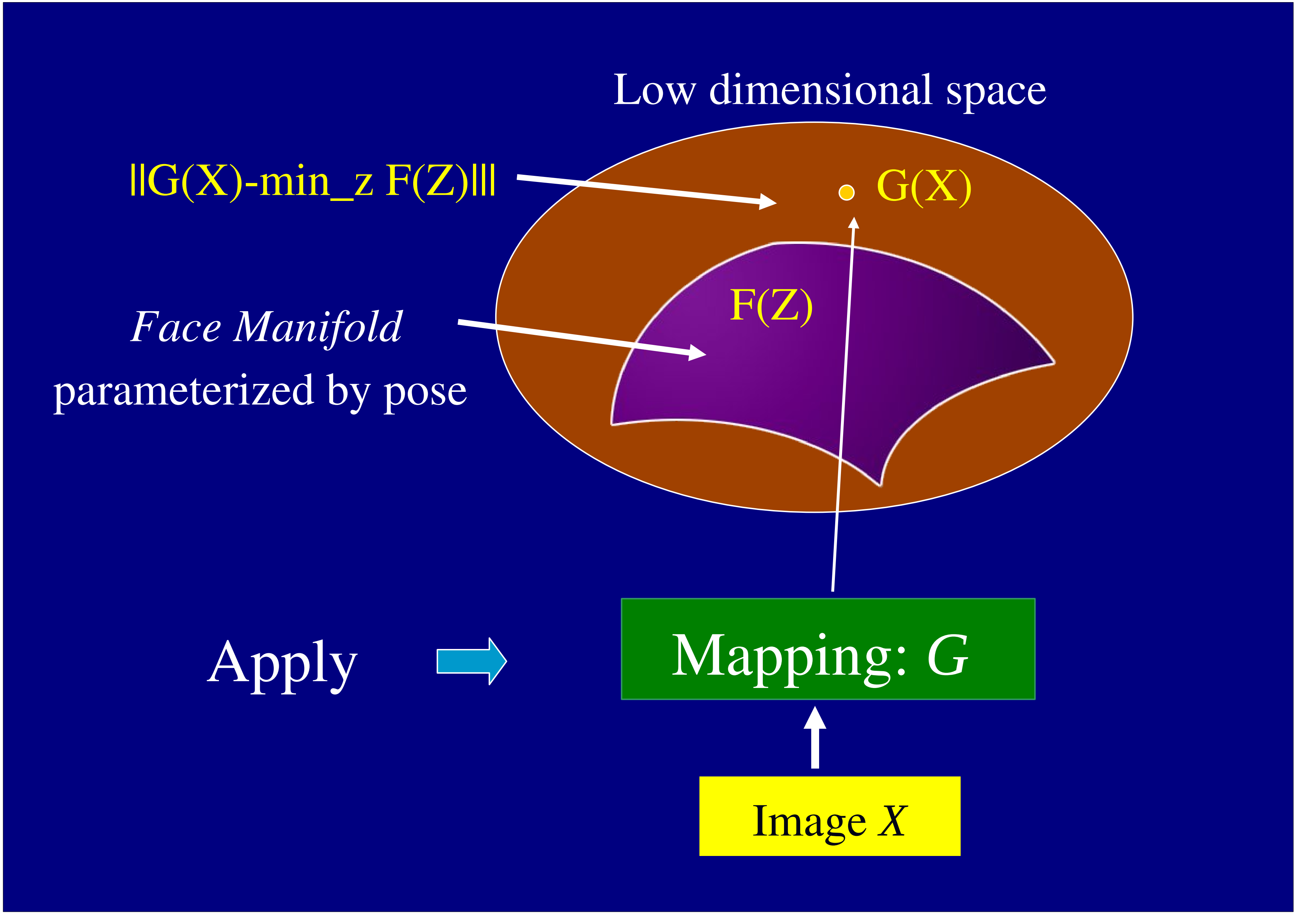


Small E*(W,X): face

Large E*(W,X): no face

[Osadchy, Miller, LeCun, NIPS 2004]

Face Manifold



Probabilistic Approach: Density model of joint P(face,pose)

Probability that image

X is a face with pose Z

$$P(X,Z) = \frac{\exp(-E(W,Z,X))}{\int_{X,Z\in\text{images,poses}} \exp(-E(W,Z,X))}$$

Given a training set of faces annotated with pose, find the W that maximizes the likelihood of the data under the model:

$$P(\text{faces}+\text{pose}) = \prod_{X,Z\in\text{faces}+\text{pose}} \frac{\exp(-E(W,Z,X))}{\int_{X,Z\in\text{images,poses}} \exp(-E(W,Z,X))}$$

Equivalently, minimize the negative log likelihood:

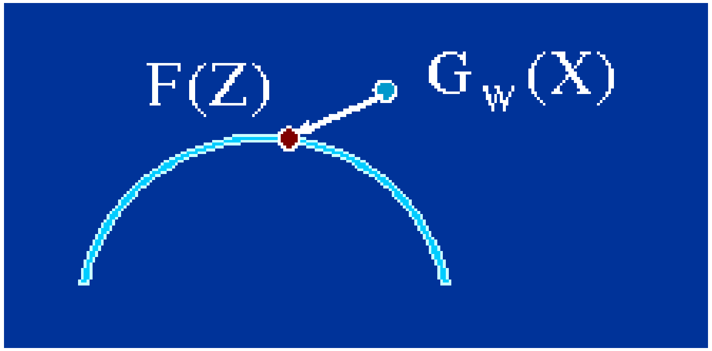
$$\mathcal{L}(W,\text{faces}+\text{pose}) = \sum_{X,Z\in\text{faces}+\text{pose}} E(W,Z,X) + \log\left[\int_{X,Z\in\text{images,poses}} \exp(-E(W,Z,X))\right]$$

COMPLICATED

Energy-Based Contrastive Loss Function

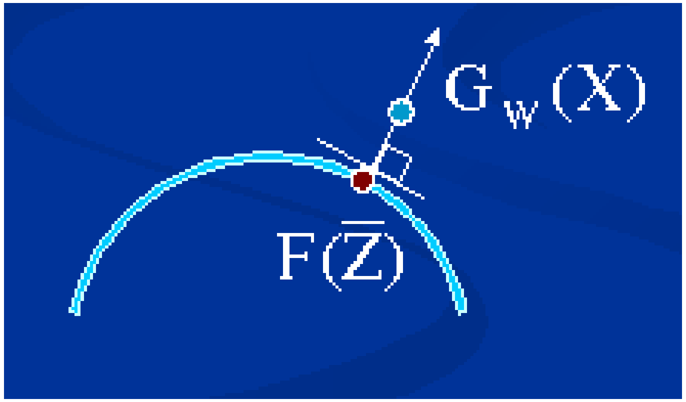
$$\mathcal{L}(W) = \frac{1}{|\mathrm{f}+\mathrm{p}|} \sum_{X,Z \in \mathrm{faces+pose}} \left[L^{+}\left(E(W,Z,X)\right)\right] + L^{-}\left(\min_{X,Z \in \mathrm{bckgnd,poses}} E(W,Z,X)\right)$$

$$L^{+}\left(E(W,Z,X)\right) = E(W,Z,X)^2 = ||G_W(X) - F(Z)||^2$$



Attract the network output Gw(X) to the location of the desired pose F(Z) on the manifold

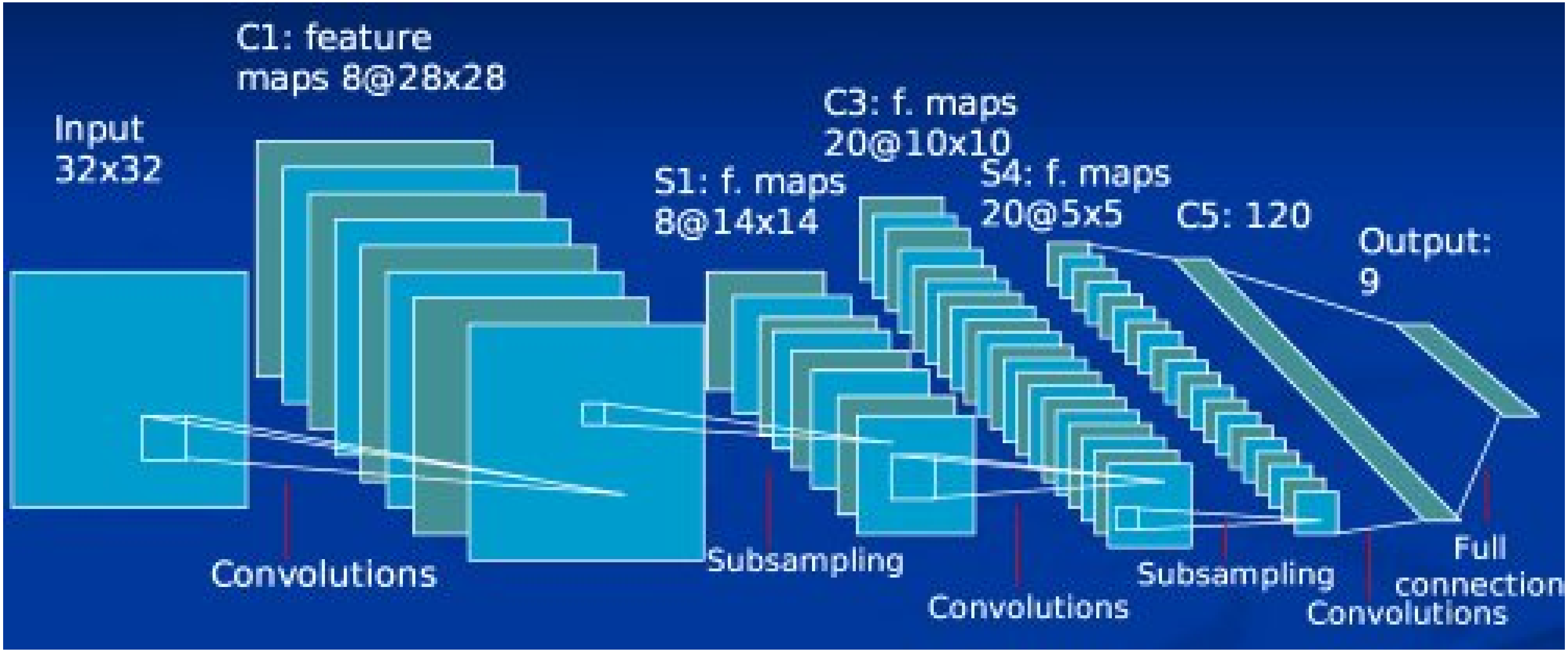
$$L^{-}\left(\min_{X,Z \in \mathrm{bckgnd,poses}} E(W,Z,X)\right) = K \exp\left(-\min_{X,Z \in \mathrm{bckgnd,poses}} ||G_W(X) - F(Z)||\right)$$



Repel the network output Gw(X) away from the face/pose manifold

Convolutional Network Architecture

[LeCun et al. 1988, 1989, 1998, 2005]



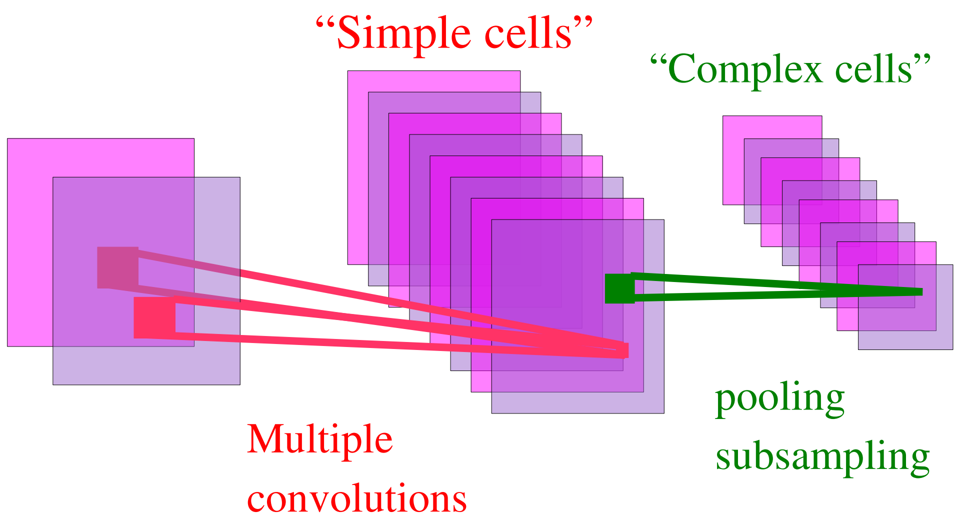
Hierarchy of local filters (convolution kernels),

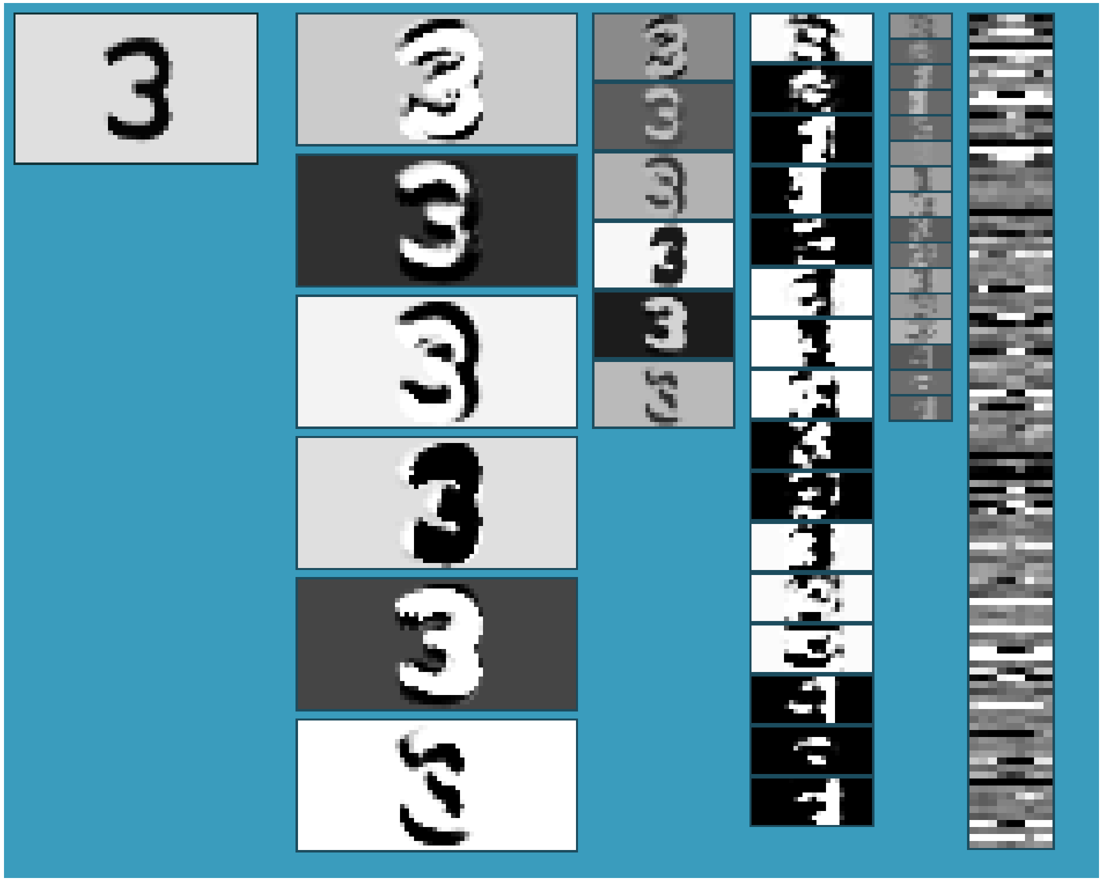
sigmoid pointwise non-linearities, and spatial subsampling

All the filter coefficients are learned with gradient descent (back-prop)

Alternated Convolutions and Pooling/Subsampling

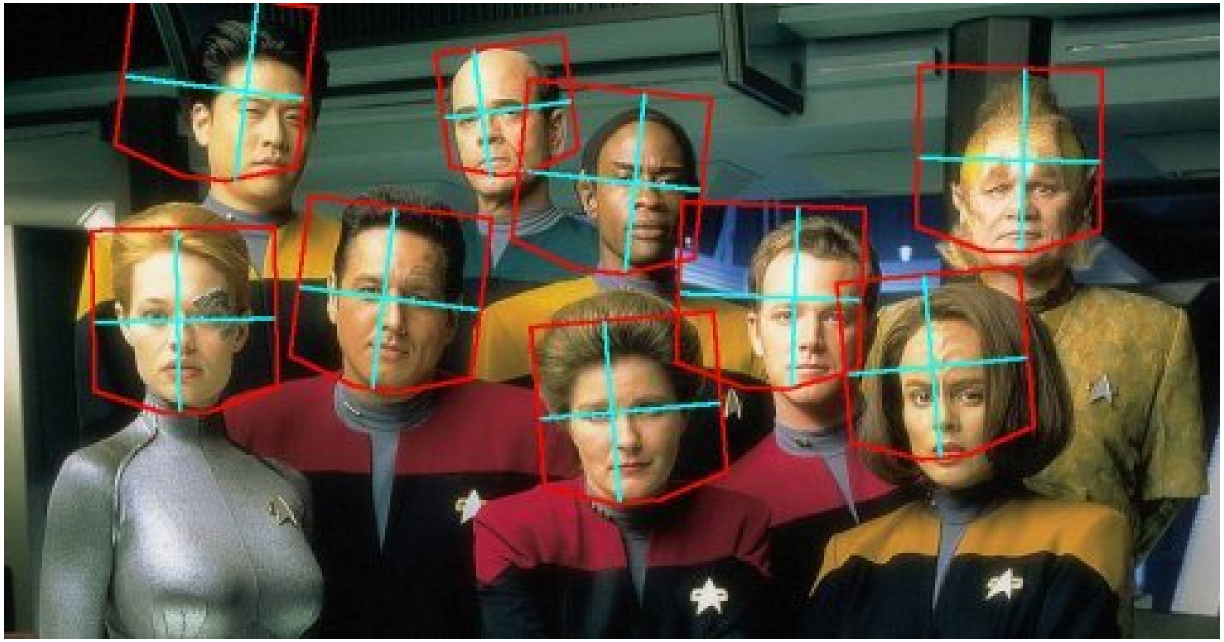
- Local features are extracted everywhere.
- pooling/subsampling layer builds robustness to variations in feature locations.
- Long history in neuroscience and computer vision:
  - Hubel/Wiesel 1962,
  - Fukushima 1971-82,
  - LeCun 1988-06
  - Poggio, Riesenhuber, Serre 02-06
  - Ullman 2002-06
  - Triggs, Lowe,....

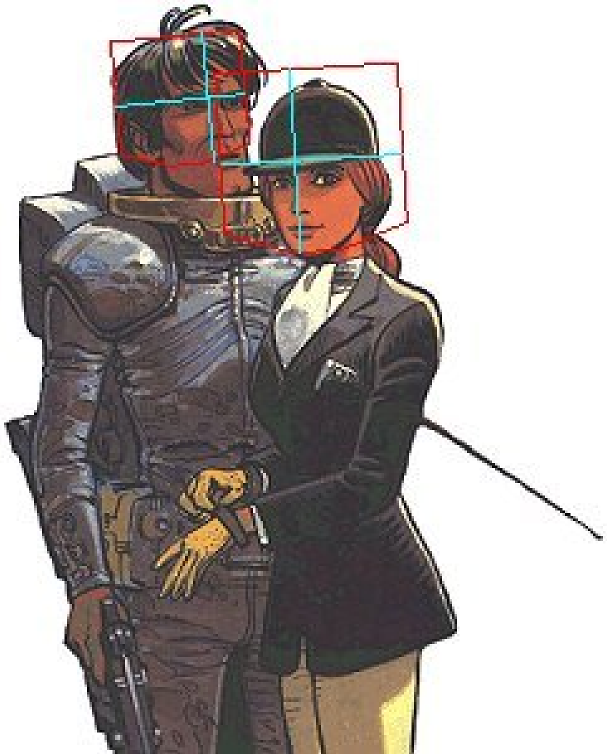




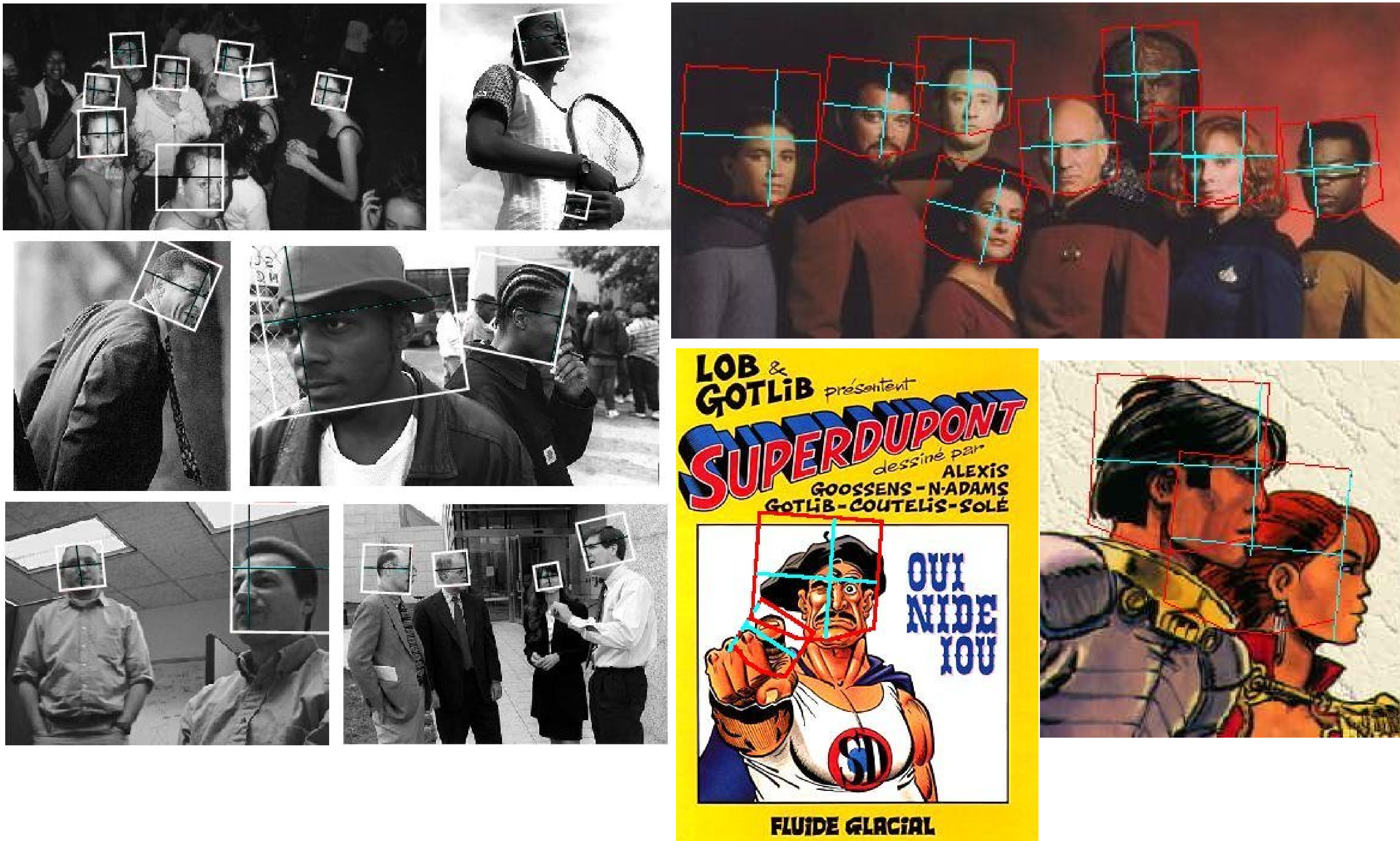
Face Detection: Results

Data Set->	TILTED		PROFILE		MIT+CMU	
False positives per image->	4.42	26.9	0.47	3.36	0.5	1.28
Our Detector	90%	97%	67%	83%	83%	88%
Jones & Viola (tilted)	90%	95%	x		x	
Jones & Viola (profile)	x		70%	83%	x	

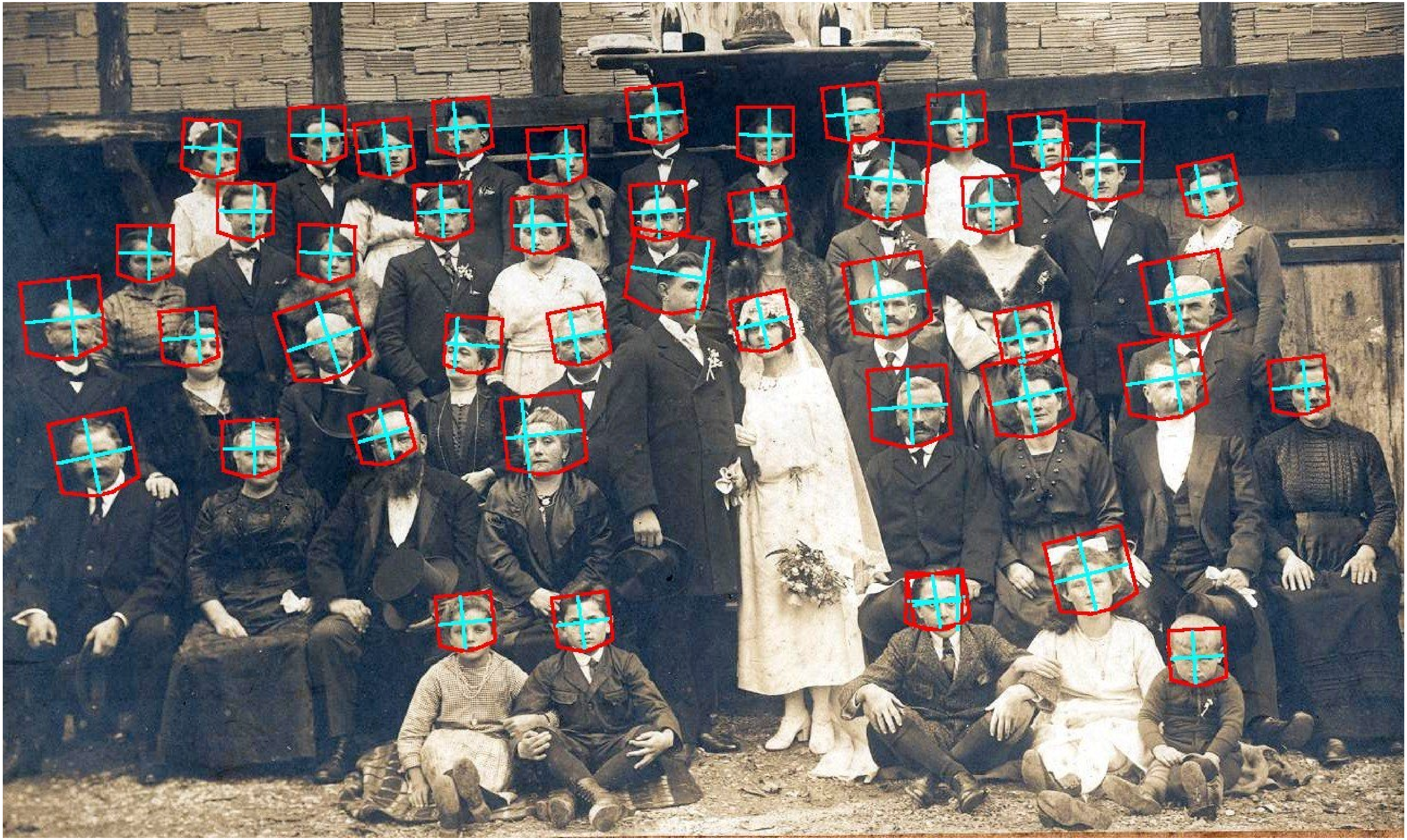




Face Detection and Pose Estimation: Results



Face Detection with a Convolutional Net

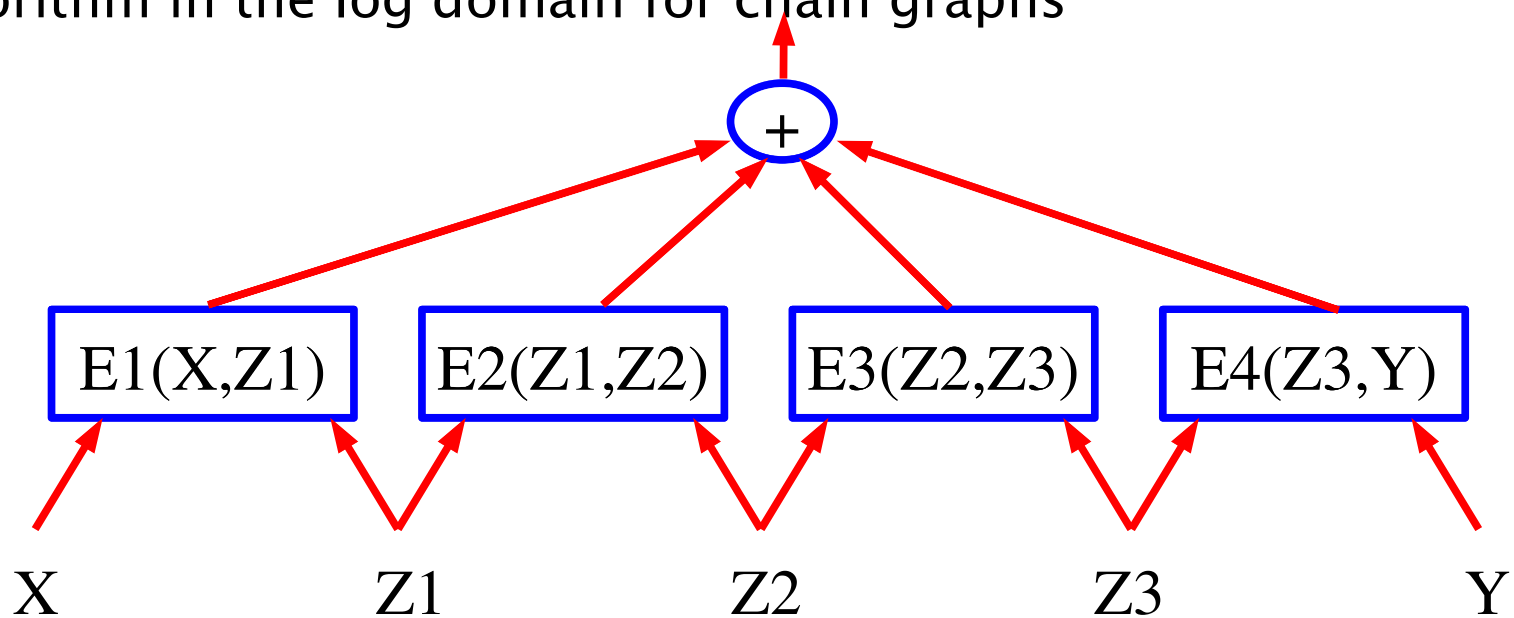


Efficient Inference: Energy-Based Factor Graphs

- **Graphical models** have given us efficient inference algorithms, such as **belief propagation** and its numerous variations.
- Traditionally, graphical models are viewed as probabilistic models
- At first glance, is seems difficult to dissociate graphical models (Bayesian networks) from the probabilistic view.
- **Energy-Based Factor Graphs** are an extension of graphical models to non-probabilistic settings.
- An EBFG is an energy function that can be written as a **sum of "factor" functions** that take different subsets of variables as inputs.
- Basically, most algorithms for probabilistic factor graphs (such as belief prop) have a counterpart for EBFG:
  - Operations are performed in the log domain
  - The normalization steps are left out.

Energy-Based Factor Graphs

- **When the energy is a sum of partial energy functions (or when the probability is a product of factors):**
  - An EBM can be seen as an unnormalized factor graph in the log domain
  - Our favorite efficient inference algorithms can be used for inference (without the normalization step).
  - Min-sum algorithm (instead of max-product), Viterbi for chain graphs
  - (Log/sum/exp)-sum algorithm (instead of sum-product), Forward algorithm in the log domain for chain graphs

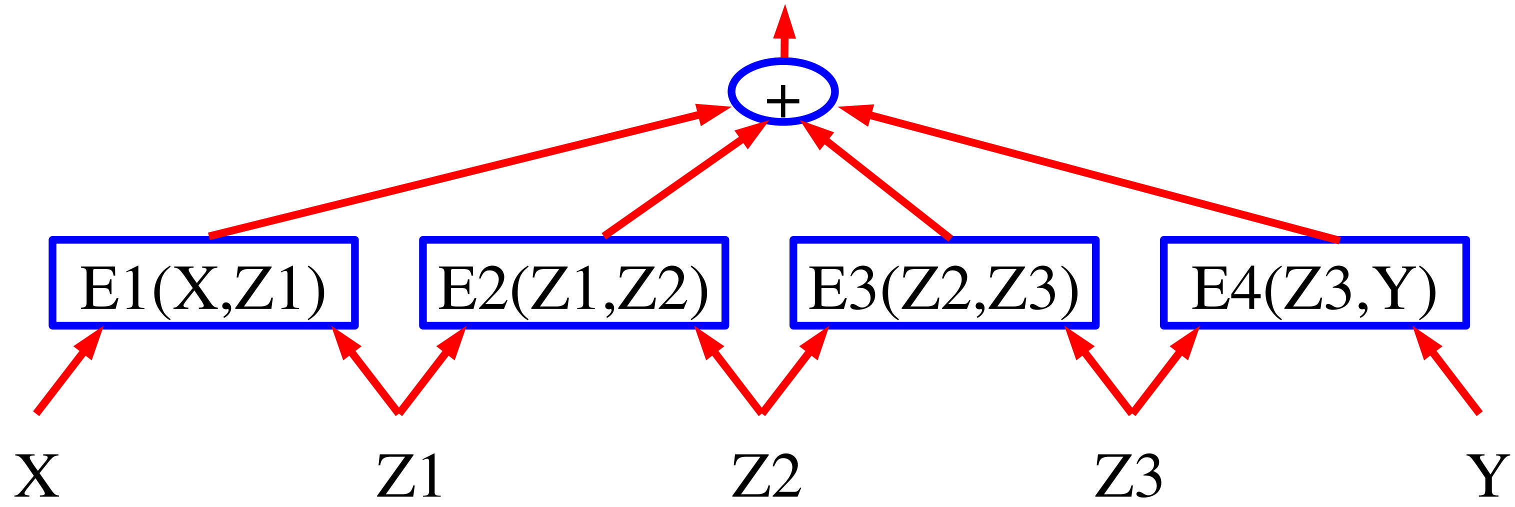


EBFG for Structured Outputs: Sequences, Graphs, Images

- **Structured outputs**
  - When Y is a complex object with components that must satisfy certain constraints.
- **Typically, structured outputs are sequences of symbols that must satisfy "grammatical" constraints**
  - spoken/handwritten word recognition
  - spoken/written sentence recognition
  - DNA sequence analysis
  - Parts of Speech tagging
  - Automatic Machine Translation
- **In General, structured outputs are collections of variables in which subsets of variables must satisfy constraints**
  - Pixels in an image for image restoration
  - Labels of regions for image segmentations
- **We represent the constraints using an Energy-Based Factor Graph.**

Energy-Based Factor Graphs: Three Inference Problems

- X: input, Y: output, Z: latent variables
- Minimization over Y and Z
  - $$E(Y,X) = \min_{Z \in \mathcal{Z}} E(Z,Y,X). \qquad Y^* = \mathrm{argmin}_{Y \in \mathcal{Y}} E(Y,X).$$
- Min over Y, marginalization over Z
  - $$E(X,Y) = -\frac{1}{\beta} \log \int_{z \in \mathcal{Z}} e^{-\beta E(z,Y,X)} \qquad Y^* = \mathrm{argmin}_{Y \in \mathcal{Y}} E(Y,X).$$
- Marginal Distribution over Y
  - $$P(Y|X) = \frac{e^{-\beta E(Y,X)}}{\int_{y \in \mathcal{Y}} e^{-\beta E(y,X)}},$$

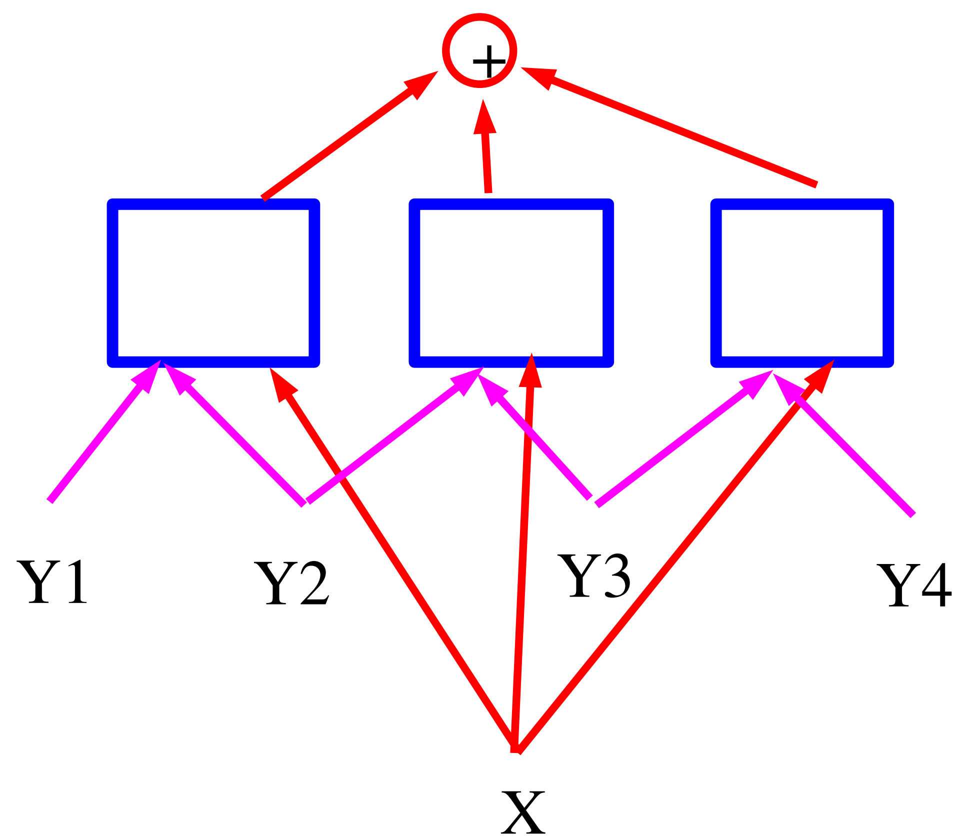


Energy-Based Factor Graphs: simple graphs

Sequence Labeling

- Output is a sequence Y1,Y2,Y3,Y4......
- NLP parsing, MT, speech/handwriting recognition, biological sequence analysis
- The factors ensure grammatical consistency
- They give low energy to consistent sub-sequences of output symbols
- The graph is generally simple (chain or tree)/
- Inference is easy (dynamic programming)

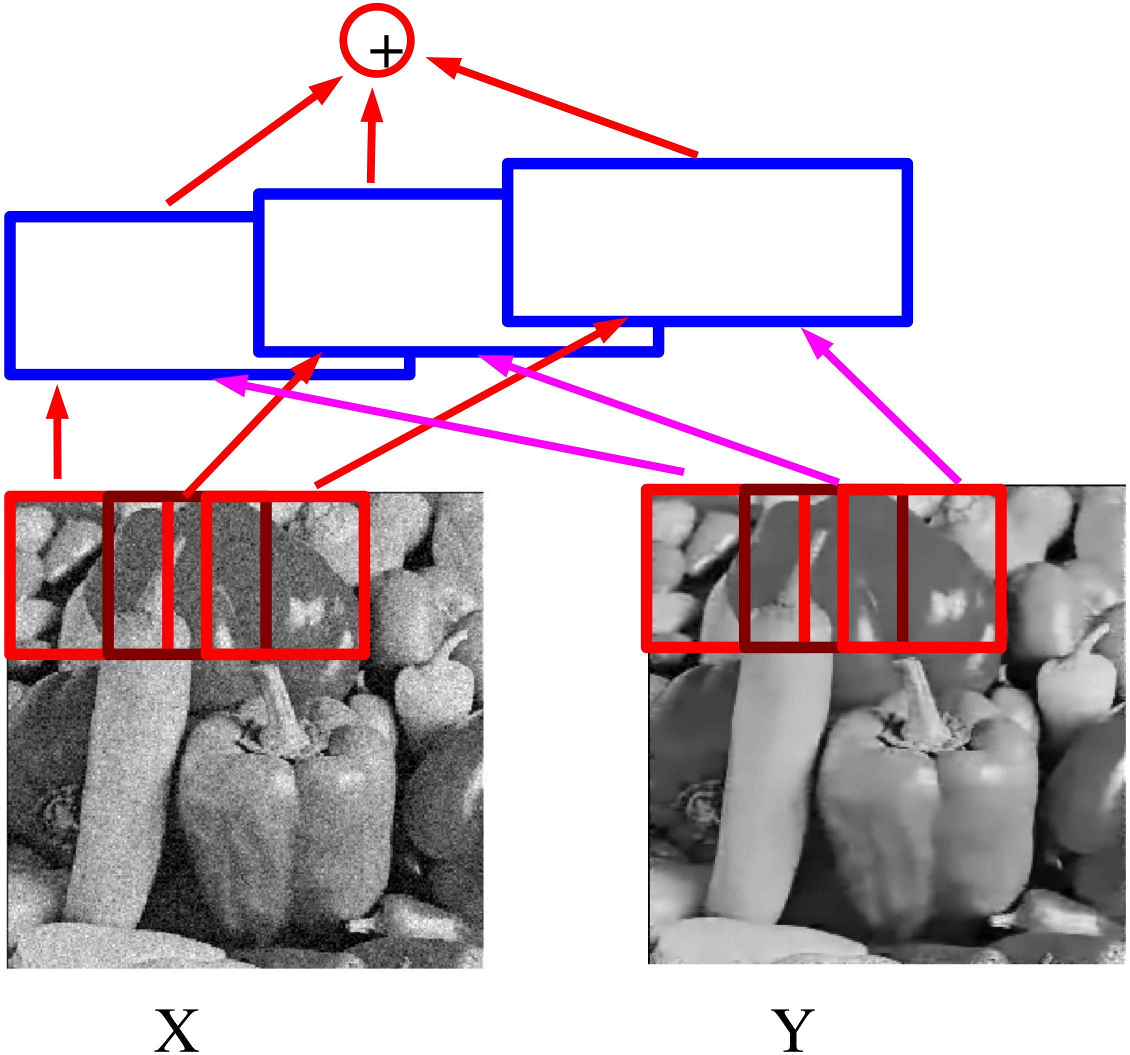
$$Y^* = \operatorname{argmin}_{Y \in \mathcal{Y}, Z \in \mathcal{Z}} E(Z, Y, X).$$



Energy-Based Factor Graphs: complex/loopy graphs

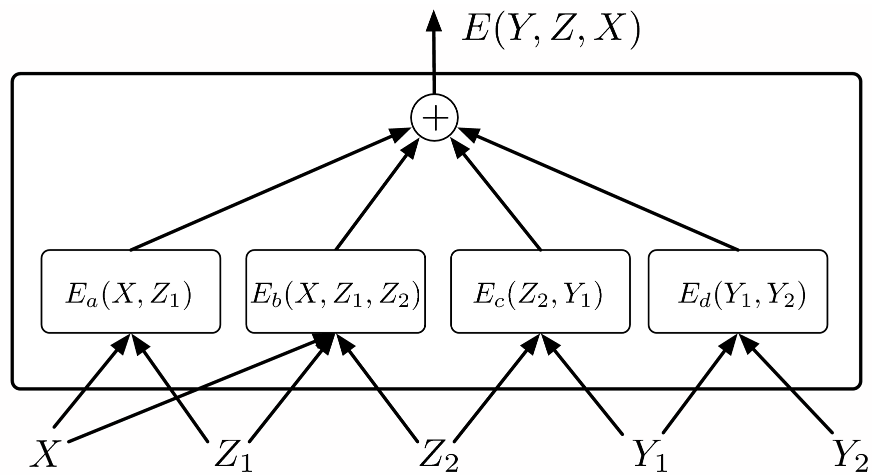
- **Image restoration**
  - The factors ensure local consistency on small overlapping patches
  - They give low energy to “clean” patches, given the noisy versions
  - The graph is loopy when the patches overlap.
  - Inference is difficult, particularly when the patches are large,and when the number of greyscale values is large

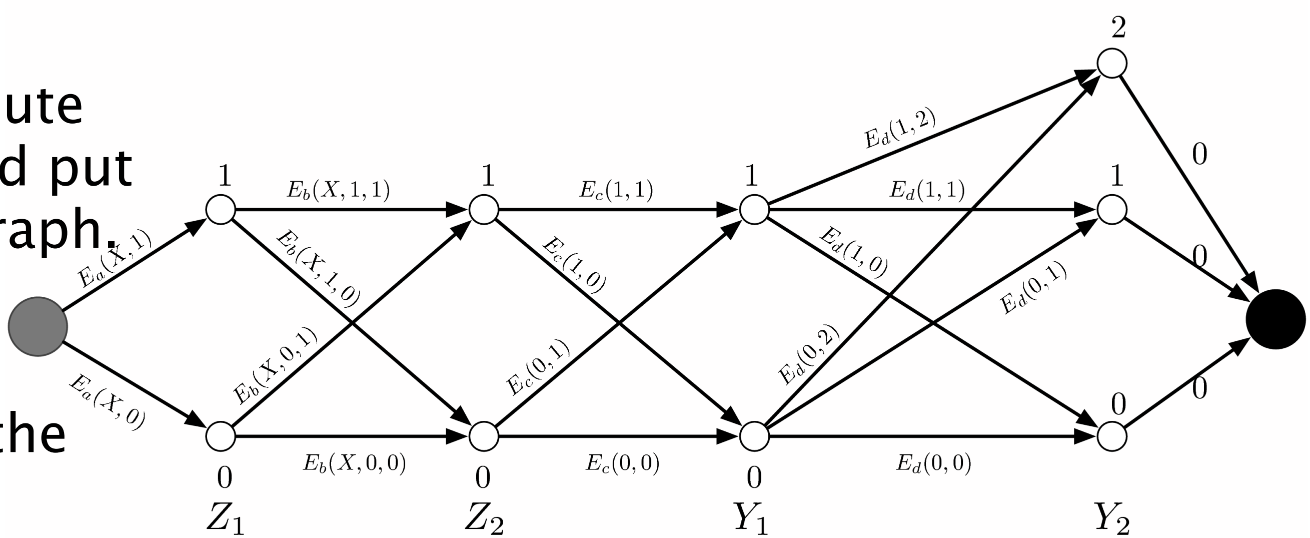
$$Y^* = \operatorname{argmin}_{Y \in \mathcal{Y}} E(Y, X).$$



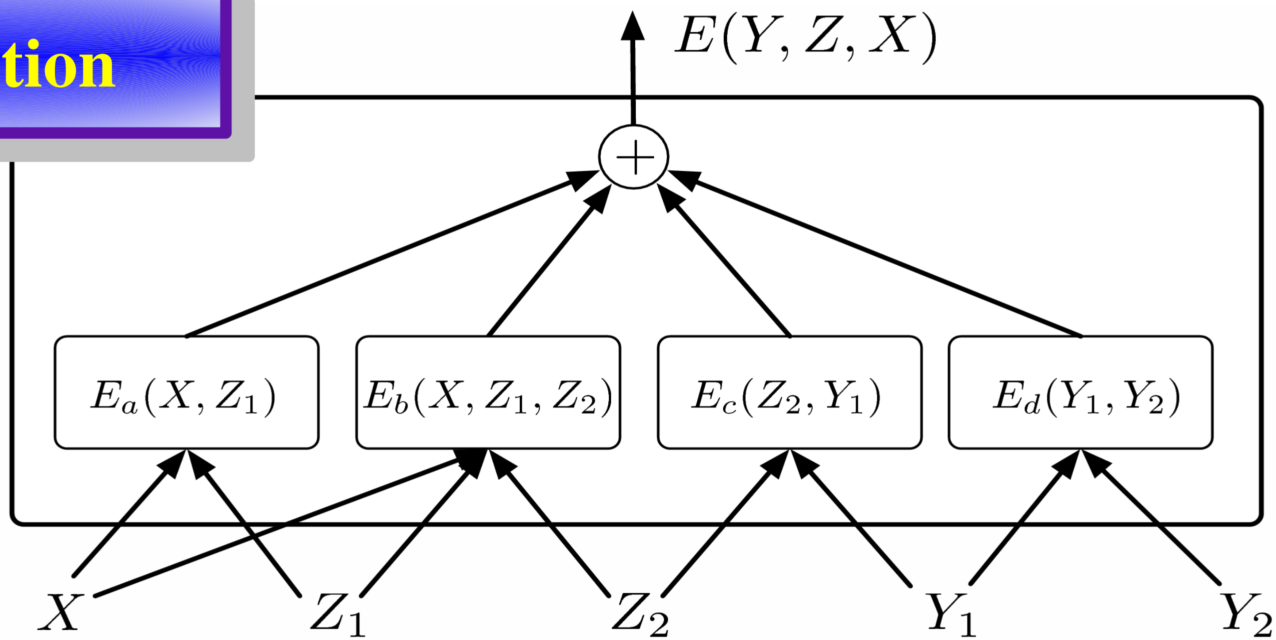
Efficient Inference in simple EBFG

- **The energy is a sum of "factor" functions, the graph is a chain**
- **Example:**
  - Z1, Z2, Y1 are binary
  - Z2 is ternary
  - A naïve exhaustive inference would require 2x2x2x3 energy evaluations (= 96 factor evaluations)
  - BUT: Ea only has 2 possible input configurations, Eb and Ec have 4, and Ed 6.
  - Hence, we can precompute the 16 factor values, and put them on the arcs in a graph.
  - A path in the graph is a config of variable
  - The cost of the path is the energy of the config



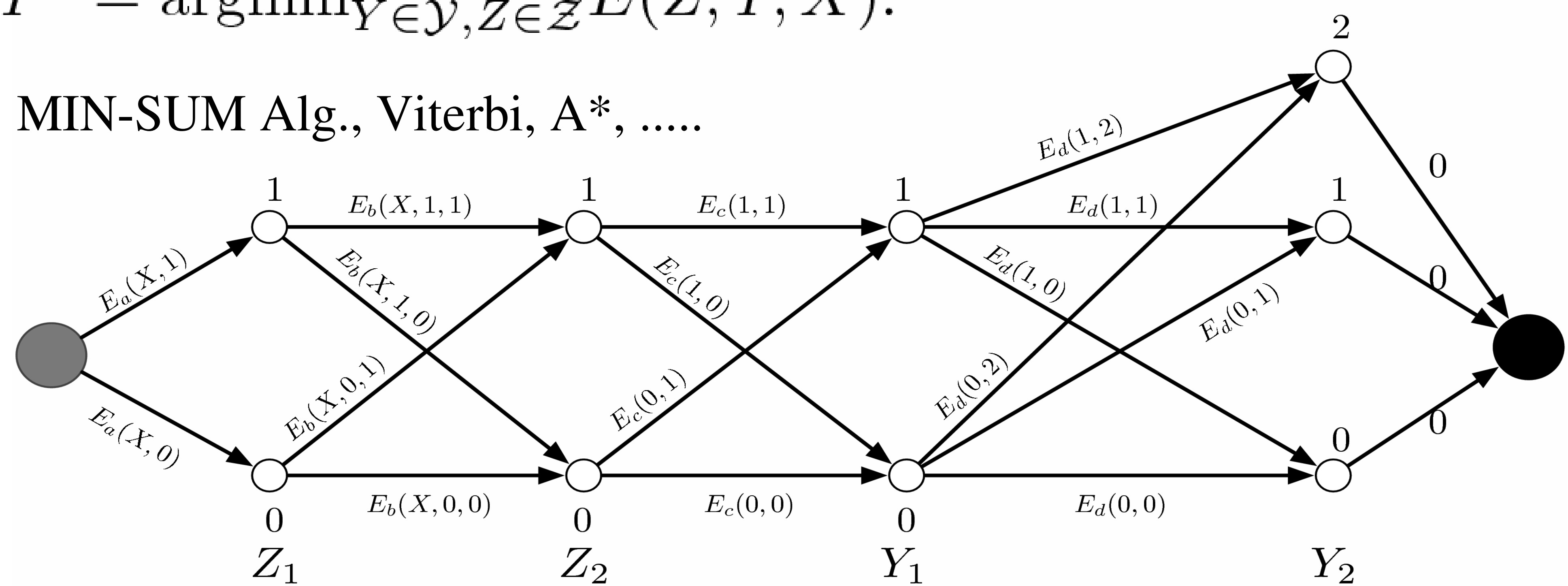


Minimization



$$Y^* = \operatorname{argmin}_{Y\in\mathcal{Y},Z\in\mathcal{Z}} E(Z,Y,X).$$

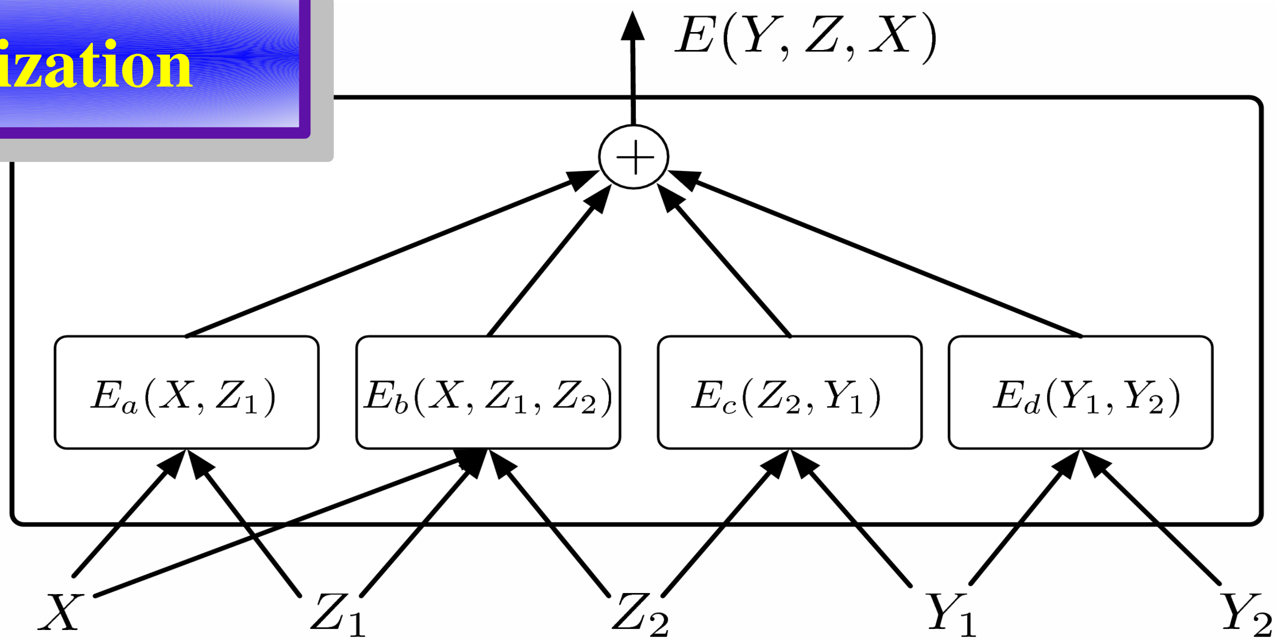
MIN-SUM Alg., Viterbi, A*, ...



Energy-Based Belief Prop: Minimization over Latent Variables

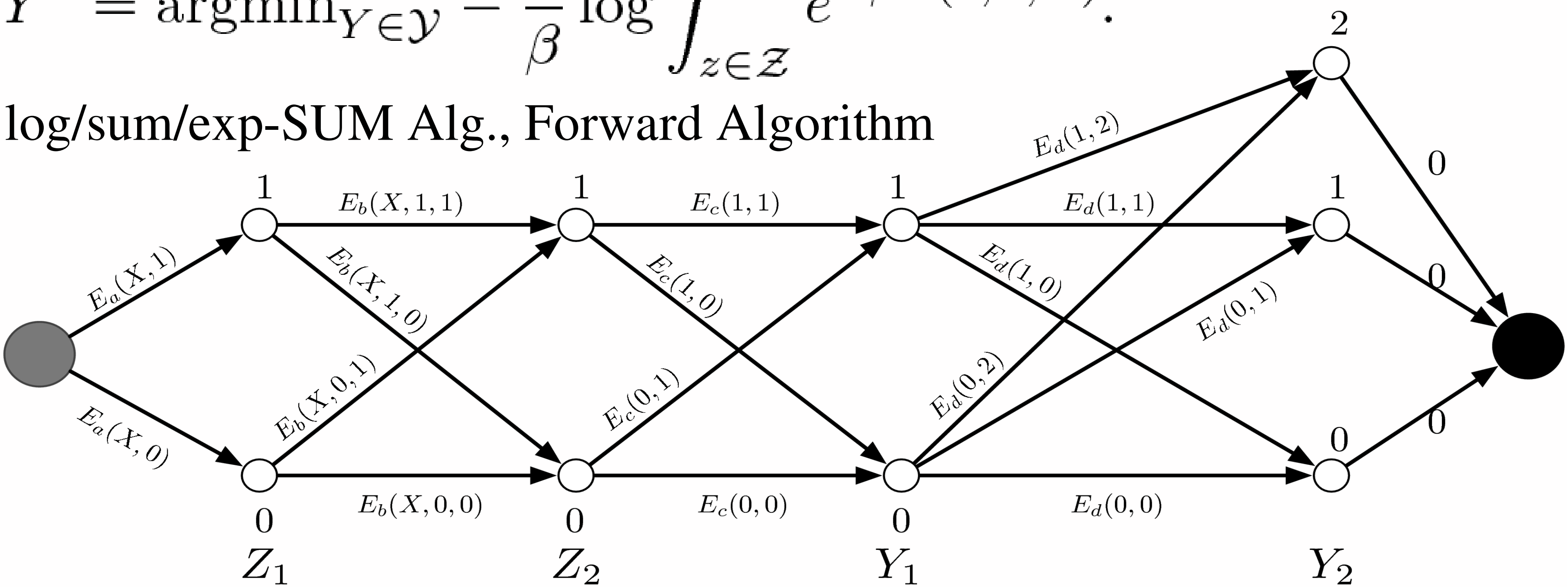
- **The previous picture shows a chain graph of factors with 2 inputs.**
- **The extension of this procedure to trees, with factors that can have more than 2 inputs is the "min-sum" algorithm (a non-probabilistic form of belief propagation)**
- **Basically, it is the sum-product algorithm with a different semi-ring algebra (min instead of sum, sum instead of product), and no normalization step.**
  - [Kschischang, Frey, Loeliger, 2001][McKay's book]

Marginalization



$$Y^* = \text{argmin}_{Y \in \mathcal{Y}} - \frac{1}{\beta} \log \int_{z \in \mathcal{Z}} e^{-\beta E(z,Y,X)}.$$

log/sum/exp-SUM Alg., Forward Algorithm



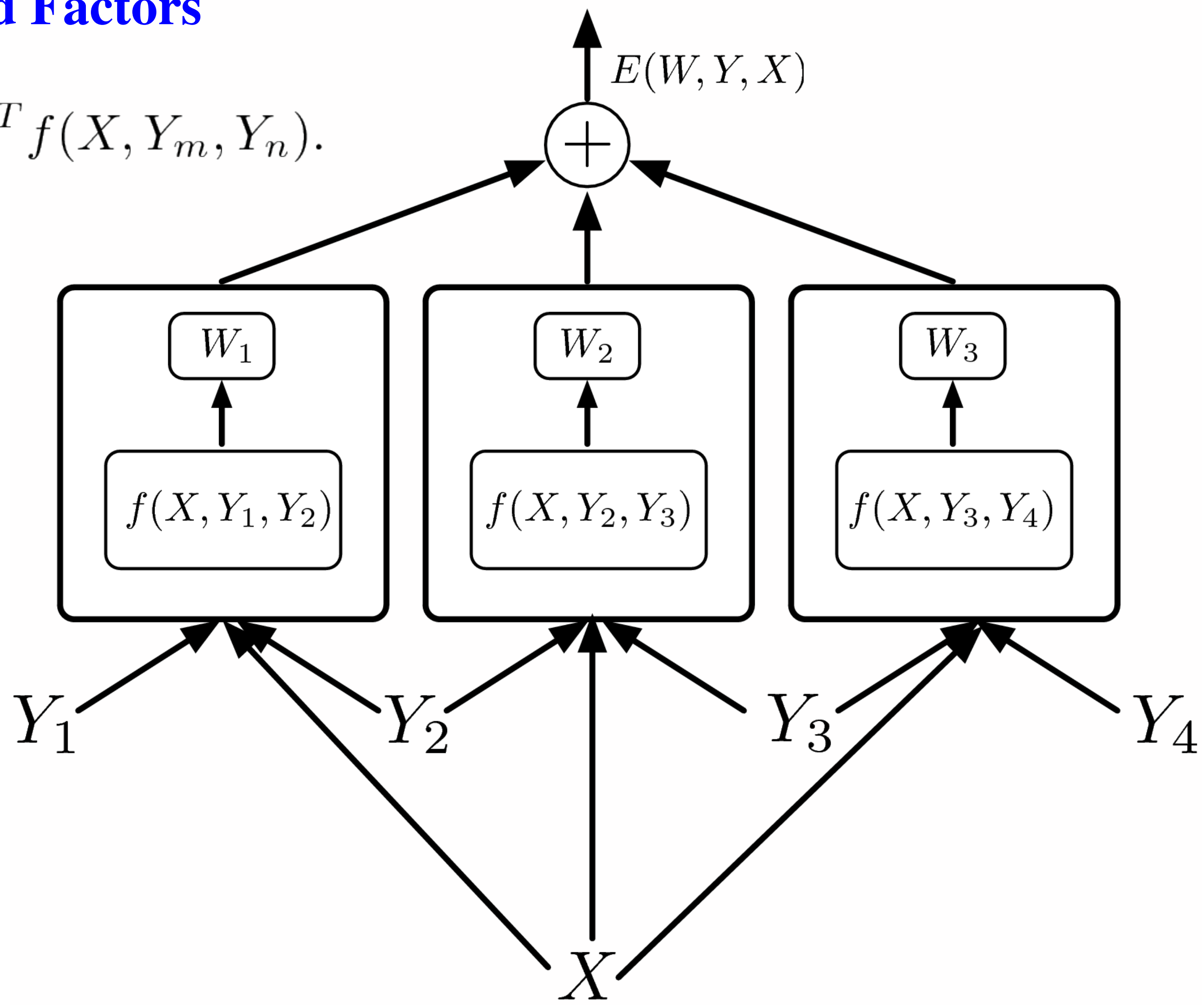
Energy-Based Belief Prop: Marginalization over Latent Variables

- **The previous picture shows a chain graph of factors with 2 inputs.**
  - Going along a path: add up the energies
  - When several paths meet: compute $-\frac{1}{\beta}\log\sum_i e^{-\beta E_{ji}}$
- **The extension of this procedure to trees, with factors that can have more than 2 inputs is the "[log/sum/exp]-sum" algorithm (a non-probabilistic form of belief propagation)**
- **Basically, it is the sum-product algorithm with a different semi-ring algebra (log/sum/exp instead of sum, sum instead of product), and no normalization step.**
  - [Kschischang, Frey, Loeliger, 2001][McKay's book]

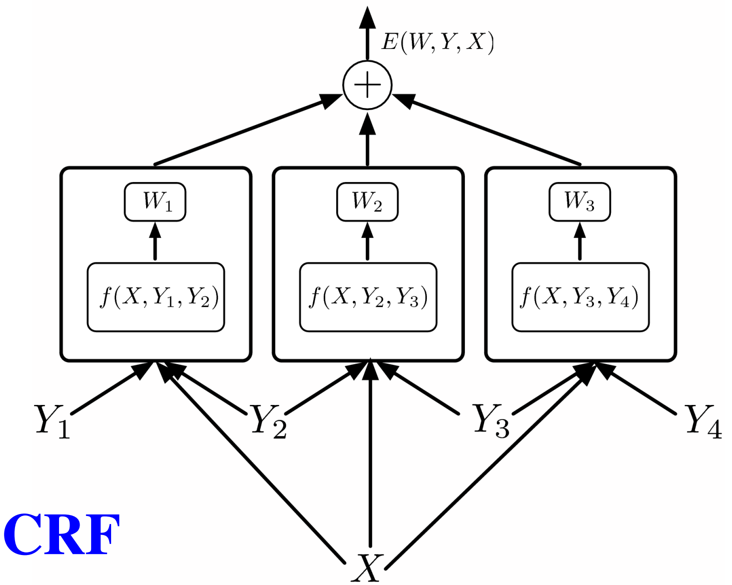
A Simple Case: Linearly Parameterized Factors: CRF, MMMN

- **Linearly Parameterized Factors**

$$E(W,Y,X) = \sum_{(m,n)\in\mathcal{F}} W^T f(X, Y_m, Y_n).$$



Linearly Parameterized Factors + Negative Log Likelihood Loss = Conditional Random Fields



- **Linearly Parameterized Factors + NLL loss = CRF**
  - [Lafferty, McCallum, Pereira, 2001]

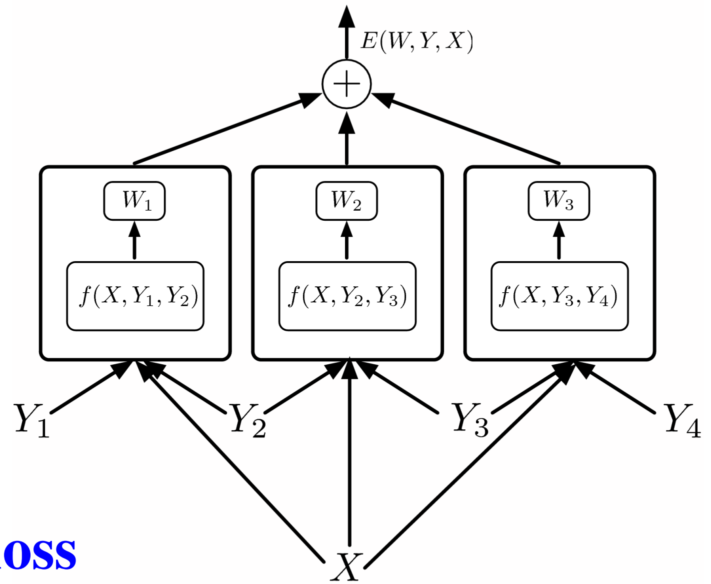
$$\mathcal{L}_{\text{nll}}(W) = \frac{1}{P}\sum_{i=1}^{P} W^T F(X^i, Y^i) + \frac{1}{\beta}\log \sum_{y\in\mathcal{Y}} e^{-\beta W^T F(X^i, y)}.$$

$$\frac{\partial \mathcal{L}_{\text{nll}}(W)}{\partial W} = \frac{1}{P}\sum_{i=1}^{P} F(X^i, Y^i) - \sum_{y\in\mathcal{Y}} F(X^i, y) P(y|X^i, W),$$

$$P(y|X^i, W) = \frac{e^{-\beta W^T F(X^i, y)}}{\sum_{y'\in\mathcal{Y}} e^{-\beta W^T F(X^i, y')}}.$$

simplest/best learning procedure: stochastic gradient

Linearly Parameterized Factors + Perceptron Loss = Sequence Perceptron



- **Linearly Parameterized Factors + Perceptron loss**
  - [LeCun, Bottou, Bengio, Haffner 1998, Collins 2000, Collins 2001]

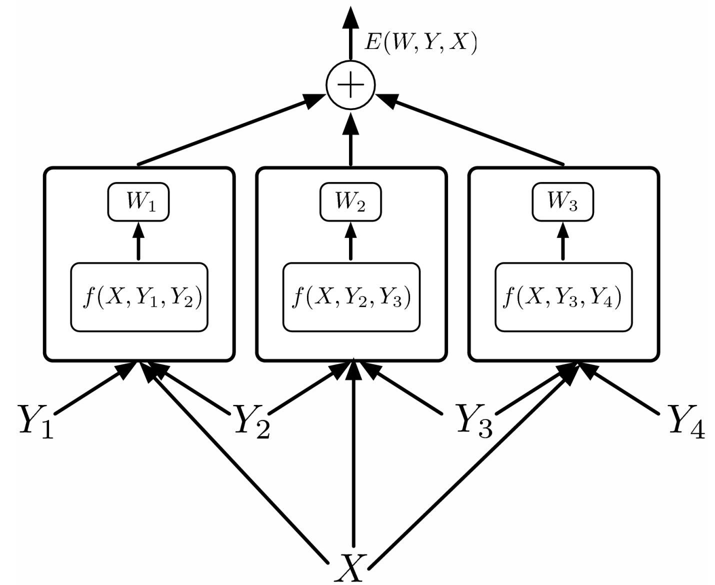
$$\mathcal{L}_{\text{perceptron}}(W) = \frac{1}{P}\sum_{i=1}^{P} E(W, Y^i, X^i) - E(W, Y^{*i}, X^i),$$

$$\mathcal{L}_{\text{perceptron}}(W) = \frac{1}{P}\sum_{i=1}^{P} W^T \left(F(X^i, Y^i) - F(X^i, Y^{*i})\right).$$

$$W \leftarrow W - \eta \left(F(X^i, Y^i) - F(X^i, Y^{*i})\right).$$

(LeCun 1998 used non-linear factors)

Linearly Parameterized Factors + Hinge Loss = Max Margin Markov Networks



- **Linearly Parameterized Factor + Hinge loss**
  - [Altun et a. 2003, Taskar et al. 2003]

$$\mathcal{L}_{\text{hinge}}(W) = \frac{1}{P}\sum_{i=1}^{P} \max(0, m + E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)) + \gamma ||W||^2.$$

$$\mathcal{L}_{\text{hinge}}(W) = \frac{1}{P}\sum_{i=1}^{P} \max\left(0, m + W^T \Delta F(X^i, Y^i)\right) + \gamma ||W||^2,$$

$$\Delta F(X^i, Y^i) = F(X^i, Y^i) - F(X^i, \bar{Y}^i)$$

Simple gradient descent rule:

$$\text{If } \Delta F(X^i, Y^i) > -m \text{ then } W \leftarrow W - \eta \Delta F(X^i, Y^i) - 2\gamma W$$

Can be performed in the dual (like an SVM)

Non-Linear Factors

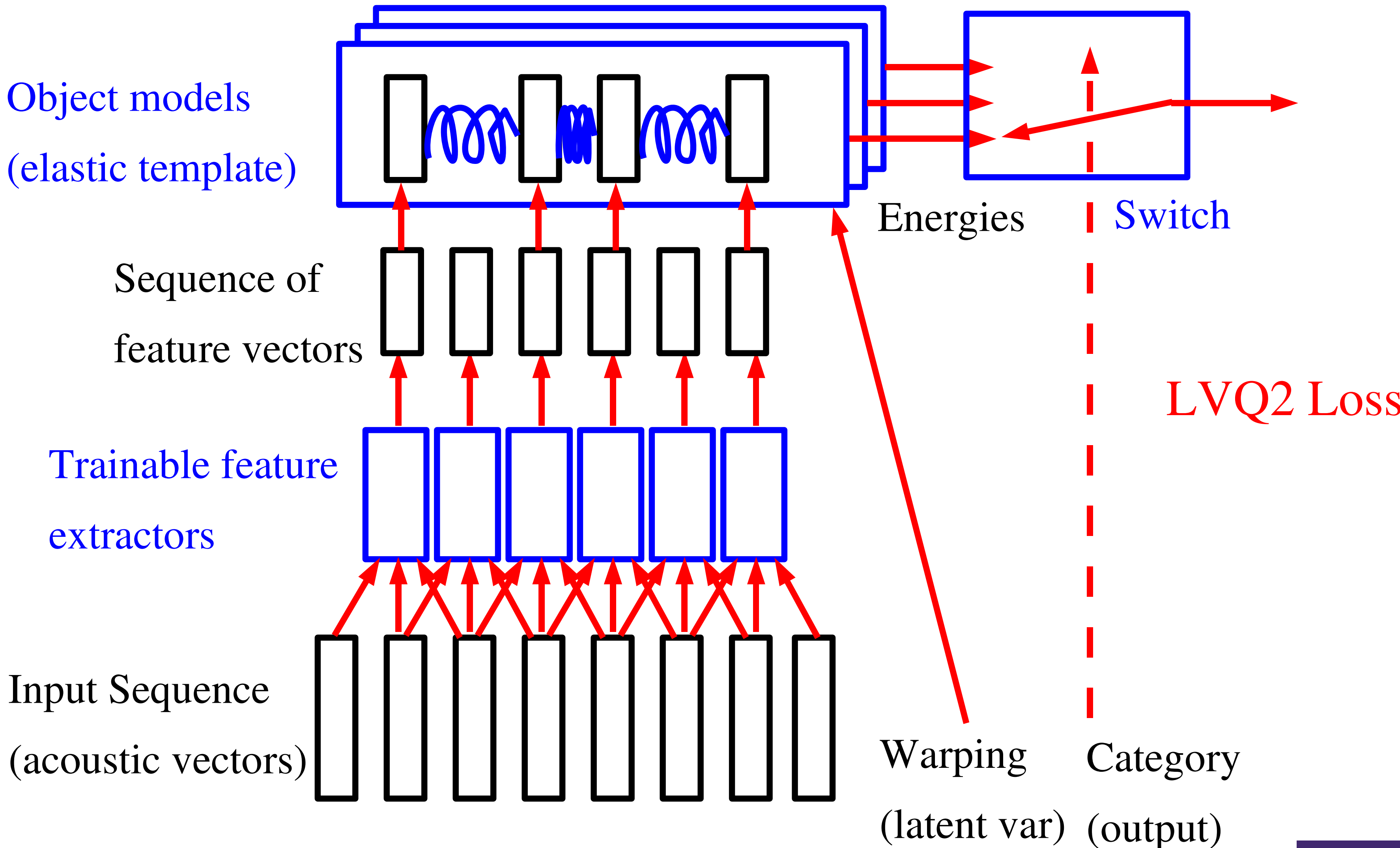
- **Energy-Based sequence labeling systems trained discriminatively have been used since the early 1990's**
- **Almost all of them used non-linear factors, such as multi-layer neural nets or mixtures of Gaussians.**
- **They were used mostly for speech and handwriting recognition**
- **There is a huge literature on the subject that has been somewhat ignored or forgotten by the NIPS and NLP communities.**
- **Why use non linear factors?**
  - :-(the loss function is non-convex
  - :-o You have to use simple gradient-based optimization algorithms, such as stochastic gradient descent (but that's what works best anyway, even in the convex case)
  - :-) linear factors simply don't cut it for speech and handwriting (including SVM-like linear combinations of kernel functions)

Deep Factors / Deep Graph: ASR with TDNN/HMM

- **Discriminative Automatic Speech Recognition system with HMM and various acoustic models**
  - Training the acoustic model (feature extractor) and a (normalized) HMM in an integrated fashion.
- **With Minimum Empirical Error loss**
  - Ljolje and Rabiner (1990)
- **with NLL:**
  - Bengio (1992)
  - Haffner (1993)
  - Bourlard (1994)
- **With MCE**
  - Juang et al. (1997)
- **Late normalization scheme (un-normalized HMM)**
  - Bottou pointed out the **label bias problem** (1991)
  - Denker and Burges proposed a solution (1995)

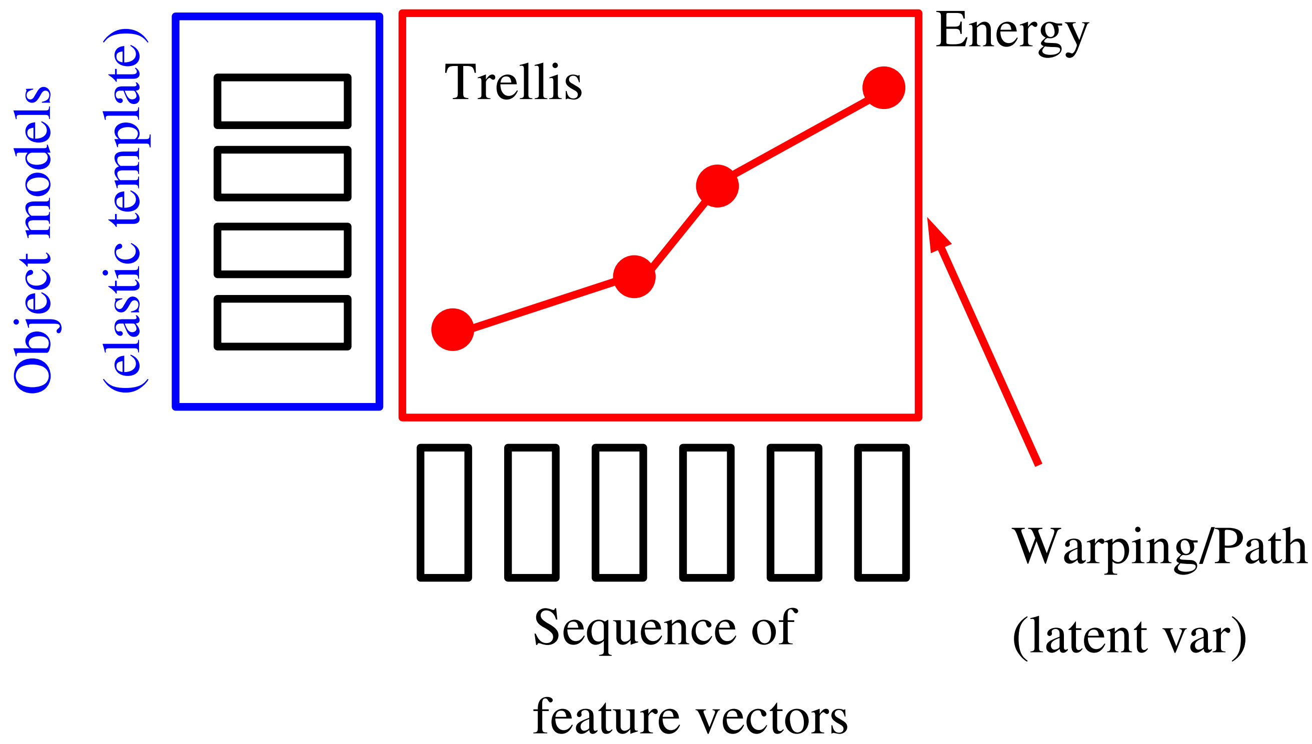
Example 1: Integrated Disc. Training with Sequence Alignment

- **Spoken word recognition with trainable elastic templates and trainable feature extraction [Driancourt&Bottou 1991, Bottou 1991, Driancourt 1994]**



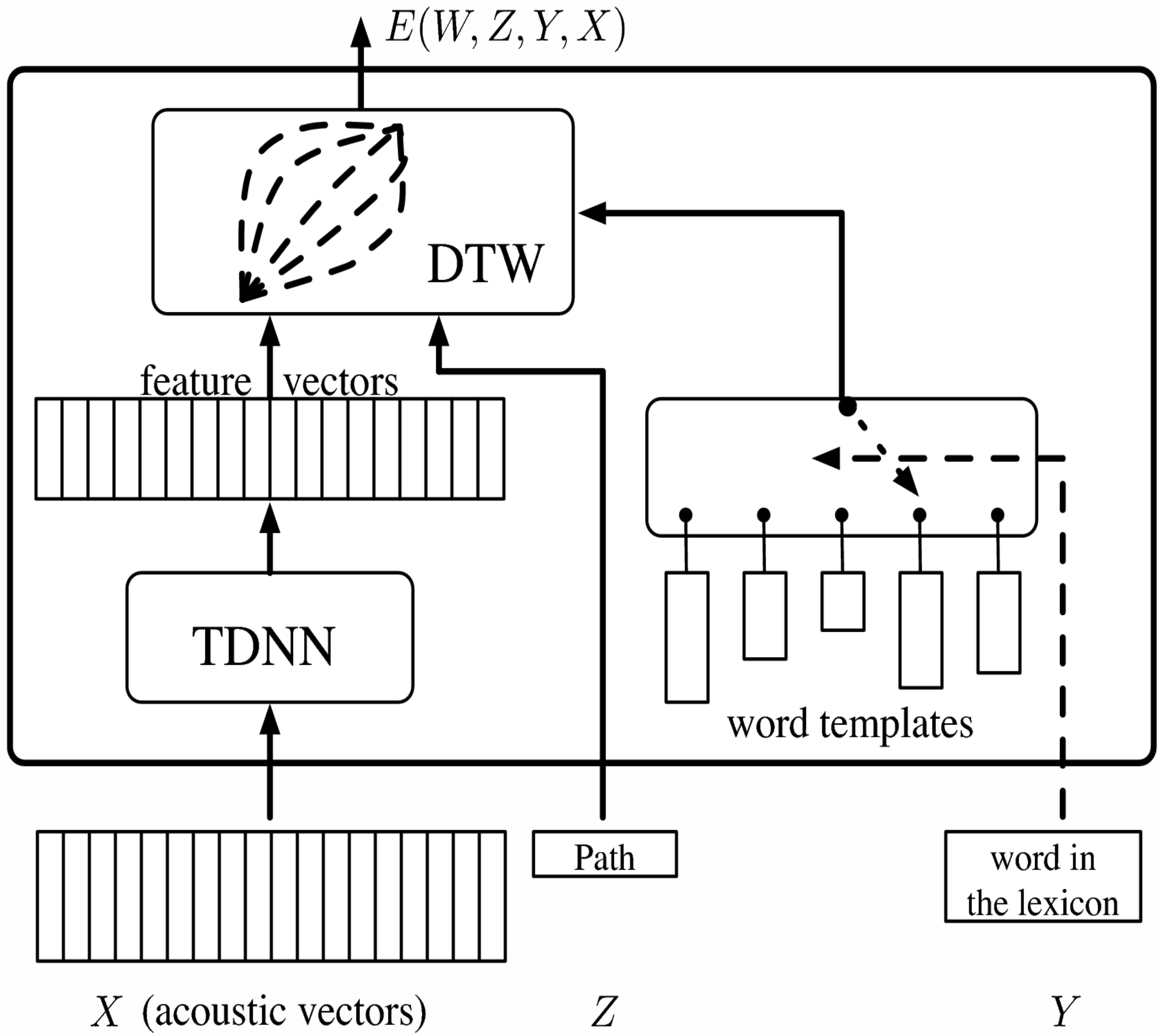
Example: 1-D Constellation Model (a.k.a. Dynamic Time Warping)

- Spoken word recognition with trainable elastic templates and trainable feature extraction [Driancourt&Bottou 1991, Bottou 1991, Driancourt 1994]
- Elastic matching using dynamic time warping (Viterbi algorithm on a trellis).
- The corresponding EBFG is implicit (it changes for every new sample).



Deep Factors / Deep Graph: ASR with TDNN/DTW

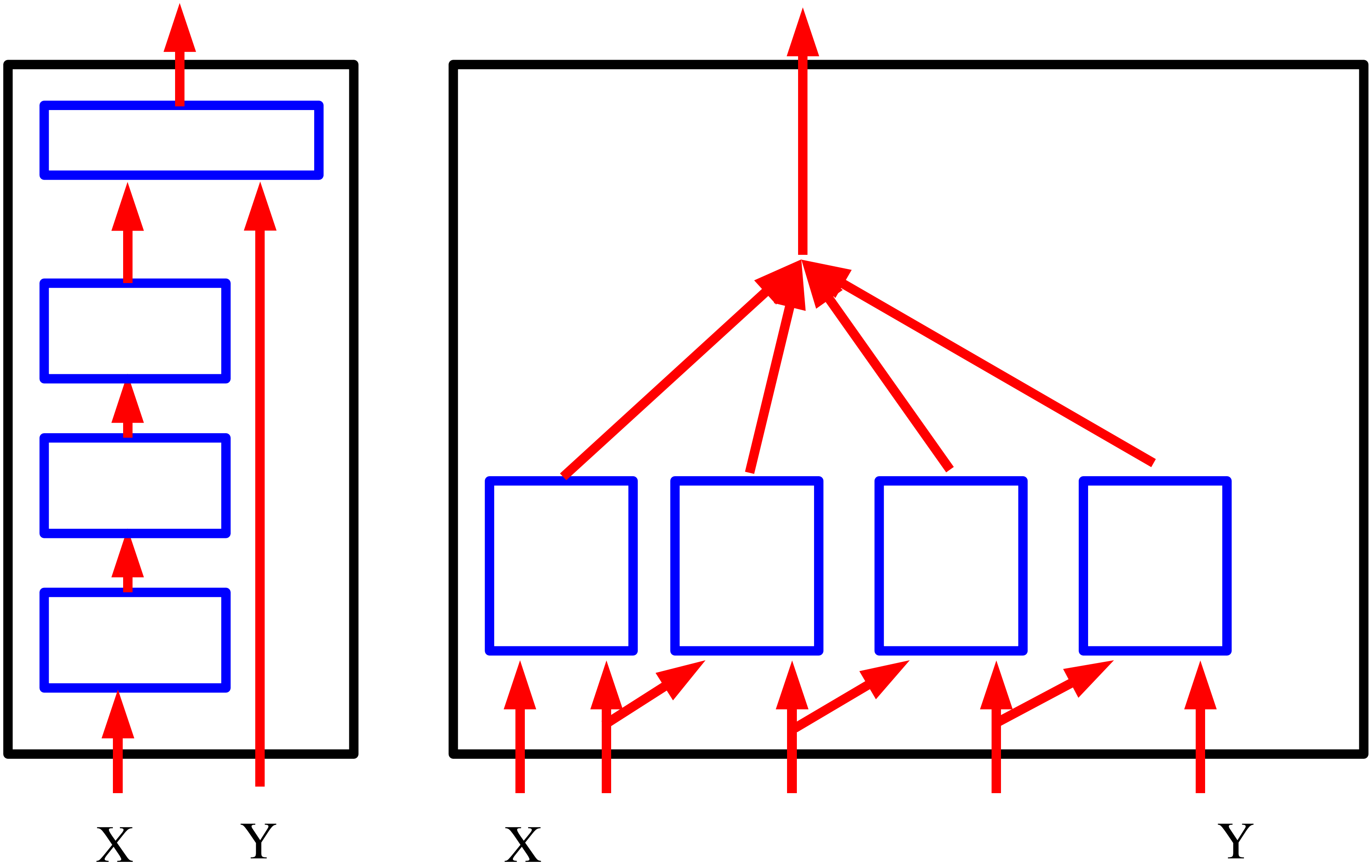
- **Trainable Automatic Speech Recognition system with convolutional nets (TDNN) and dynamic time warping (DTW)**



- **Training the feature extractor as part of the whole process.**
- **with the LVQ2 Loss :**
  - Driancourt and Bottou's speech recognizer (1991)
- **with NLL:**
  - Bengio's speech recognizer (1992)
  - Haffner's speech recognizer (1993)

Two types of "deep" architectures

- **Factors are deep / graph is deep**

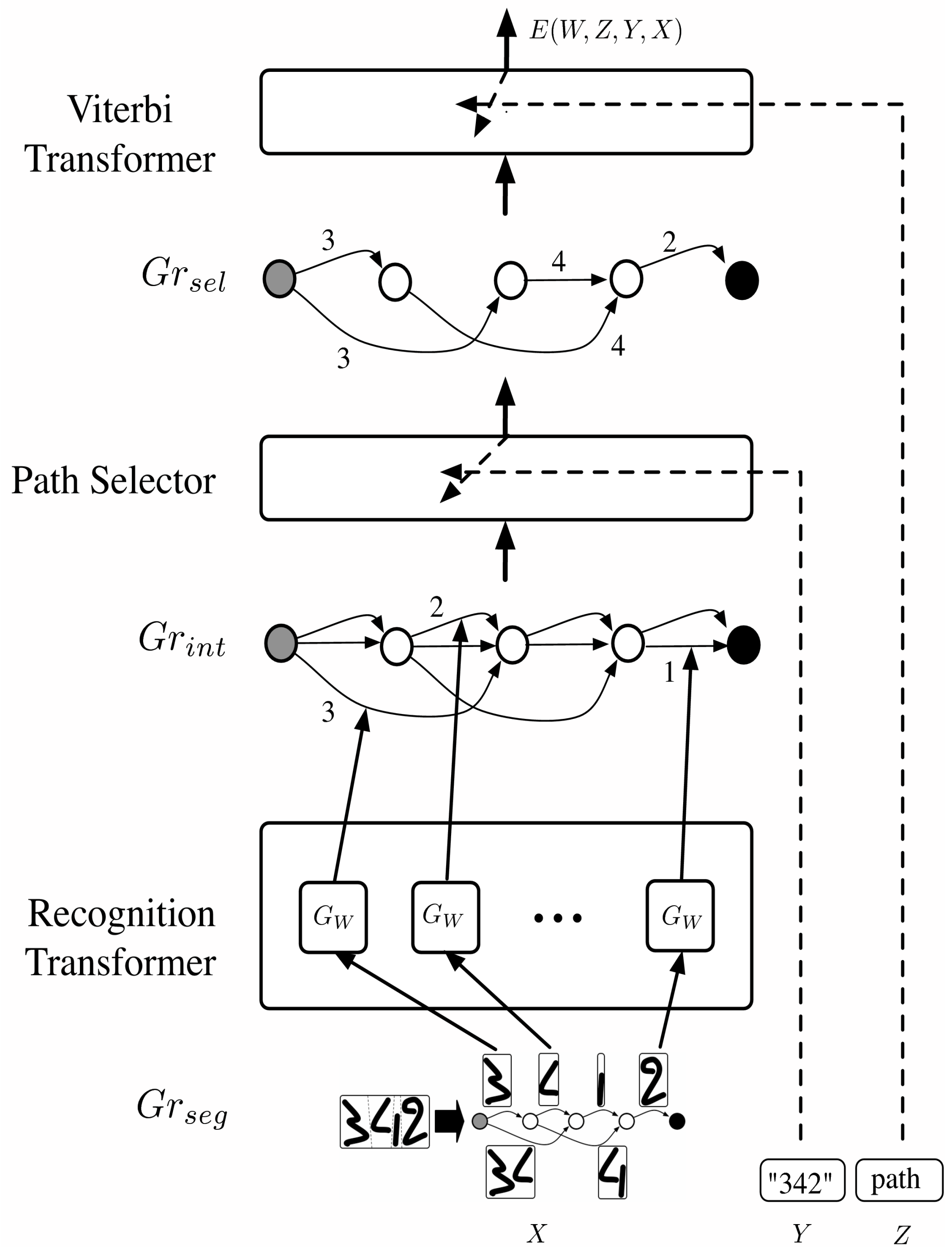


Complex Trellises: procedural representation of trellises

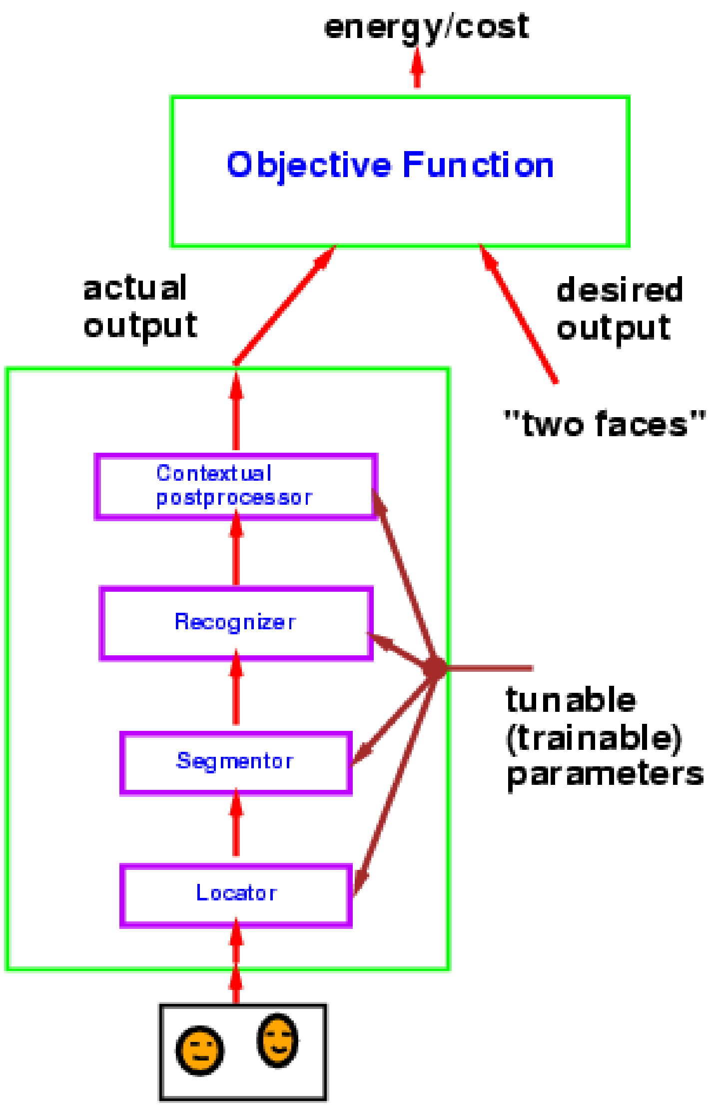
- **When the trellis is too large, we cannot store it in its entirety in memory.**
  - We must represent it proceduraly
- **The cleanest way to represent complex graphs proceduraly is through the formalism of finite-state transducer algebra**
  - [Mohri 1997, Pereira et al.]

Really Deep Factors / Really Deep Graph

- **Handwriting Recognition with Graph Transformer Networks**
- **Un-normalized hierarchical HMMs**
  - Trained with Perceptron loss [LeCun, Bottou, Bengio, Haffner 1998]
  - Trained with NLL loss [Bengio, LeCun 1994], [LeCun, Bottou, Bengio, Haffner 1998]
- **Answer = sequence of symbols**
- **Latent variable = segmentation**

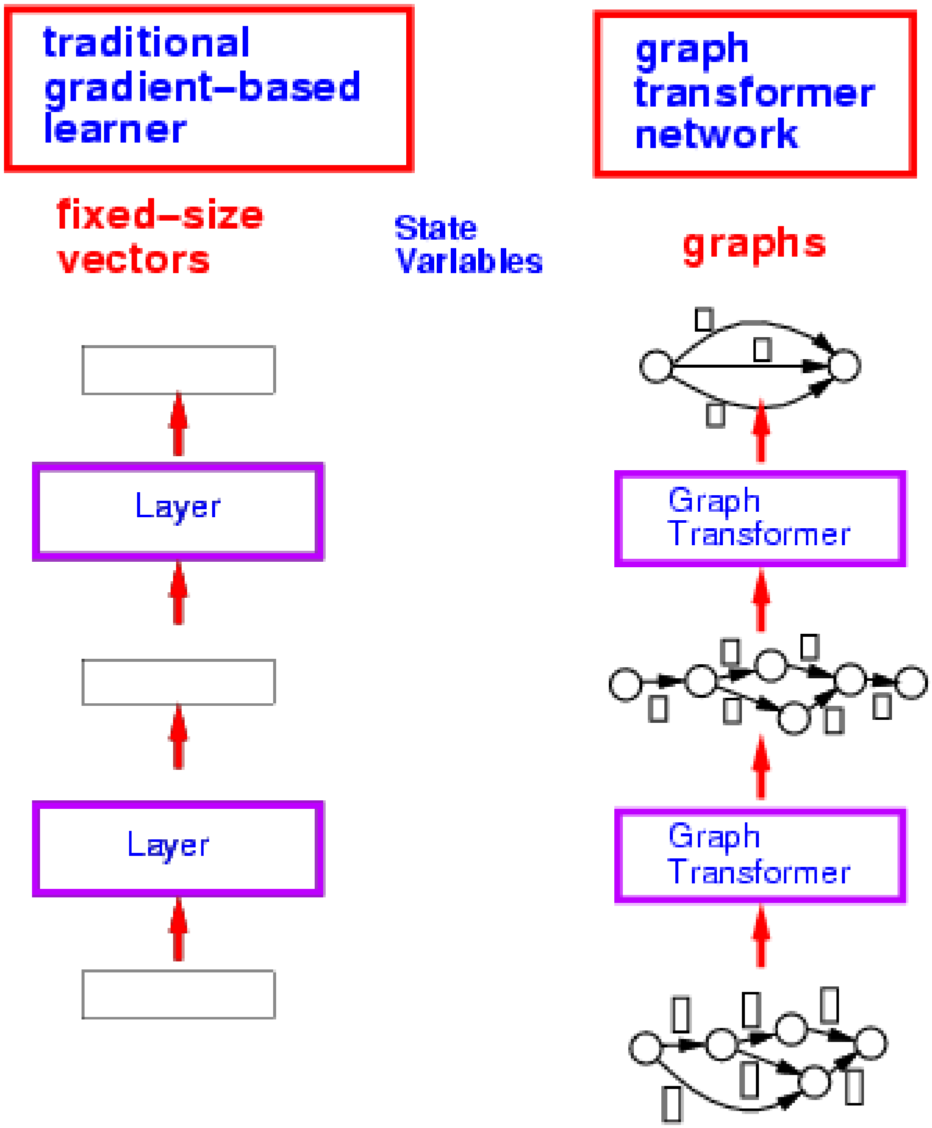


End-to-End Learning.



- Making every single module in the system trainable.
- Every module is trained simultaneously so as to optimize a global loss function.

Using Graphs instead of Vectors.

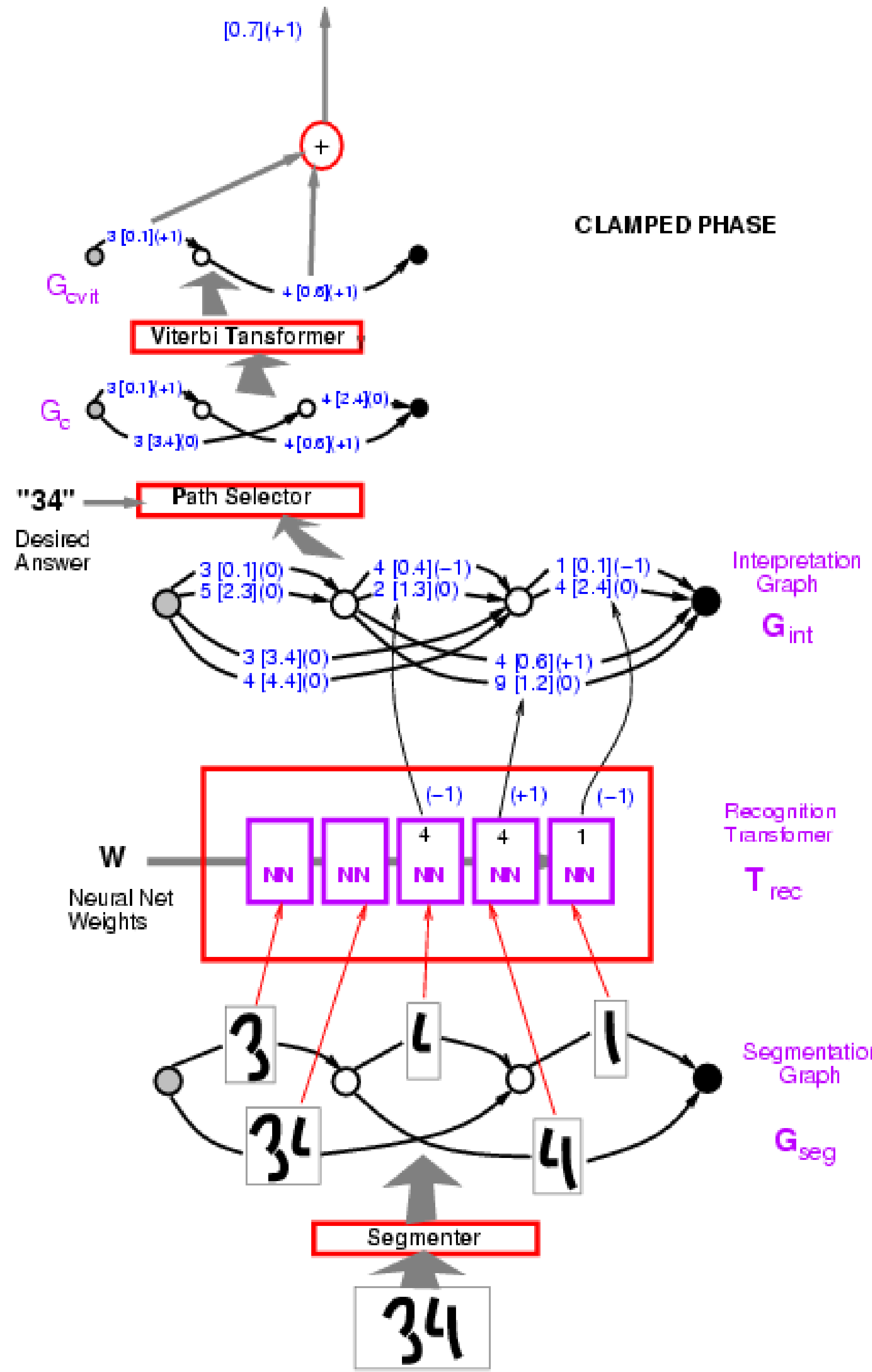


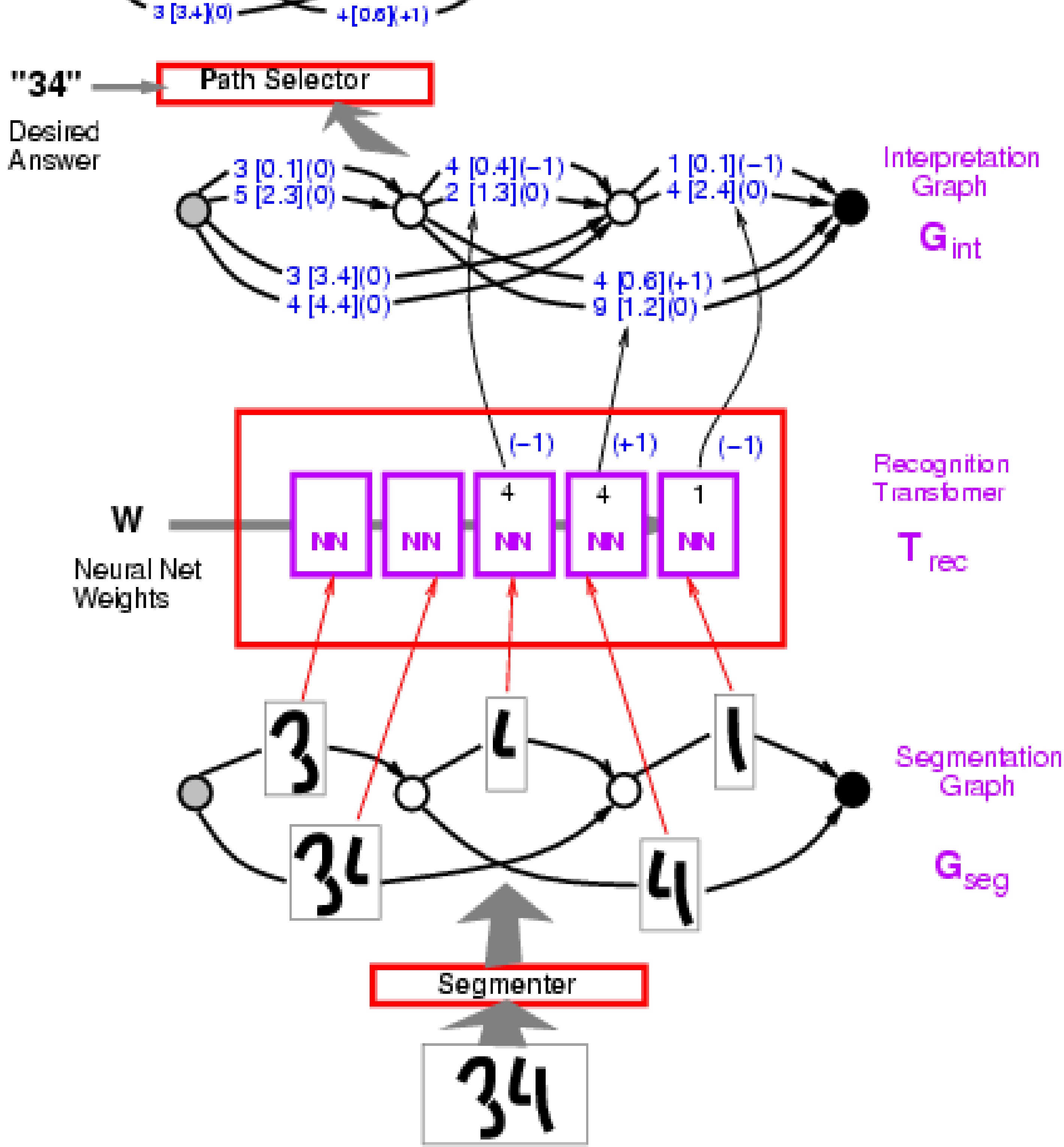
- **Whereas traditional learning machines manipulate fixed-size vectors, Graph Transformer Networks manipulate graphs.**

Graph Transformer Networks

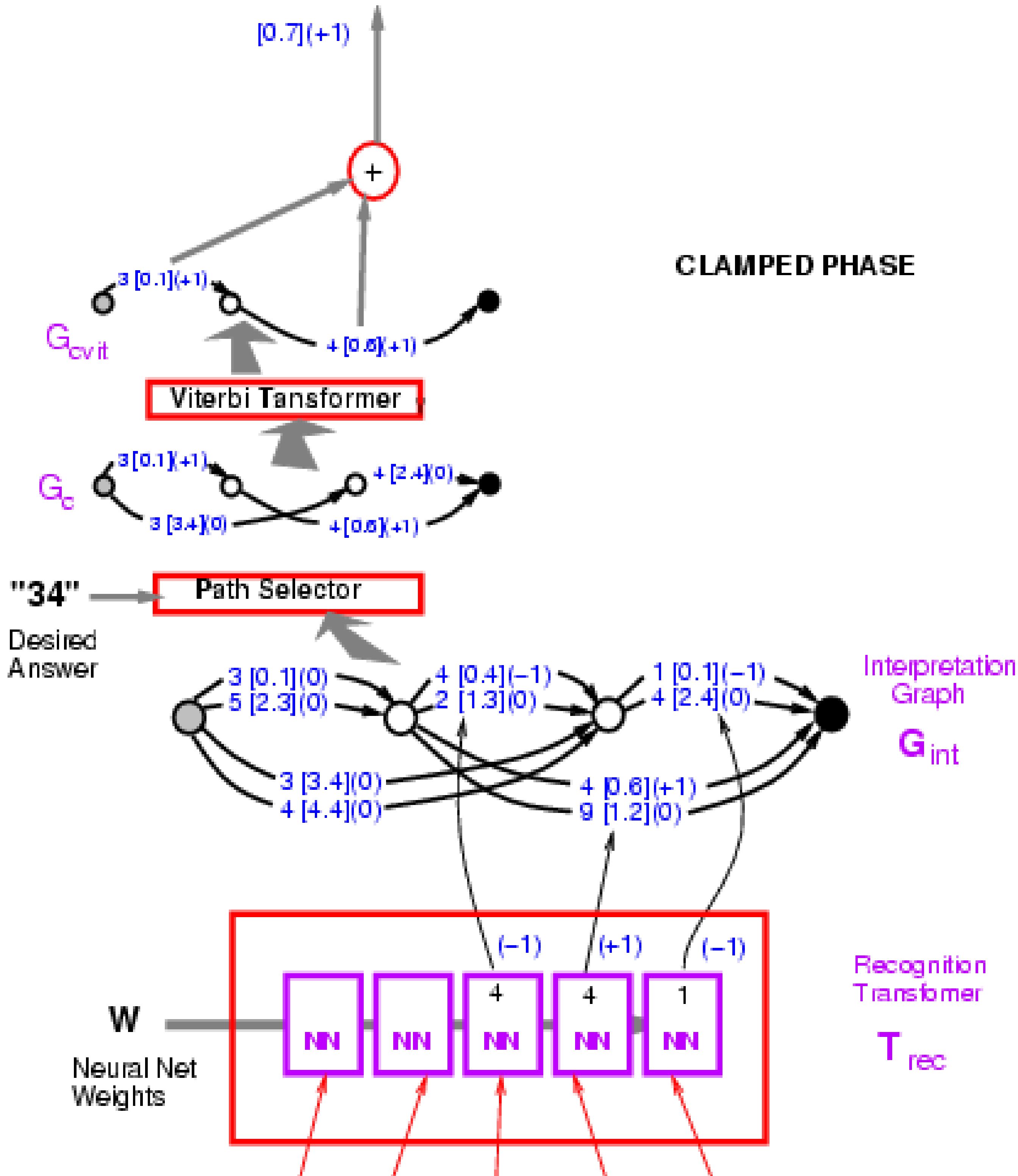
- **Variables:**
  - X: input image
  - Z: path in the interpretation graph/segmentation
  - Y: sequence of labels on a path
- **Loss function: computing the energy of the desired answer:**

$$E(W, Y, X)$$





3 [3.4](0)
4 [0.6](+1)
"34"
Desired Answer
Path Selector
3 [0.1](0)
5 [2.3](0)
4 [0.4](−1)
2 [1.3](0)
1 [0.1](−1)
4 [2.4](0)
3 [3.4](0)
4 [4.4](0)
4 [0.6](+1)
9 [1.2](0)
Interpretation Graph
G_{int}
W
Neural Net Weights
(−1)
(+1)
(−1)
4
4
1
NN
NN
NN
NN
NN
Recognition Transfomer
T_{rec}
Segmentation Graph
G_{seg}
Segmenter

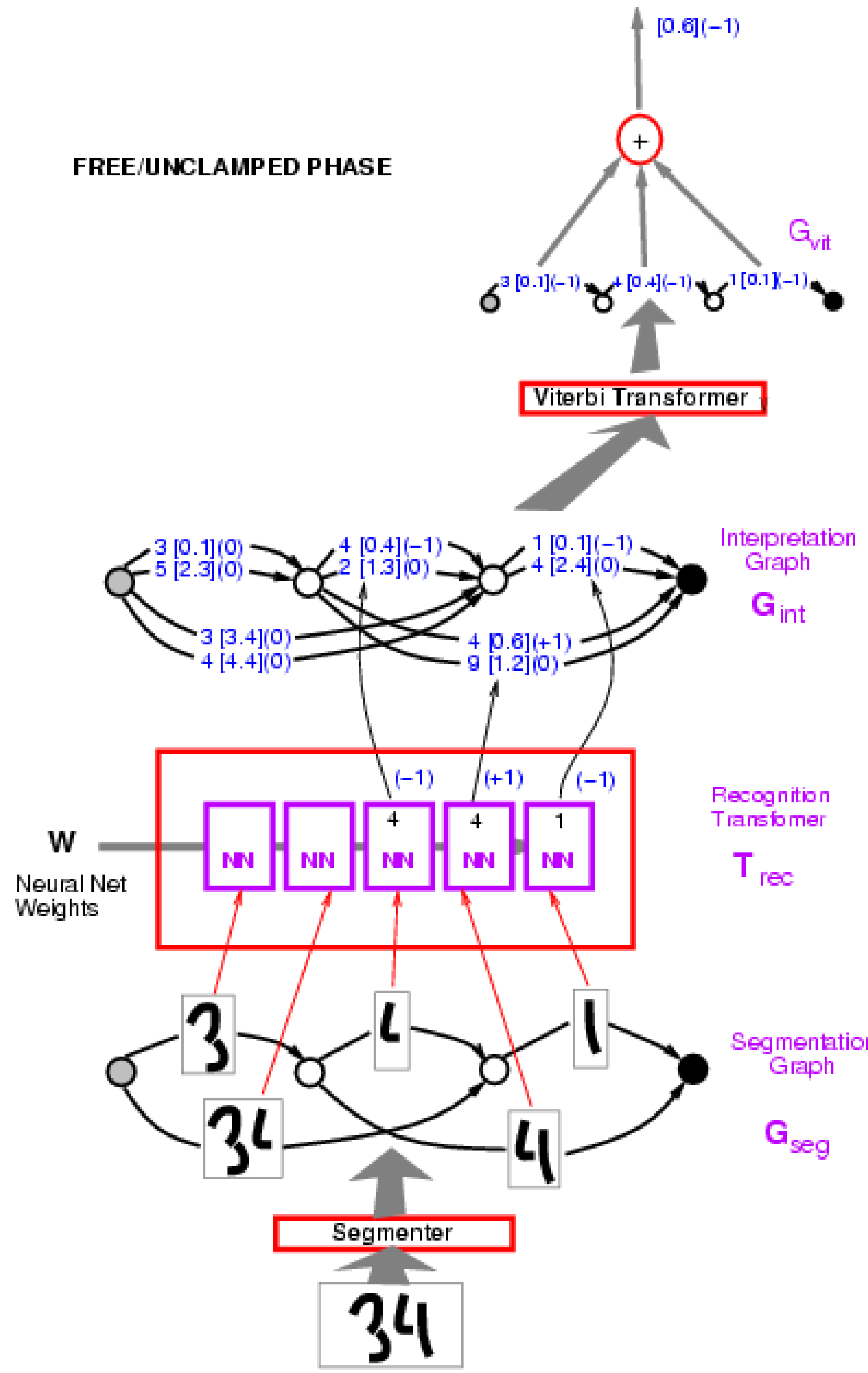


[0.7](+1)
+
CLAMPED PHASE
3 [0.1](+1)
4 [0.6](+1)
G_{cvit}
Viterbi Tansformer
3 [0.1](+1)
4 [2.4](0)
G_c
3 [3.4](0)
4 [0.6](+1)
"34"
Path Selector
Desired Answer
3 [0.1](0)
5 [2.3](0)
4 [0.4](−1)
2 [1.3](0)
1 [0.1](−1)
4 [2.4](0)
Interpretation Graph
G_{int}
3 [3.4](0)
4 [4.4](0)
4 [0.6](+1)
9 [1.2](0)
(−1)
(+1)
(−1)
4
4
1
NN
NN
NN
NN
NN
W
Neural Net Weights
Recognition Transfomer
T_{rec}

Graph Transformer Networks

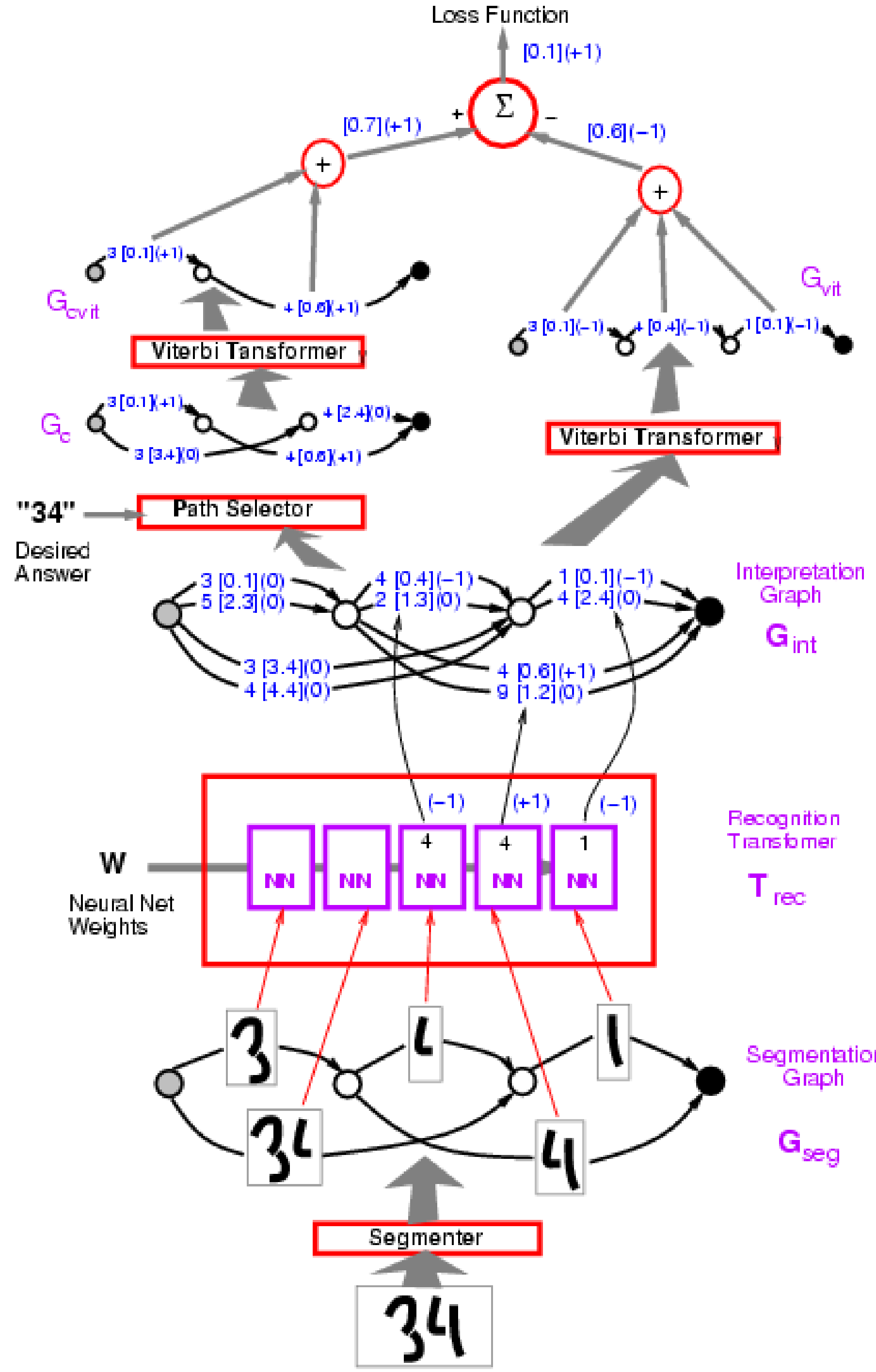
- **Variables:**
  - X: input image
  - Z: path in the interpretation graph/segmentation
  - Y: sequence of labels on a path
- **Loss function: computing the constrastive term:**

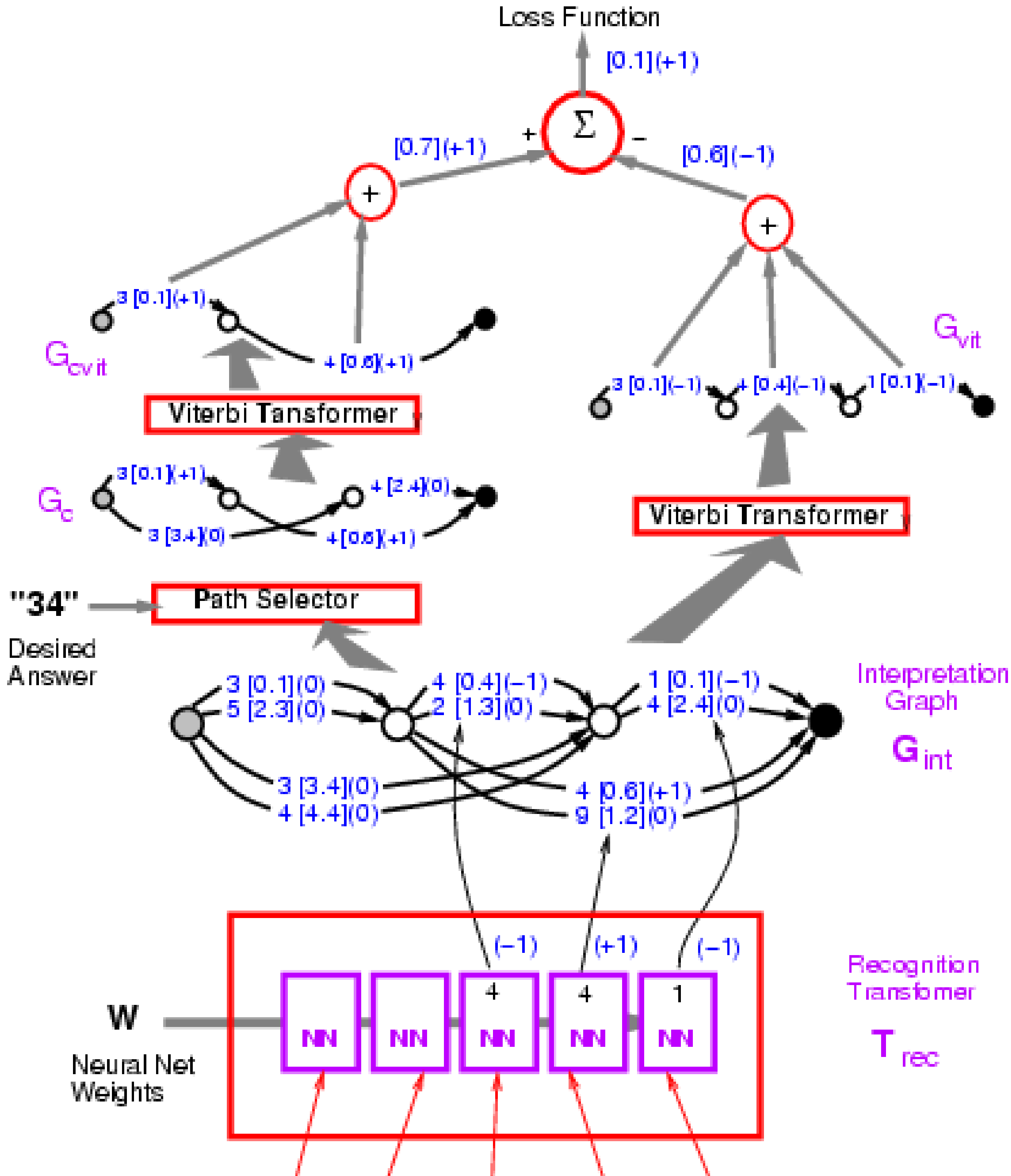
$$E(W, \check{Y}, X)$$



Graph Transformer Networks

- **Example: Perceptron loss**
- **Loss = Energy of desired answer – Energy of best answer.**
  - (no margin)




Loss Function
[0.1](+1)
+
Σ
–
[0.7](+1)
[0.6](–1)
+
+
3 [0.1](+1)
4 [0.6](+1)
G_{cvit}
Viterbi Tansformer
G_{vit}
3 [0.1](–1)
4 [0.4](–1)
1 [0.1](–1)
3 [0.1](+1)
4 [2.4](0)
G_c
3 [3.4](0)
4 [0.6](+1)
Viterbi Transformer
"34"
Desired Answer
Path Selector
3 [0.1](0)
5 [2.3](0)
4 [0.4](–1)
2 [1.3](0)
1 [0.1](–1)
4 [2.4](0)
Interpretation Graph
G_{int}
3 [3.4](0)
4 [4.4](0)
4 [0.6](+1)
9 [1.2](0)
(–1)
(+1)
(–1)
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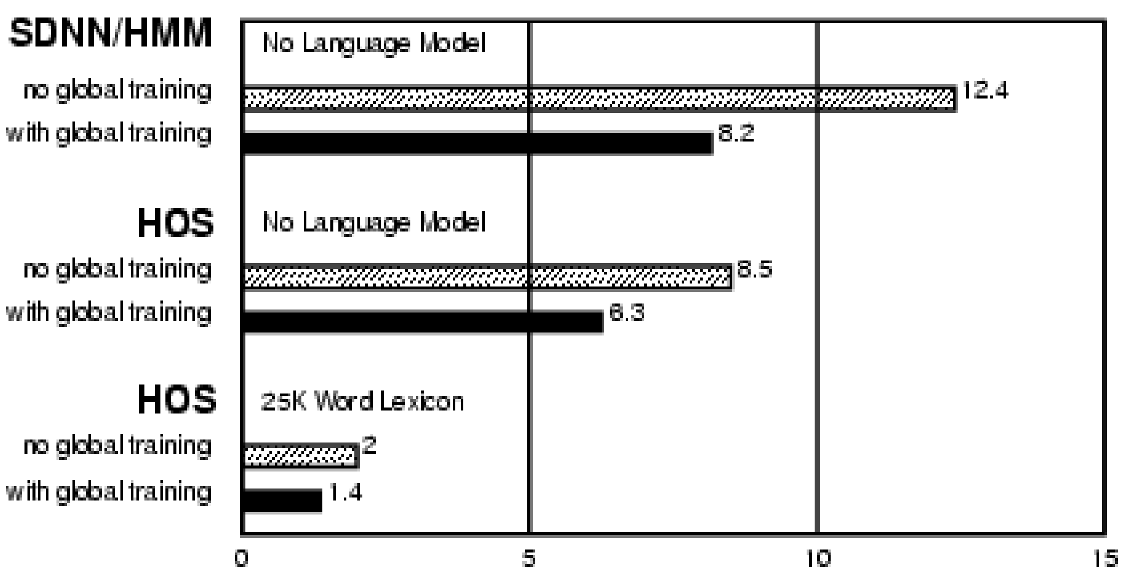
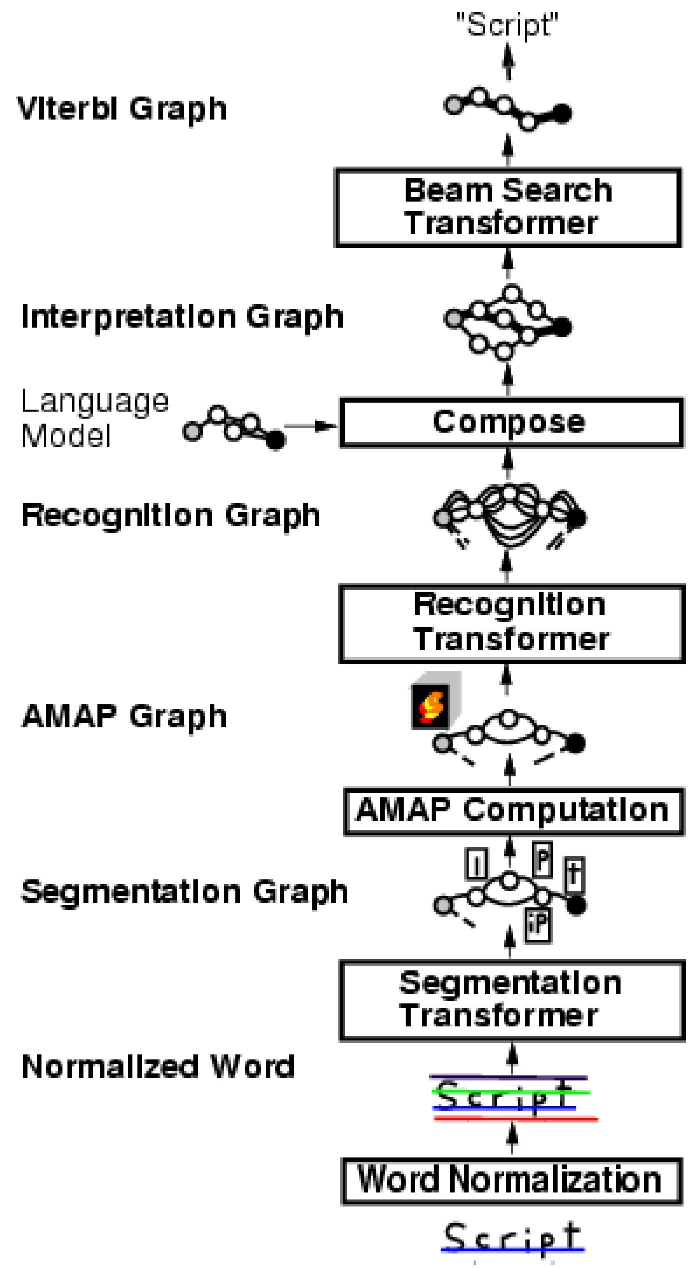
4
1
W
NN
NN
NN
NN
NN
Neural Net Weights
Recognition Transfomer
T_{rec}

Global Training Helps

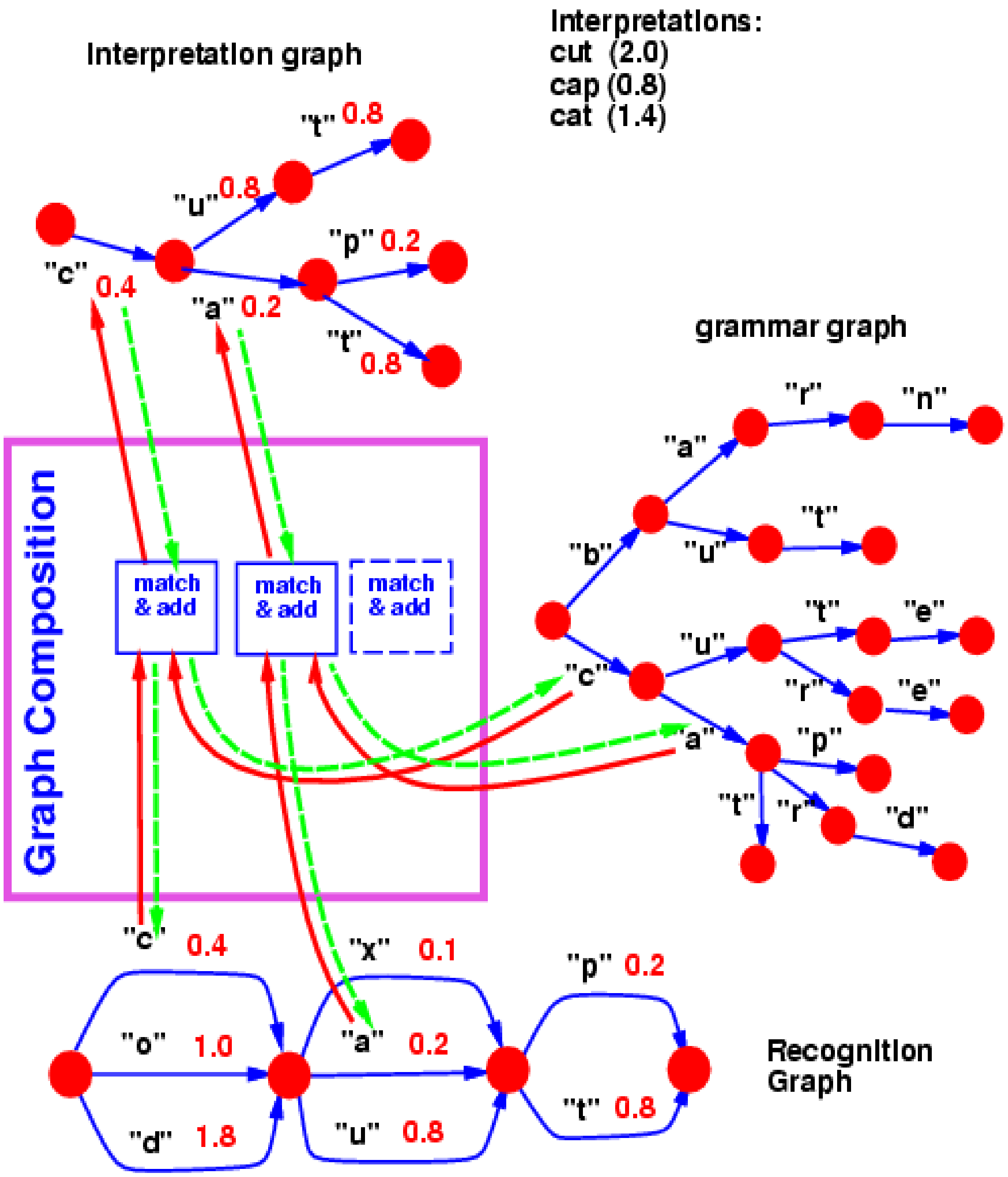
- **Pen-based handwriting recognition (for tablet computer)**
  - [Bengio&LeCun 1995]
  - Trained with NLL loss (aka MMI)





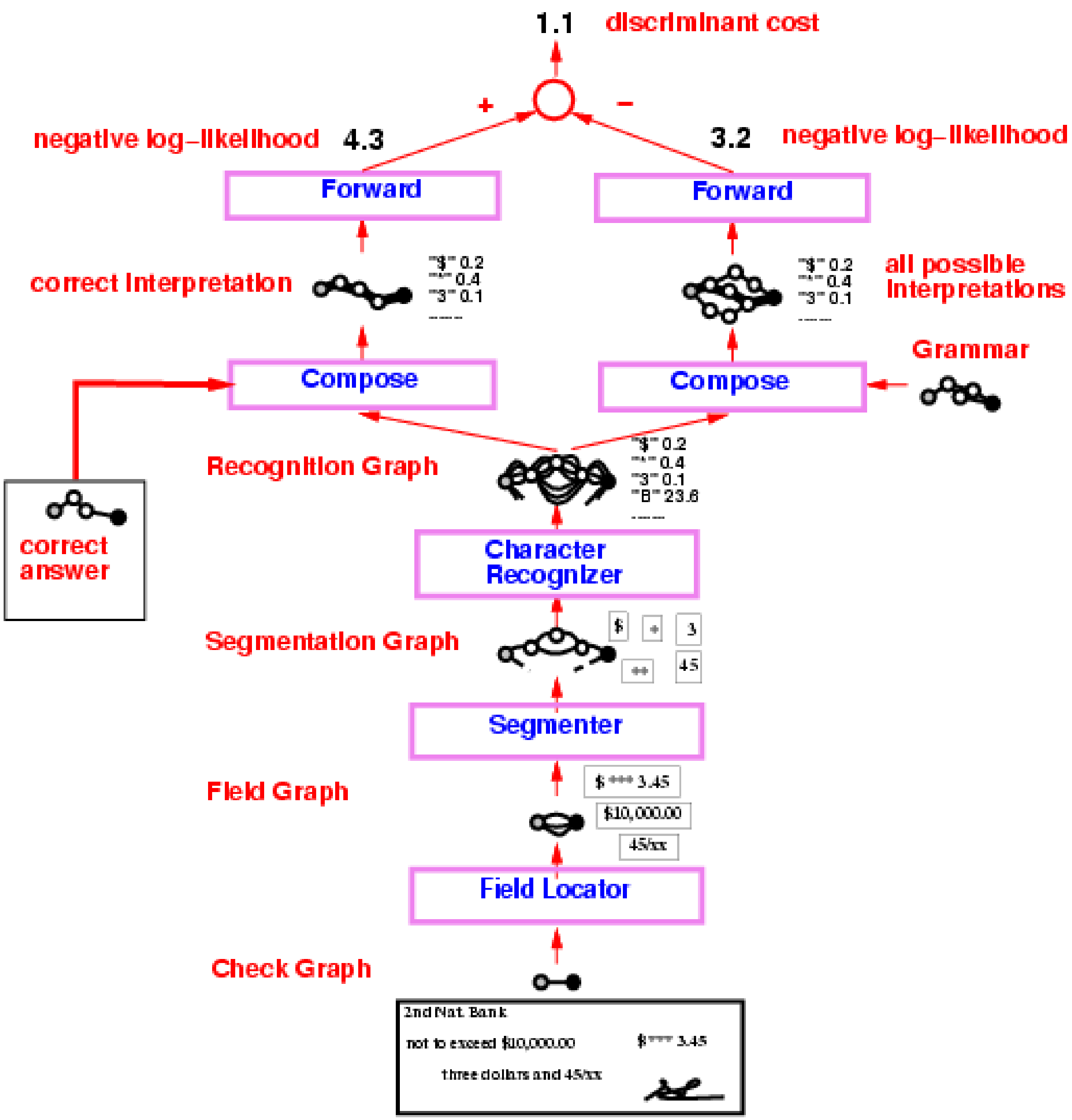
Graph Composition, Transducers.

- The composition of two graphs can be computed, the same way the dot product between two vectors can be computed.
- General theory: semi-ring algebra on weighted finite-state transducers and acceptors.



Check Reader

- Graph transformer network trained to read check amounts.
- Trained globally with Negative-Log-Likelihood loss.
- 50% percent corrent, 49% reject, 1% error (detectable later in the process.
- Fielded in 1996, used in many banks in the US and Europe.
- Processes an estimated 10% of all the checks written in the US.



Learning when the space of Y is huge

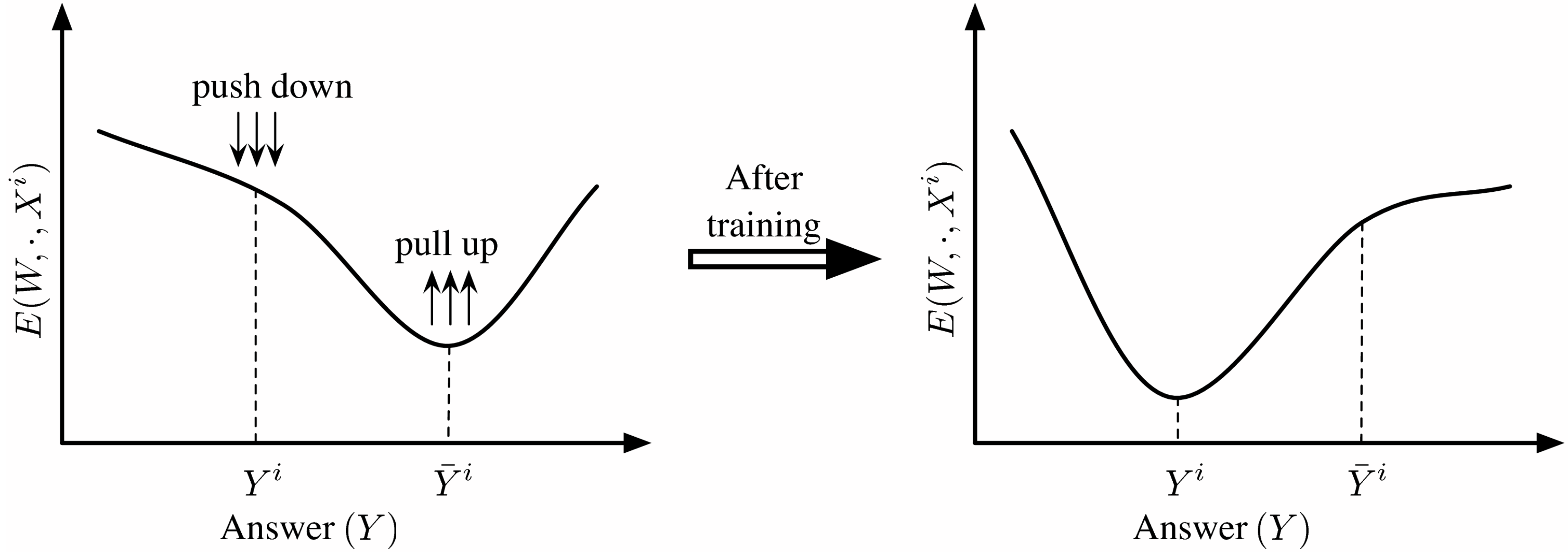
- learning when Y is in a high-dimensional continuous spaces
- Image restoration, Image segmentation
- Unsupervised learning in high-dimensional space

Learning when the space of Y is huge

- **Solutions:**
- **Use an energy function such that contrastive term in the loss is either constant or easy to compute**
  - e.g. Energy is quadratic: convex (inference is easy), integral of exponential is easily computable or constant.
- **Approximate the derivative of the contrastive term in the loss with a variational approximation**
- **Simple sampling approximation:**
  - Pull down on the energy of the training samples
  - Pull up on the energies of other configurations that have low energy (that are threatening)
  - Question: how do we pick those configurations?
  - One idea: **contrastive Divergence**

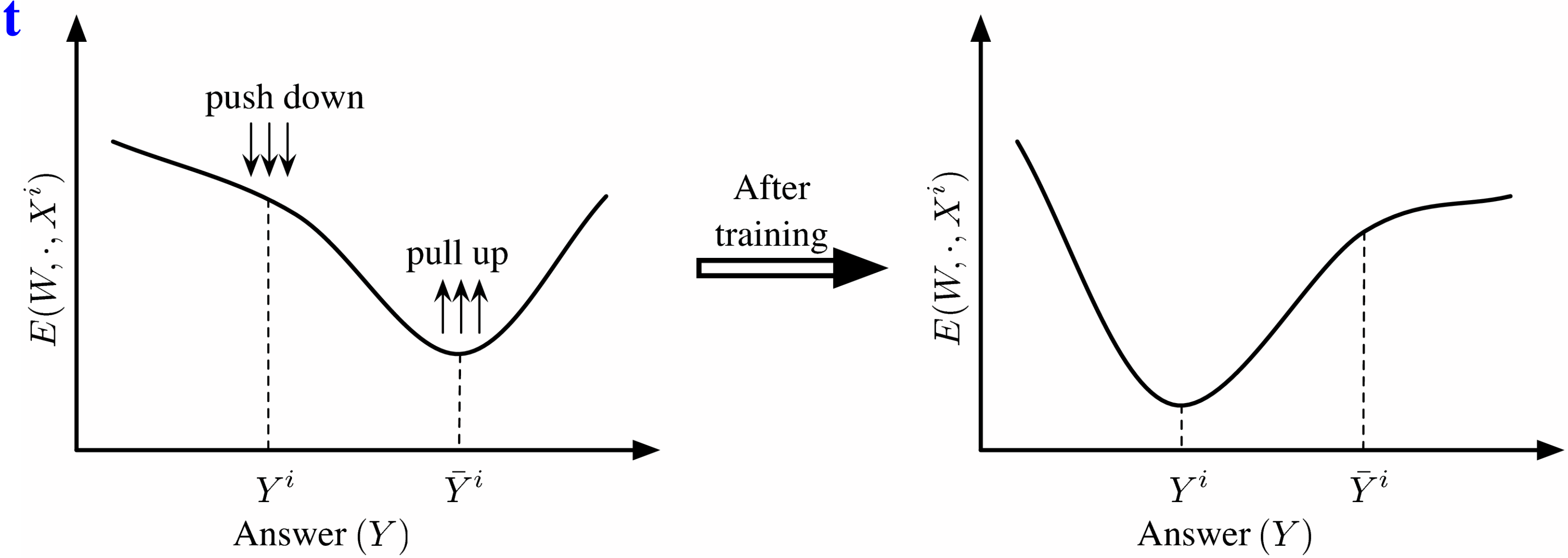
Contrastive Divergence

- To generate the “bad” configurations:
- 1. Start from the correct value of Y
- 2. Pull down the energy of the correct value
- 3. To obtain a “bad” configuration, go down the energy surface with “some noise”
- 4. pull up the energy of the obtained configuration



Contrastive Divergence

- To generate the "bad" configurations:
- Hybrid Monte-Carlo Sampling: simulate a ball rolling down the energy surface in Y space.
- Kick the ball in the a random direction (with a random momentum), and run the simulation for a few iterations.
- The final configuration is quite likely to have lower energy than t

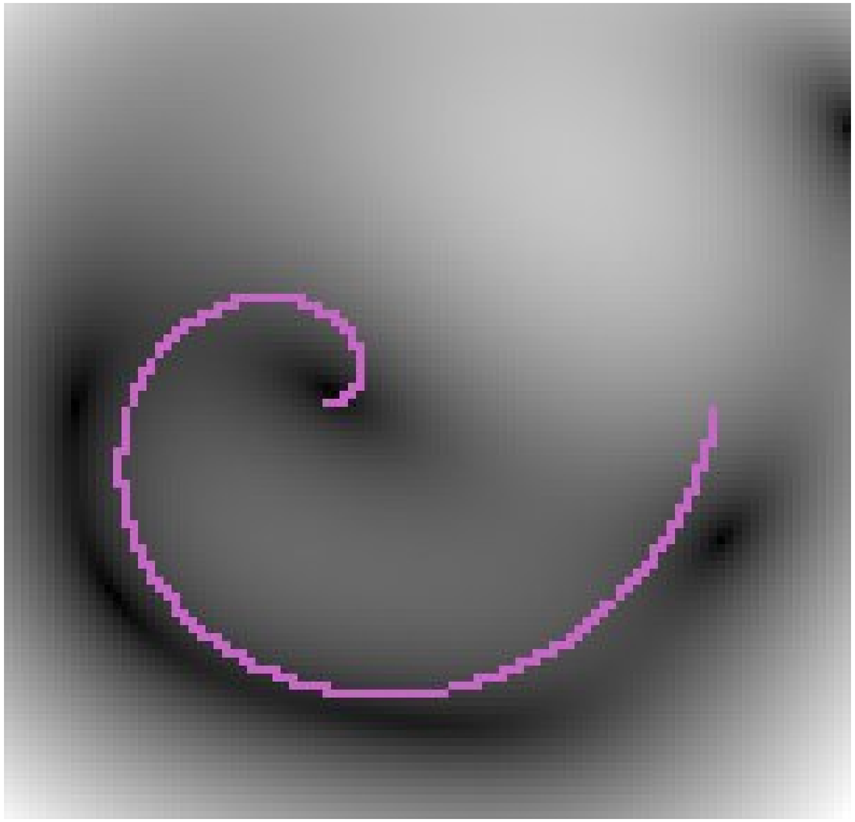


Energy-Based Unsupervised Learning with Margin Loss

- **Example: learning a spiral in 2D**
- **Energy: || Y – F(W,Y)||^2 where F is a 2-layer neural net**

$$L(Y,W) = \alpha E(Y,W) + max(0, m - E(\bar{Y},W))$$

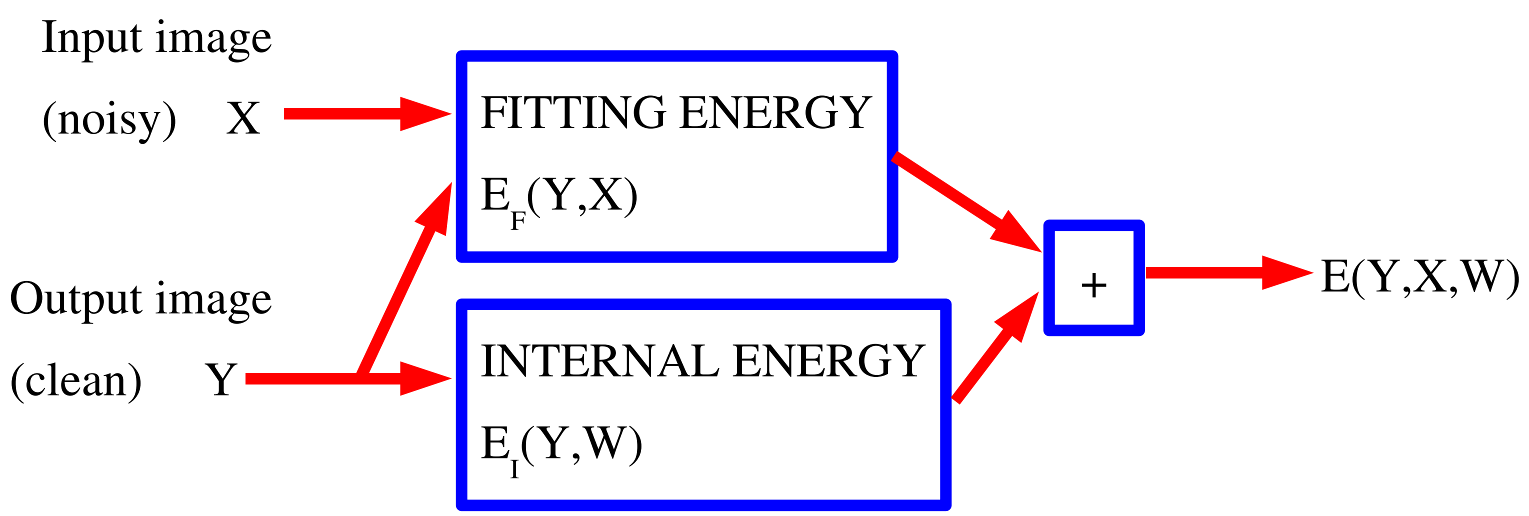
$$\bar{Y} \leftarrow \bar{Y} - \eta \frac{\delta E(\bar{Y})}{\delta Y} + \epsilon$$



Example: Conditional Product of Experts

[Ning, Delhome, LeCun, Piano, Bottou, Barbano: "Toward Automatic Phenotyping of Developing Embryos from Videos" *IEEE Trans. Image Processing*, September 2005]

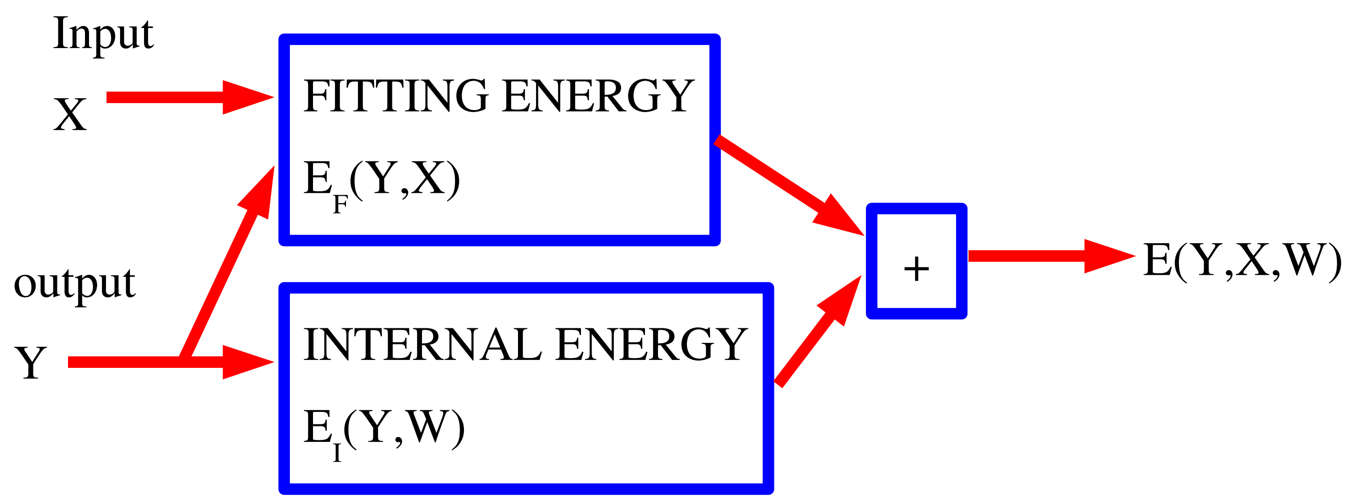
Using Energy-Based Models for image "cleanup" (segmentation, denoising,.....)



$$E(Y, X, W) = E_F(Y, X) + E_I(Y, W)$$

MAP Inference: clamp X and find a Y that minimizes E(Y,X,W)

Conditional PoE: Contrastive Divergence Training

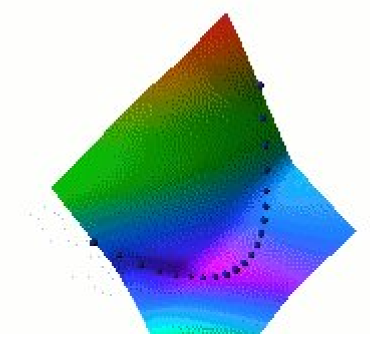


$$L_{\text{nll}}(Y^i, X^i, W) = E(Y^i, X^i, W) + \frac{1}{\beta} \log \left[\sum_y \exp(-\beta E(y, X^i, W)) \right]$$

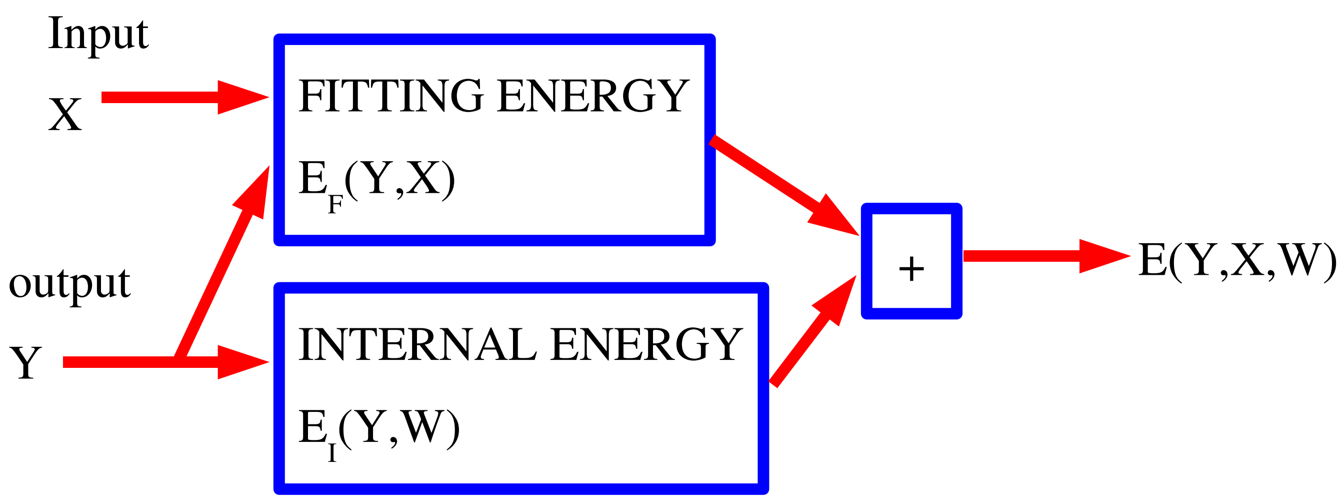
The negative log-likelihood loss has an intractable sum over all possible configurations of Y

Hinton's method:

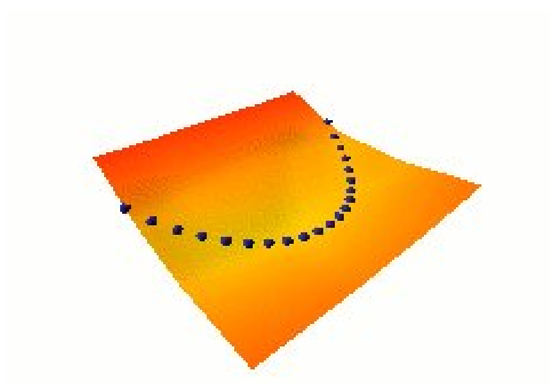
- use MCMC to approximate the derivative of the log partition function
- realize it takes too long. Get bored waiting.
- decide to cut the number of iterations of MCMC.
- realize that it's a sensible thing to do, and call is Contrastive Divergence
- come up with a complicated justification for it.

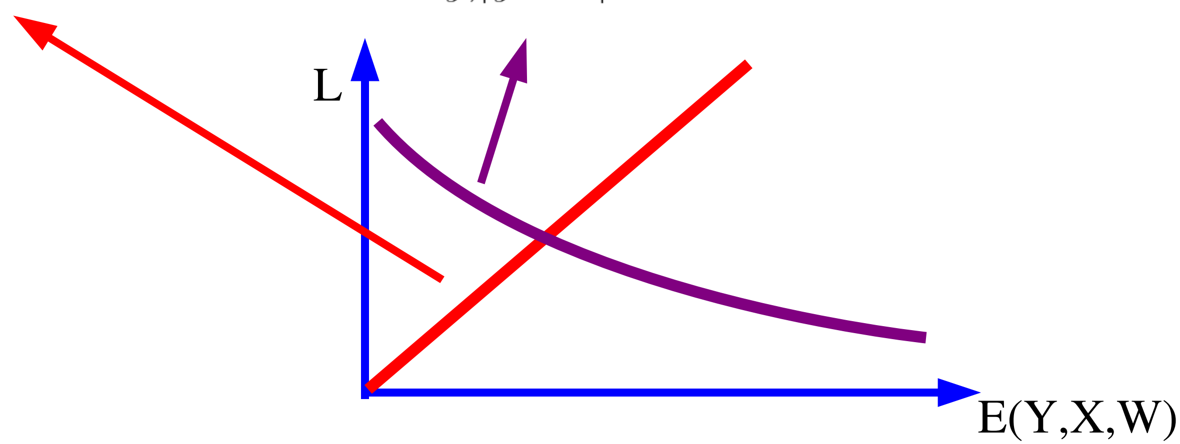


Conditional PoE: Training with the Linear-Exponential Loss



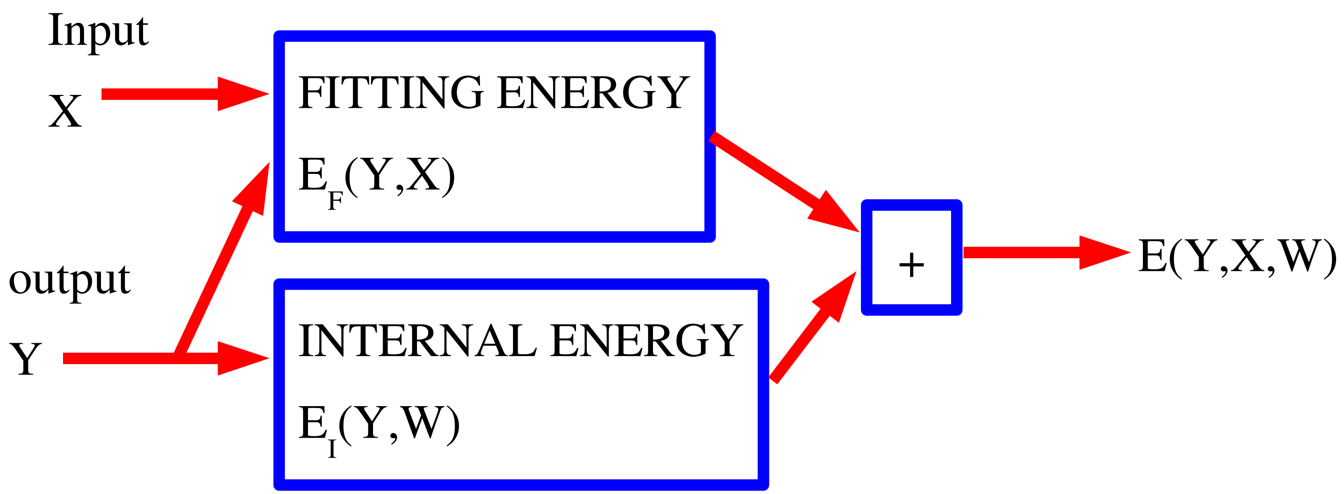
$$L(Y^i, X^i, W) = E(Y^i, X^i, W) + c.\exp(-\beta \min_{y, |y - Y^i| > \delta} E(y, X^i, W))$$





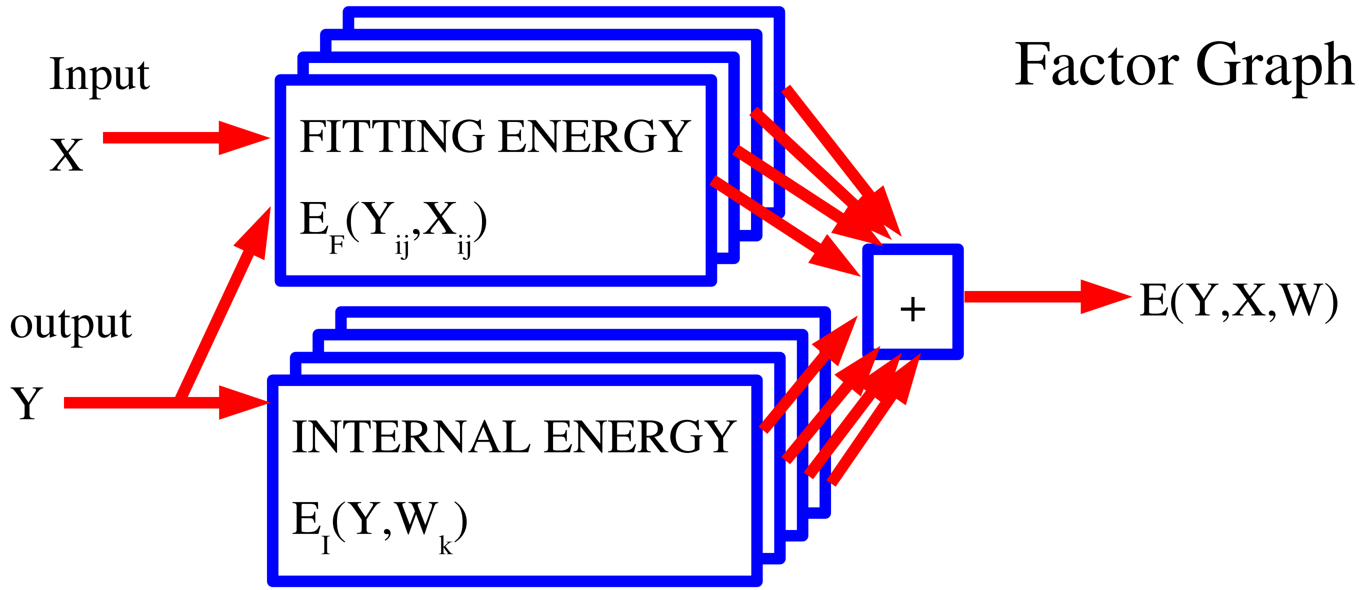
Energy-based loss: make the energy of the desired answer low, and make the energy of the most offending undesired answer high (forget about likelihoods altogether)

Conditional Product of Experts: Training



$$L(Y^i, X^i, W) = E(Y^i, X^i, W) - c.\exp(-\beta \min_{y, |y - Y^i| > \delta} E(y, X^i, W))$$

Conditional Product of Experts



$$E(Y,X,W) = \sum_{ij} E_F(Y_{ij}, X_{ij}) + \sum_k E_I(Y, W_k)$$

Fitting energy: summed over “sites” (e.g. pixels)

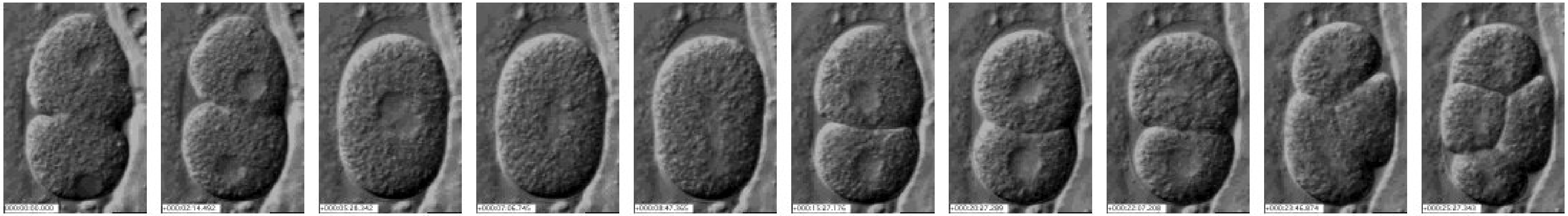
Internal energy: summed over “experts” (e.g. Features) and sites.

C. Elegans Embryo Phenotyping

[Ning, Delhome, LeCun, Piano, Bottou, Barbano

IEEE Trans. Image Processing, October 2005]

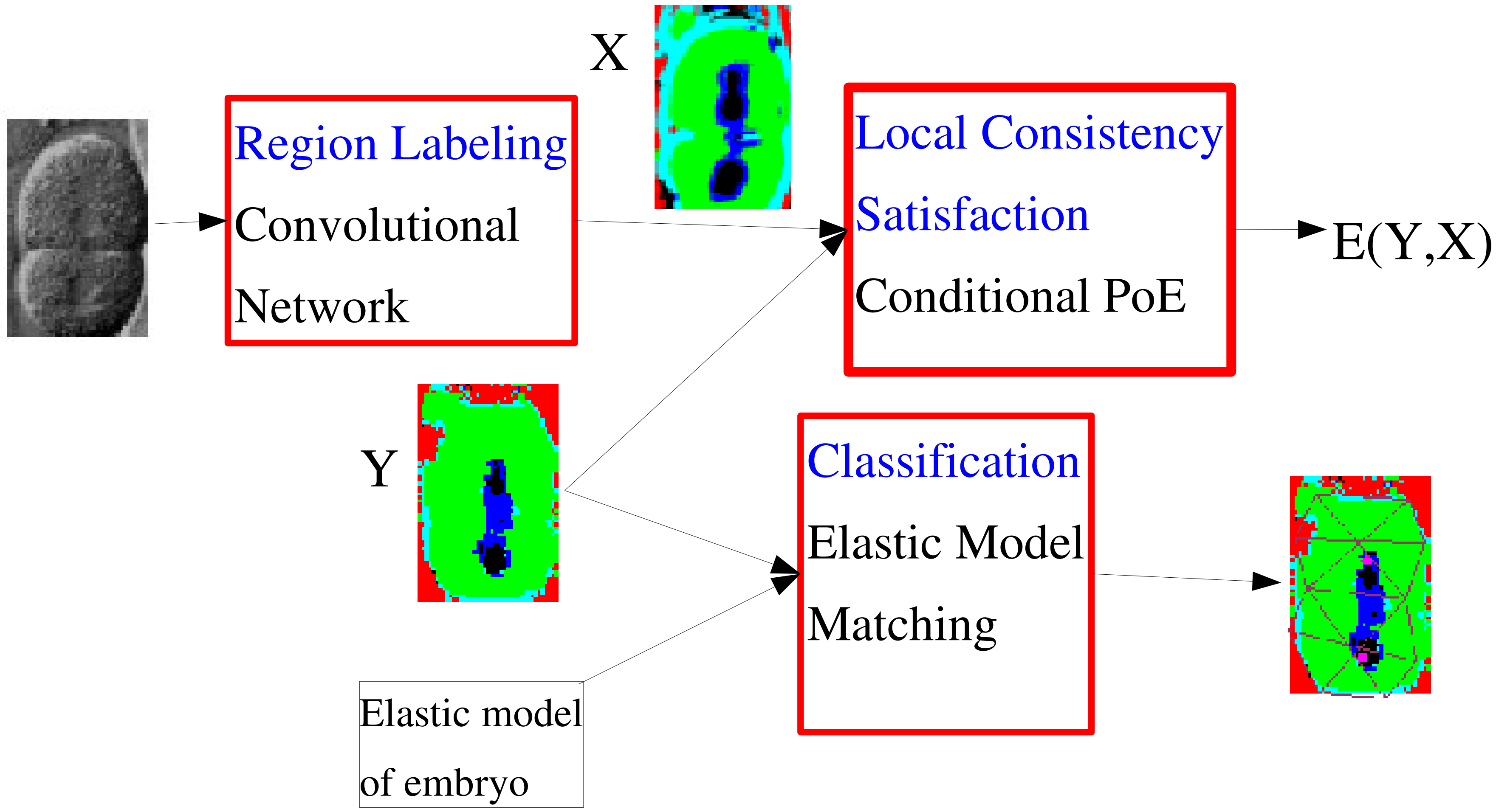
- **Analyzing results for Gene Knock-Out Experiments**
- **Automatically determining if a roundworm embryo is developing normally after a gene has been knocked out.**



Time-lapse movie

Architecture

- Region Classification with a convolutional network
- Local Consistency with a Conditional Product of Experts
- Embryo classification with elastic model matching



Region Labeling with a Convolutional Net

- Supervised training fromhand-labeled images
- 5 categories:
  - nucleus, nuclear membrane, cytoplasm, cell wall, external medium

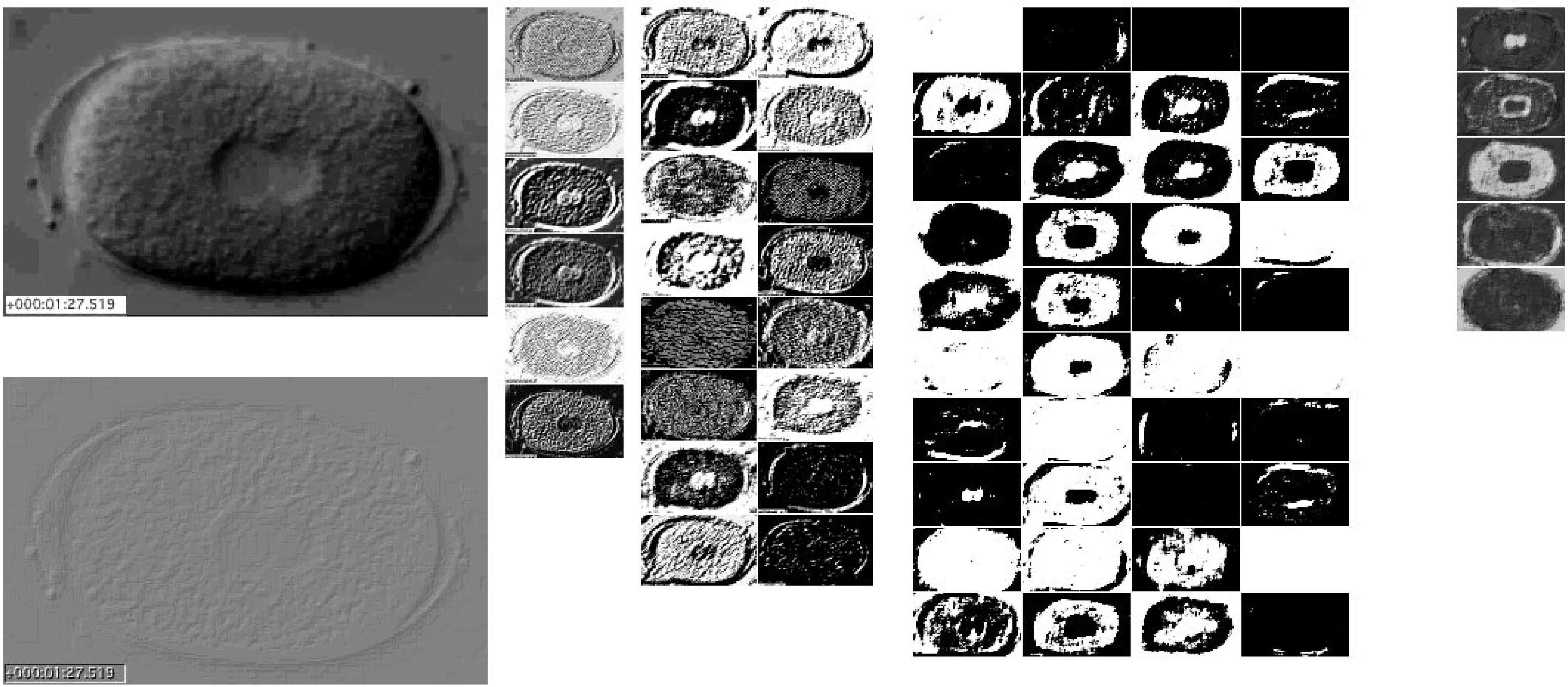
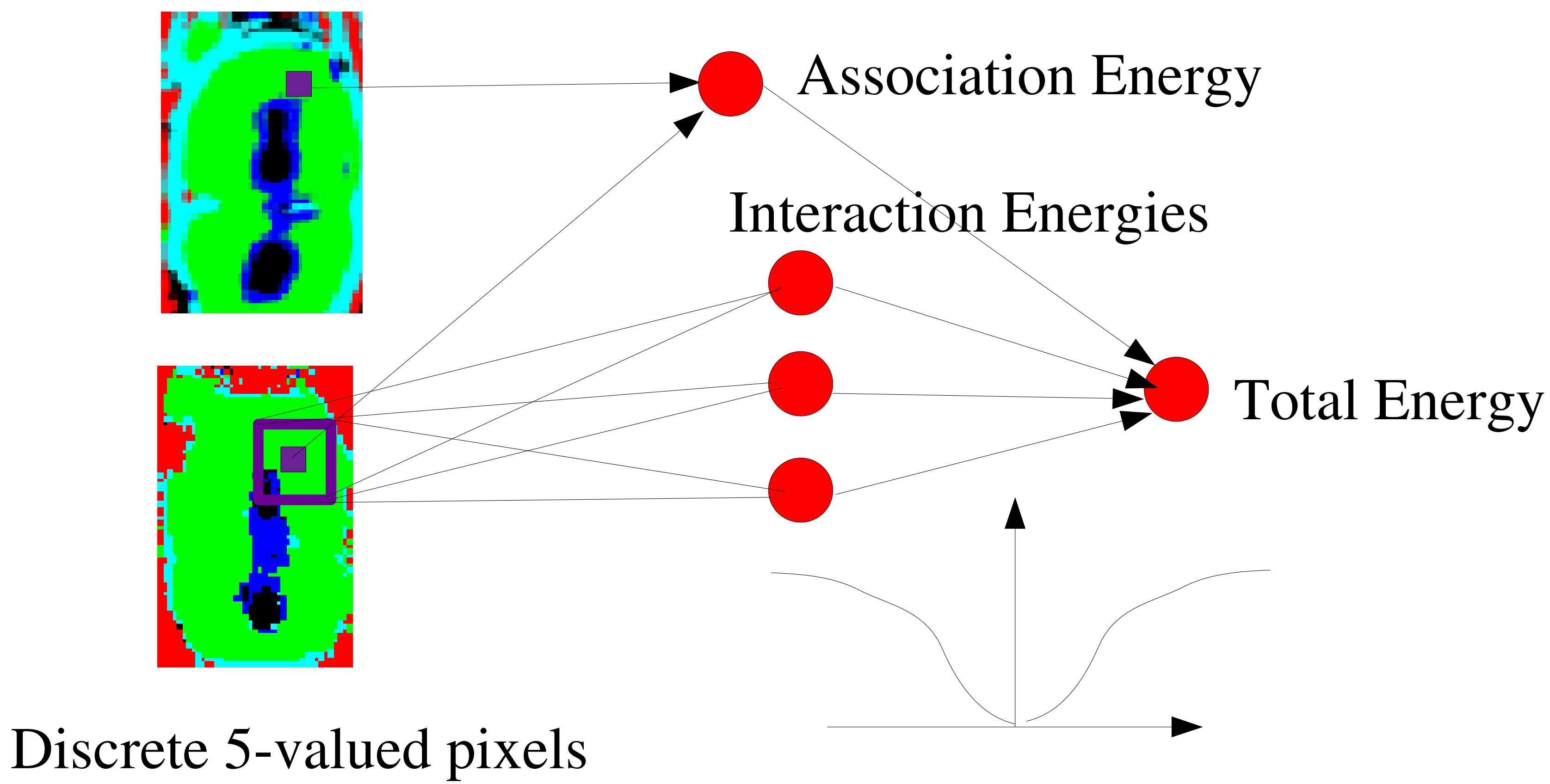


Image Segmentation with Local Consistency Constraints

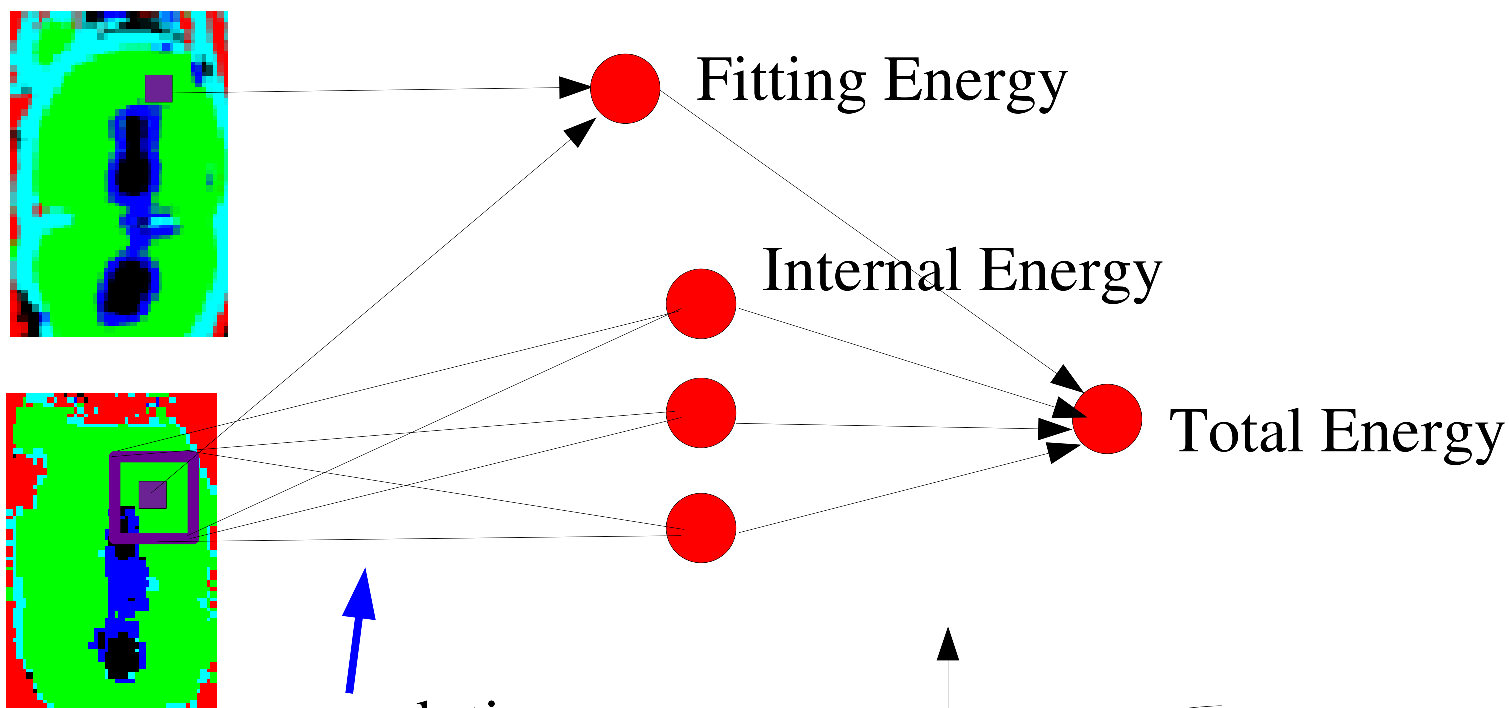
[Teh, Welling, Osindero, Hinton, 2001], [Kumar, Hebert 2003], [Zemel 2004]

- **Learn local consistency constraints with an Energy-Based Model so as to clean up images produced by the segmentor.**

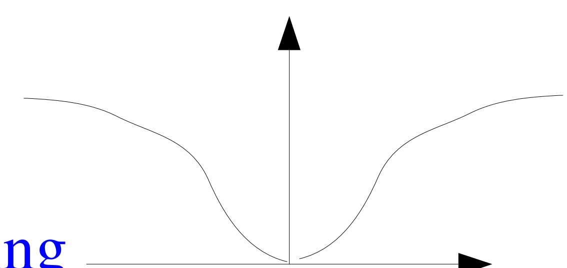


Convolutional Conditional PoE

$$E(Y,X,W) = \sum_{ij} C_{Y_{ij},X_{ij}} + \sum_{k=1}^{10} \sum_{ij} g\left(\sum_{lpq=(1,-2,-2)}^{(5,+2,+2)} W_{klpq}.Y_{l,i-p,j-q} \right)$$



Inference with Gibbs sampling



$$g(u) = \frac{u^2}{1+u^2}$$

C. Elegans Embryo Phenotyping

- **Analyzing results for Gene Knock-Out Experiments**

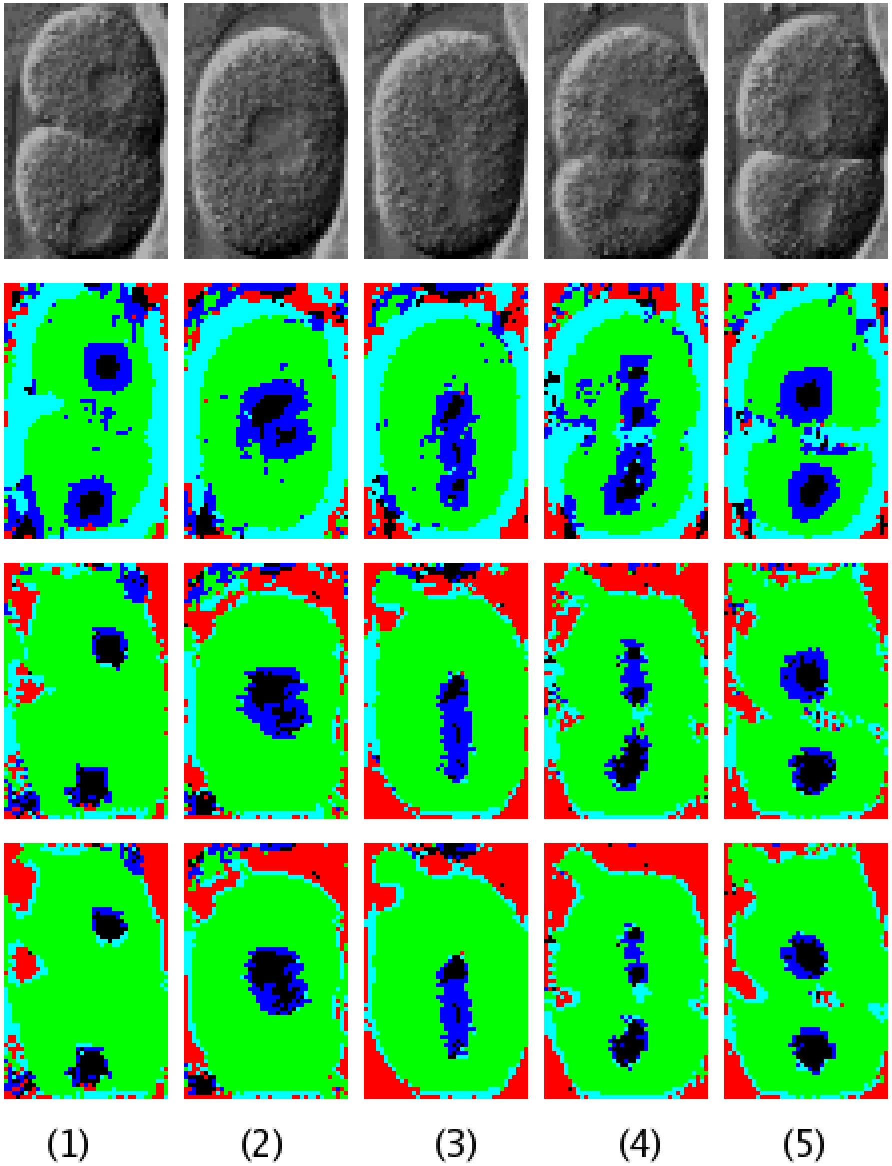
Original Images

Segmentation #1

Segmentation #2

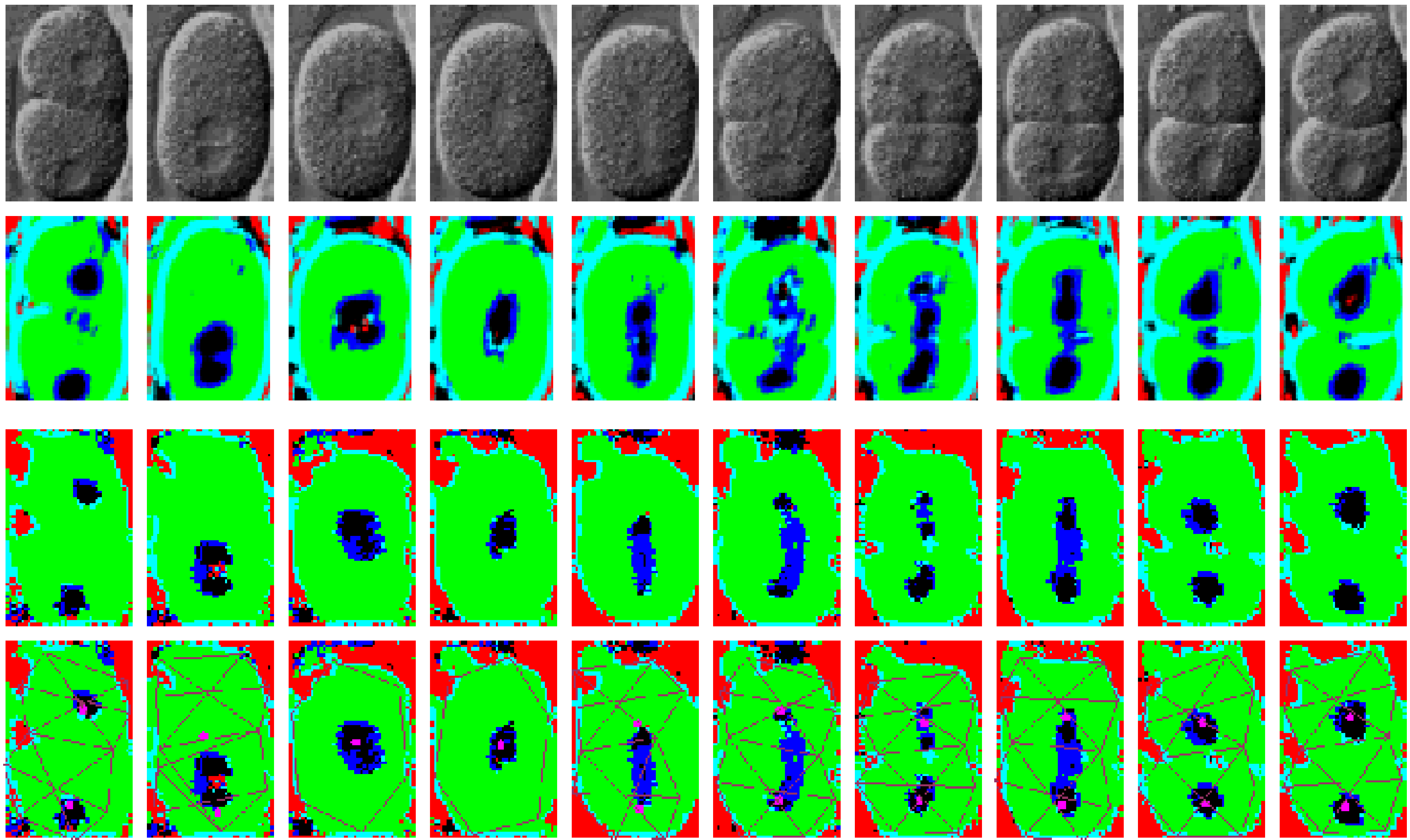
CCPoE

Cleanup



C. Elegans Embryo Phenotyping

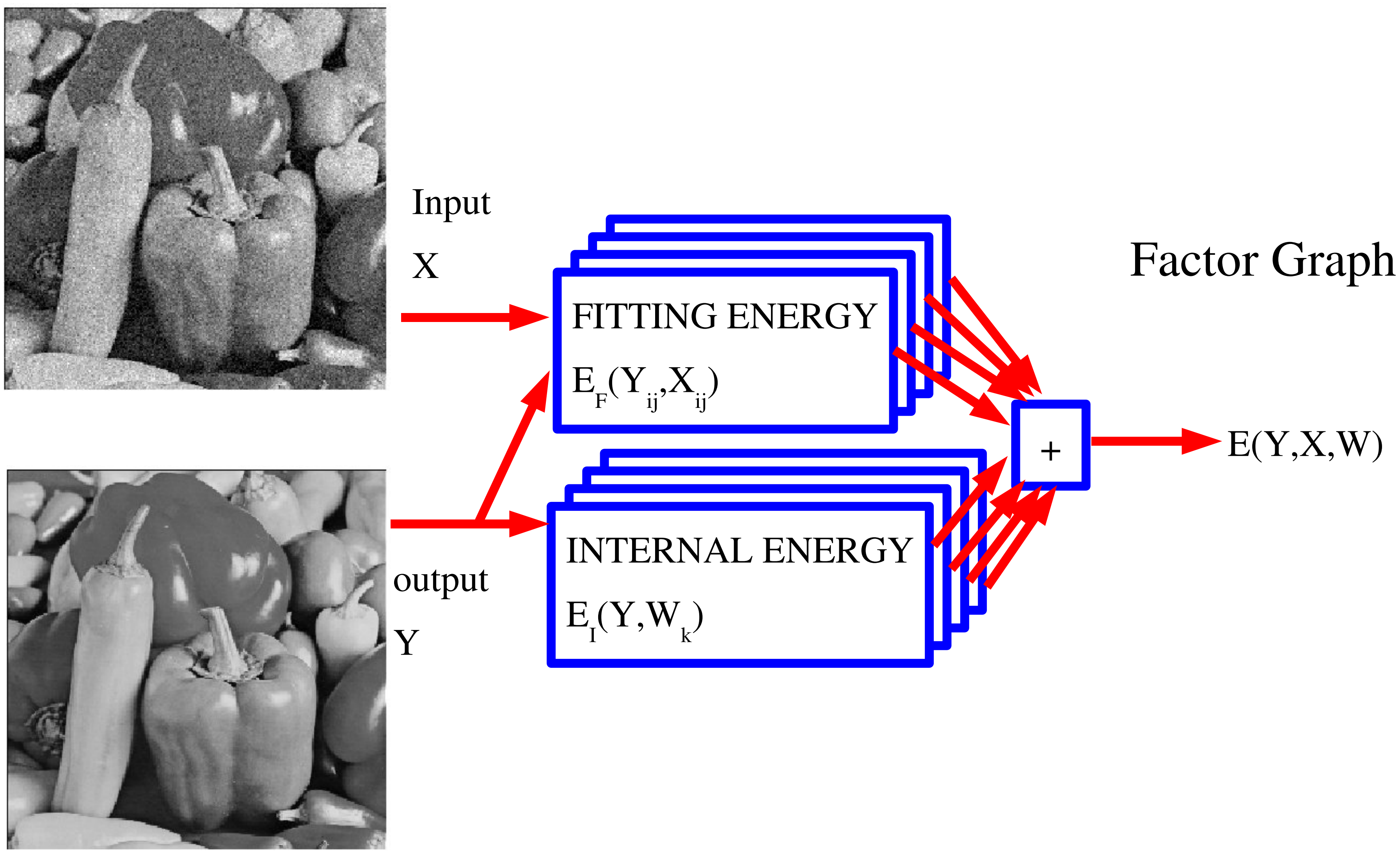
Analyzing results for Gene Knock-Out Experiments



(1) (2) (3) (4) (5) (6) (7) (8) (9) (10)

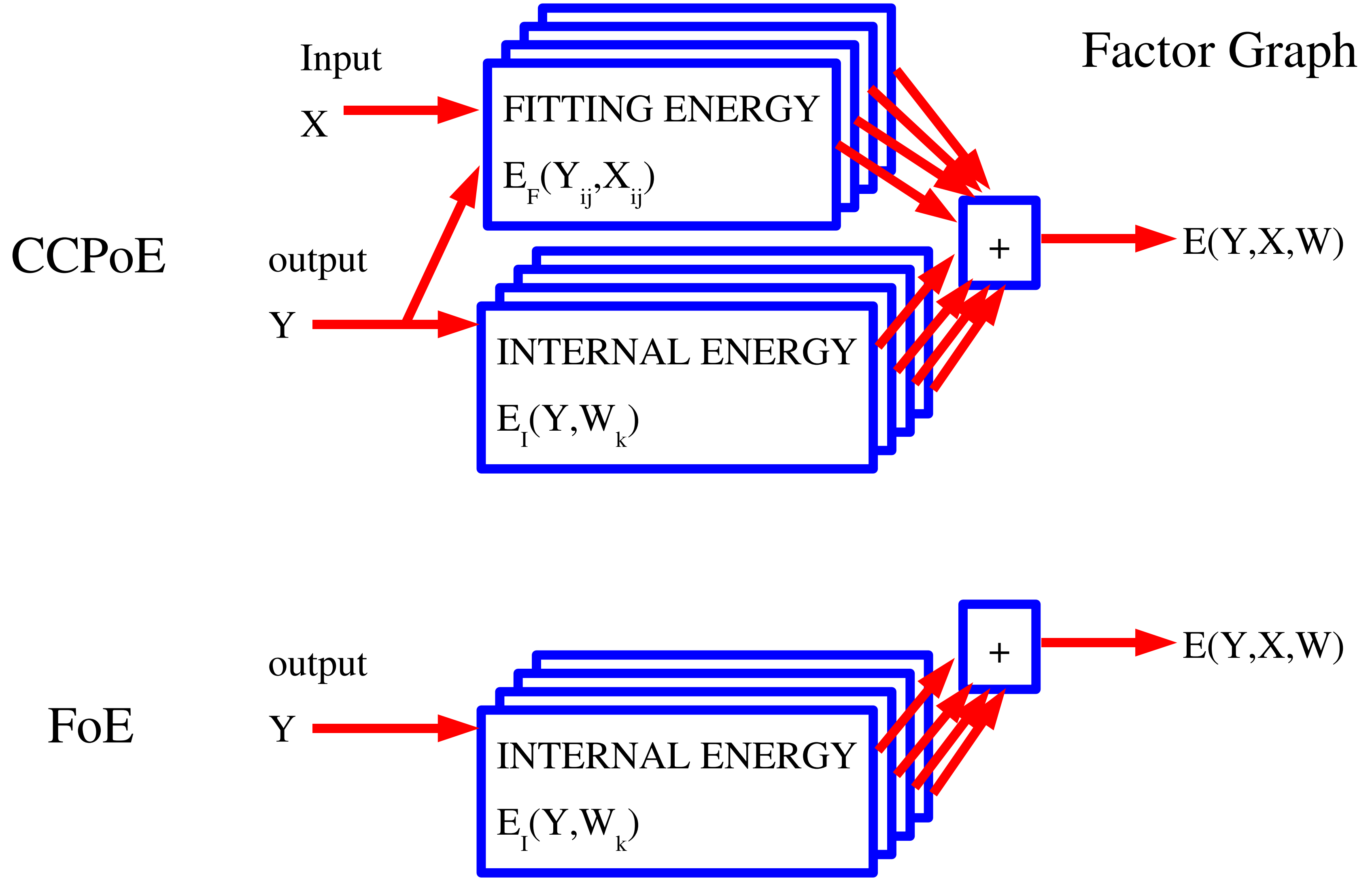
Convolutional Conditional PoE for Image Denoising

$$L(Y^i, X^i, W) = E(Y^i, X^i, W) + c.\exp(-\beta \min_{y,|y-Y^i|>\delta} E(y, X^i, W))$$

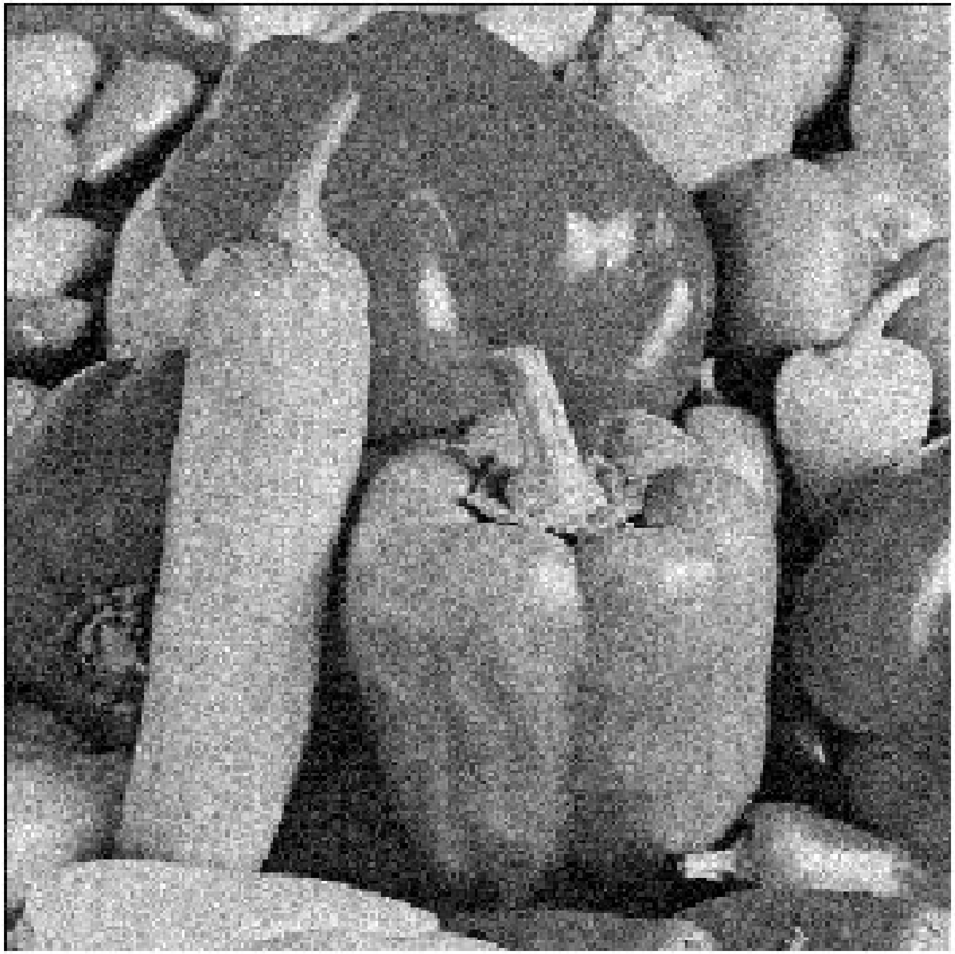


Convolutional Conditional PoE for Image Denoising

- Somewhat similar to the Field of Experts [Roth & Black CVPR 2005]



Convolutional Conditional PoE for Image Denoising



Noisy peppers PSNR=22.10

CCPoE PSNR=30.40

Convolutional Conditional PoE for Image Denoising

FoE PSNR=30.41

(Roth & Black report 30.58)

CCPoE PSNR=30.40

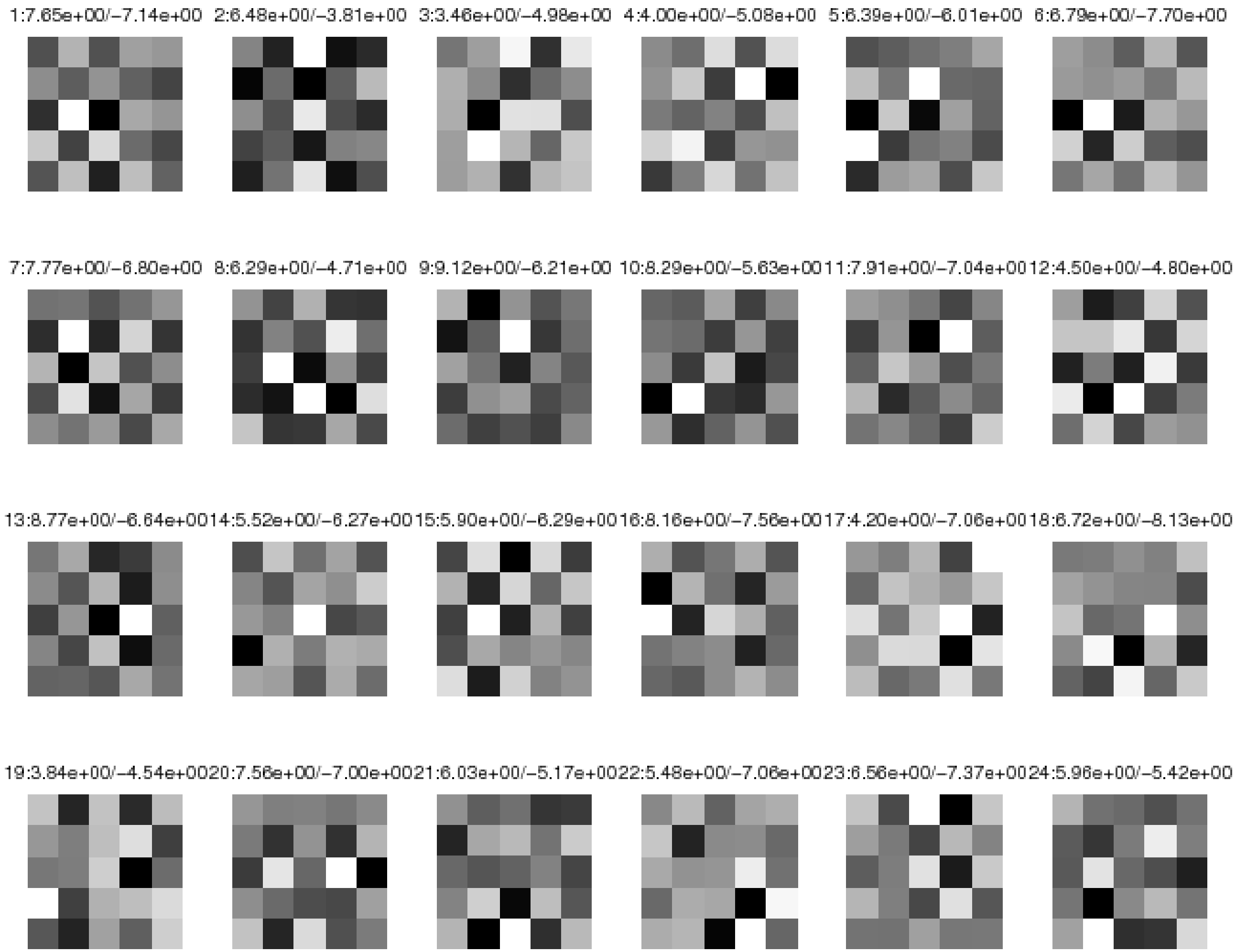
Convolutional Conditional PoE for Image Denoising

Random Kernels, PSNR= 29.70

(Gasp!)

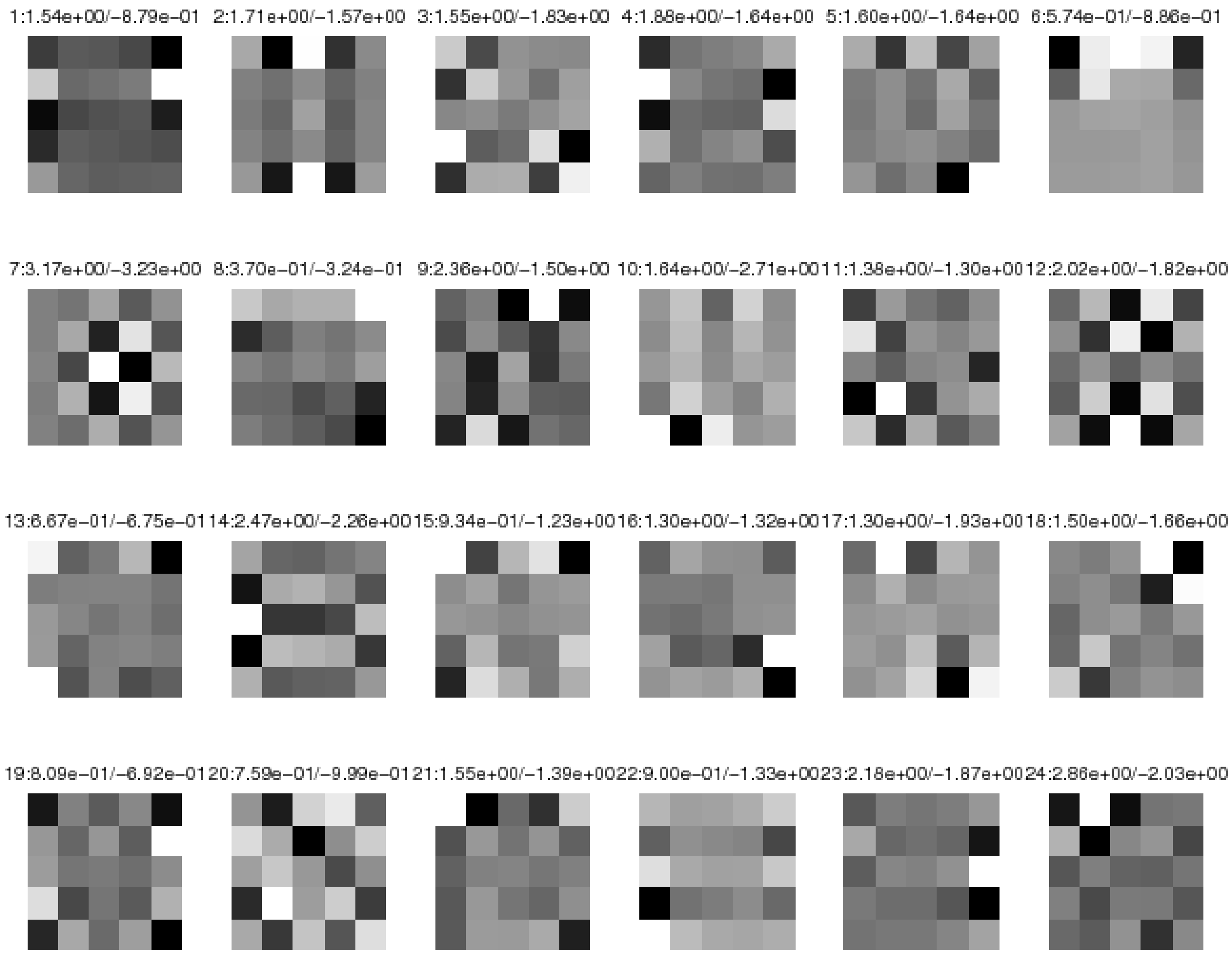
CCPoE PSNR=30.40

Convolutional Conditional PoE for Image Denoising



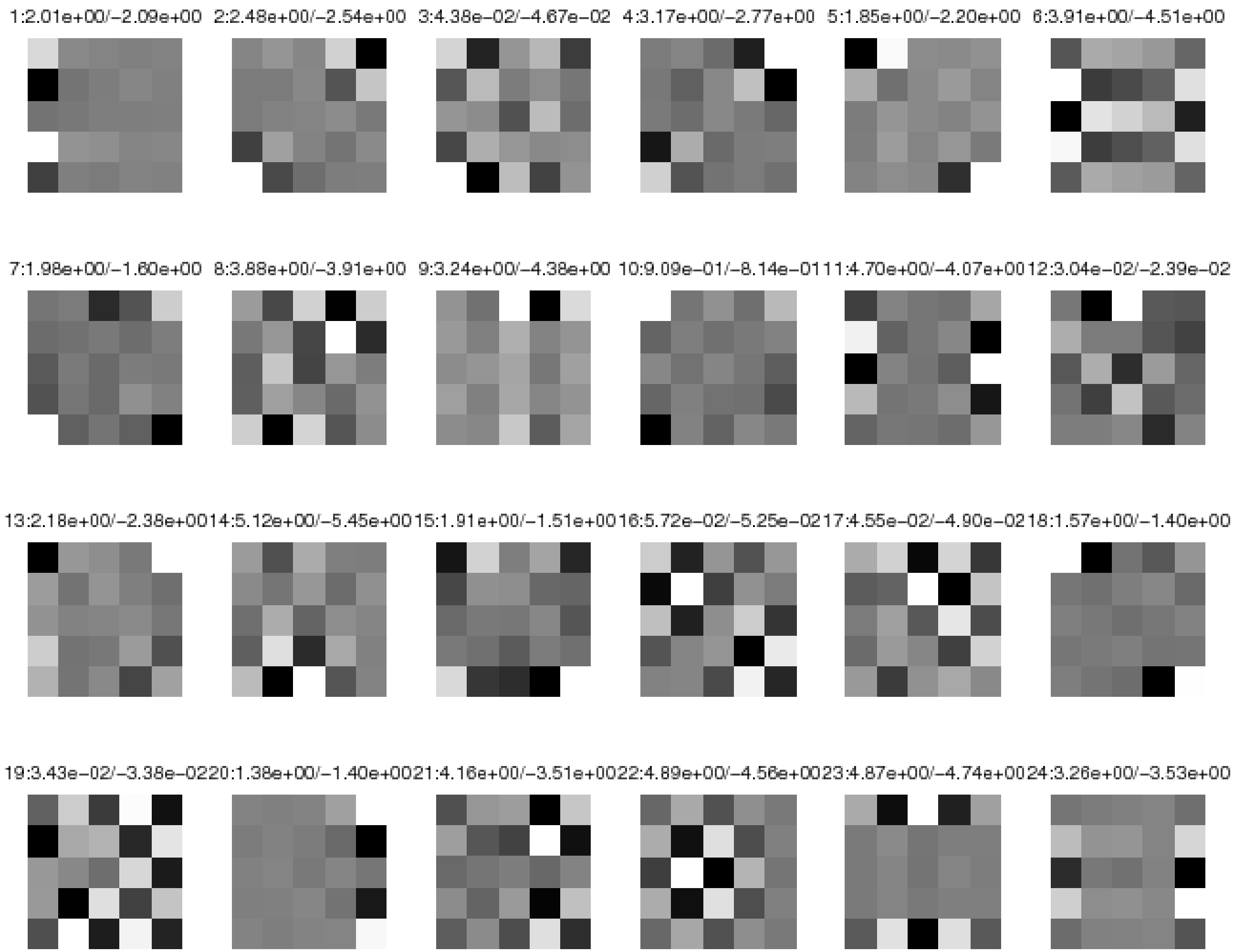
Random Kernels, PSNR= 29.70

Convolutional Conditional PoE for Image Denoising



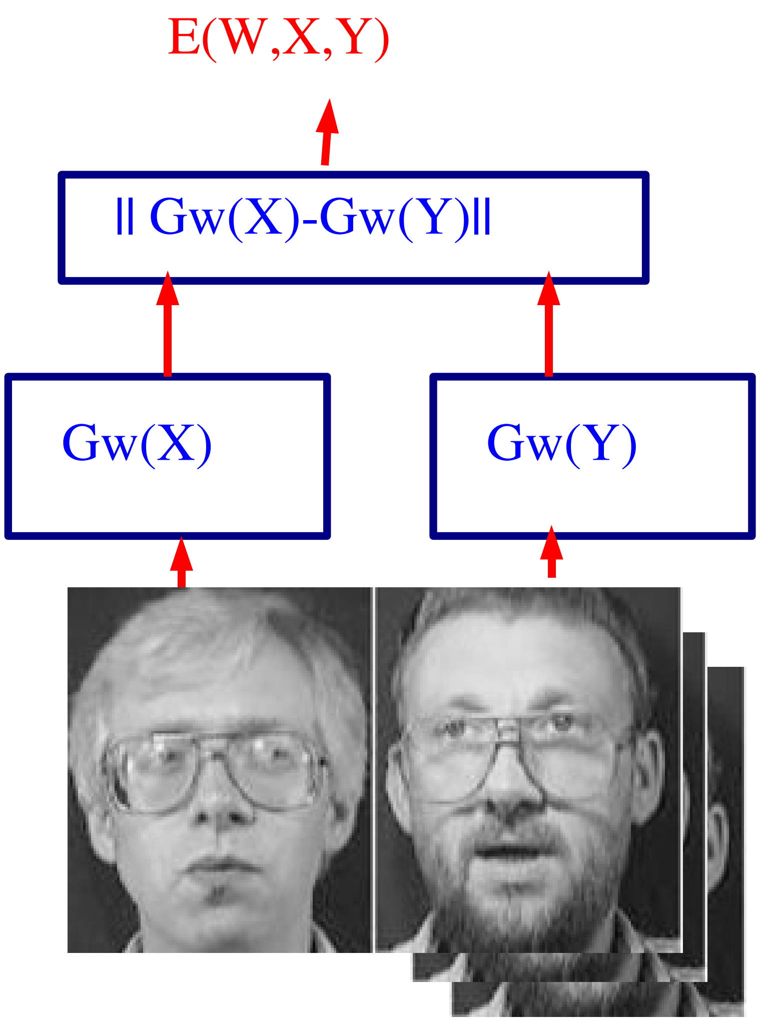
Roth & Black Kernels, PSNR= 30.58

Convolutional Conditional PoE for Image Denoising



CCPoE Kernels, PSNR= 30.40

EBM for Face Recognition

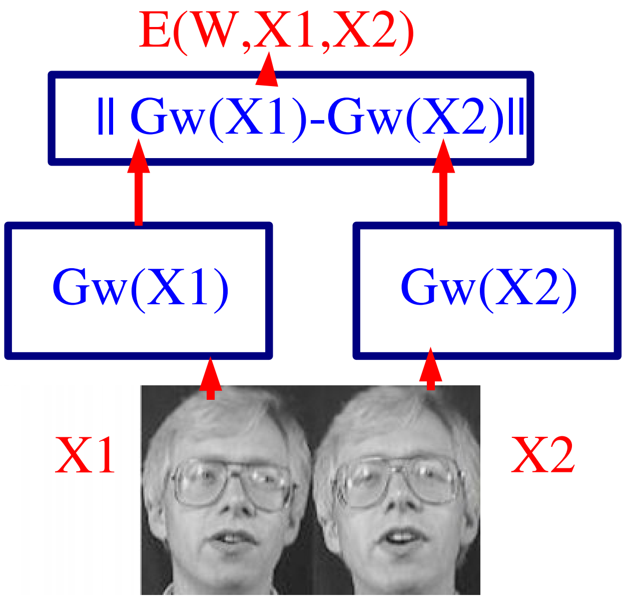


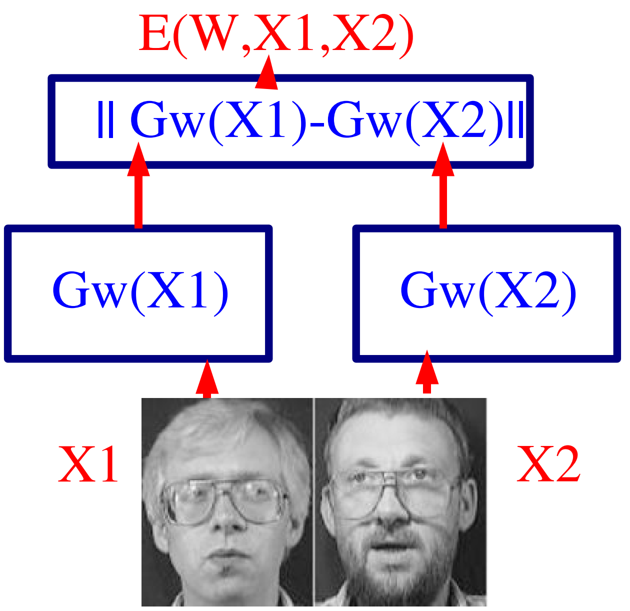
- **X and Y are images**
- **Y is a discrete variable with many possible values**
  - All the people in our gallery
- **Example of architecture:**
  - A function G(X) maps input images into a low-dimensional space in which the Euclidean distance measures dissemblance.
- **Inference:**
  - Find the Y in the gallery that minimizes the energy (find the Y that is most similar to X)
  - Minimization through exhaustive search.

Learning an Invariant Dissimilarity Metric with EBM

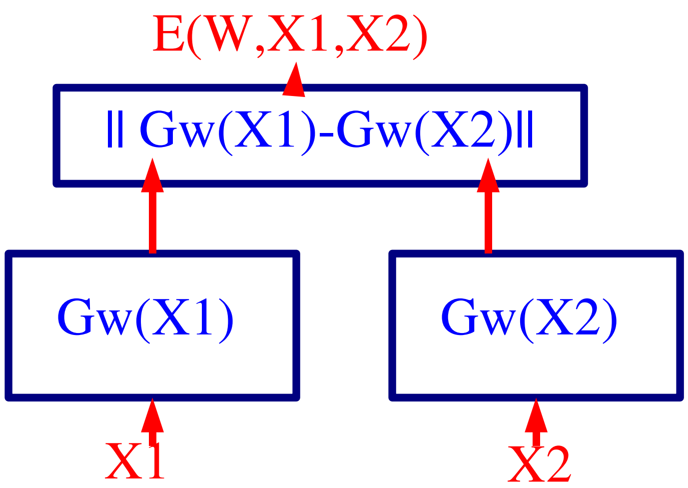
[Chopra, Hadsell, LeCun CVPR 2005]

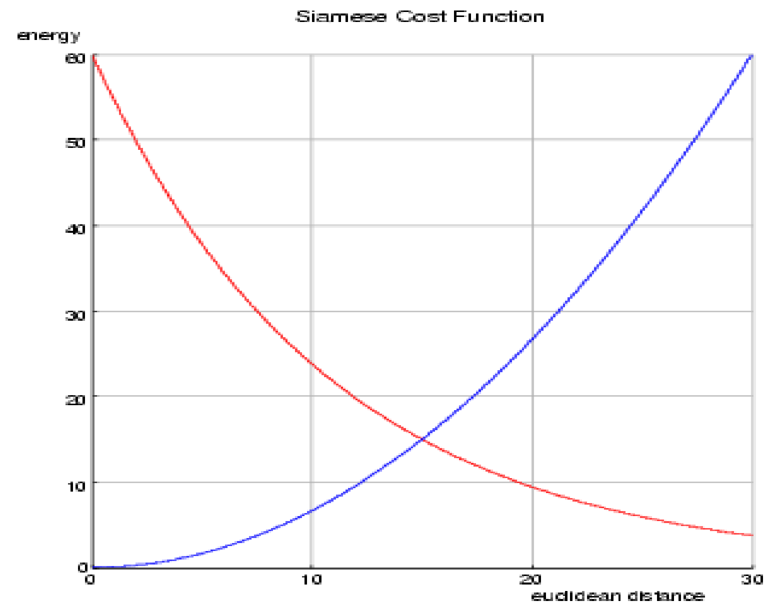
- Training a **parameterized, invariant dissimilarity metric** may be a solution to the **many-category problem**.
- Find a mapping **Gw(X)** such that the Euclidean distance **||Gw(X1)- Gw(X2)||** reflects the “semantic” distance between X1 and X2.
- Once trained, a trainable dissimilarity metric can be used to classify **new categories using a very small number of training samples** (used as prototypes).
- This is an example where probabilistic models are too constraining, because we would have to limit ourselves to models that can be normalized over the space of input pairs.
- With EBMs, we can put what we want in the box (e.g. A convolutional net).
- **Siamese Architecture**
- **Application:** face verification/recognition





Loss Function





- **Siamese models:** distance between the outputs of two identical copies of a model.
- **Energy function**: E(W,X1,X2) = ||Gw(X1)-Gw(X2)||
- If X1 and X2 are from the **same category (genuine pair)**, train the two copies of the model to produce **similar outputs (low energy)**
- If X1 and X2 are from **different categories (impostor pair)**, train the two copies of the model to produce **different outputs (high energy)**
- **Loss function: increasing function of genuine pair energy, decreasing function of impostor pair energy.**

Loss Function

- **Our Loss function for a single training pair (X1,X2)**:

$$L(W, X_1, X_2) = (1-Y) L_G(E_W(X_1, X_2)) + Y L_I(E_W(X_1, X_2))$$

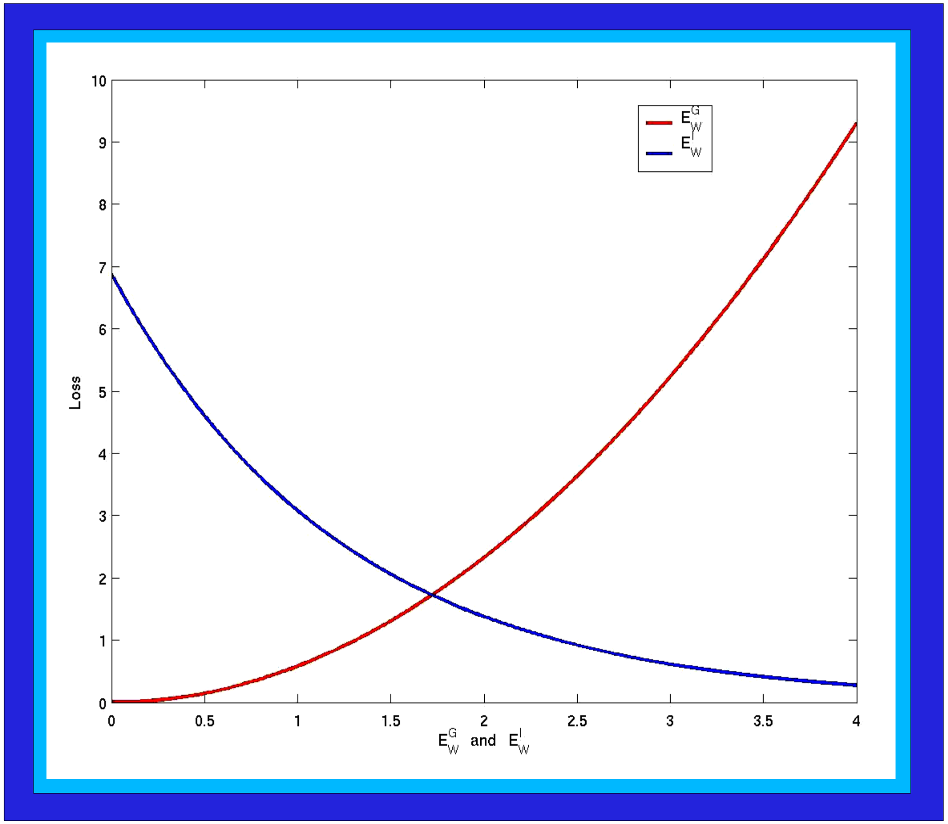
$$= (1-Y)\frac{2}{R}(E_W(X_1, X_2)^2) + (Y)\, 2R\, e^{-2.77\frac{E_W(X_1, X_2)}{R}}$$

$$E_W(X_1, X_2) = \| G_W(X_1) - G_W(X_2) \|_{L1}$$

And R is the largest possible value of

$$E_W(X_1, X_2)$$

Y=0 for a genuine pair, and Y=1 for an impostor pair.



Face Verification datasets: AT&T, FERET, and AR/Purdue

- The AT&T/ORL dataset
- Total subjects: **40**. Images per subject: **10**. Total images: **400**.
- Images had a moderate degree of variation in pose, lighting, expression and head position.
- Images from **35** subjects were used for training. Images from **5** remaining subjects for testing.
- Training set was taken from: **3500** genuine and **119000** impostor pairs.
- Test set was taken from: **500** genuine and **2000** impostor pairs.
- http://www.uk.research.att.com/facedatabase.html

AT&T/ORL Dataset

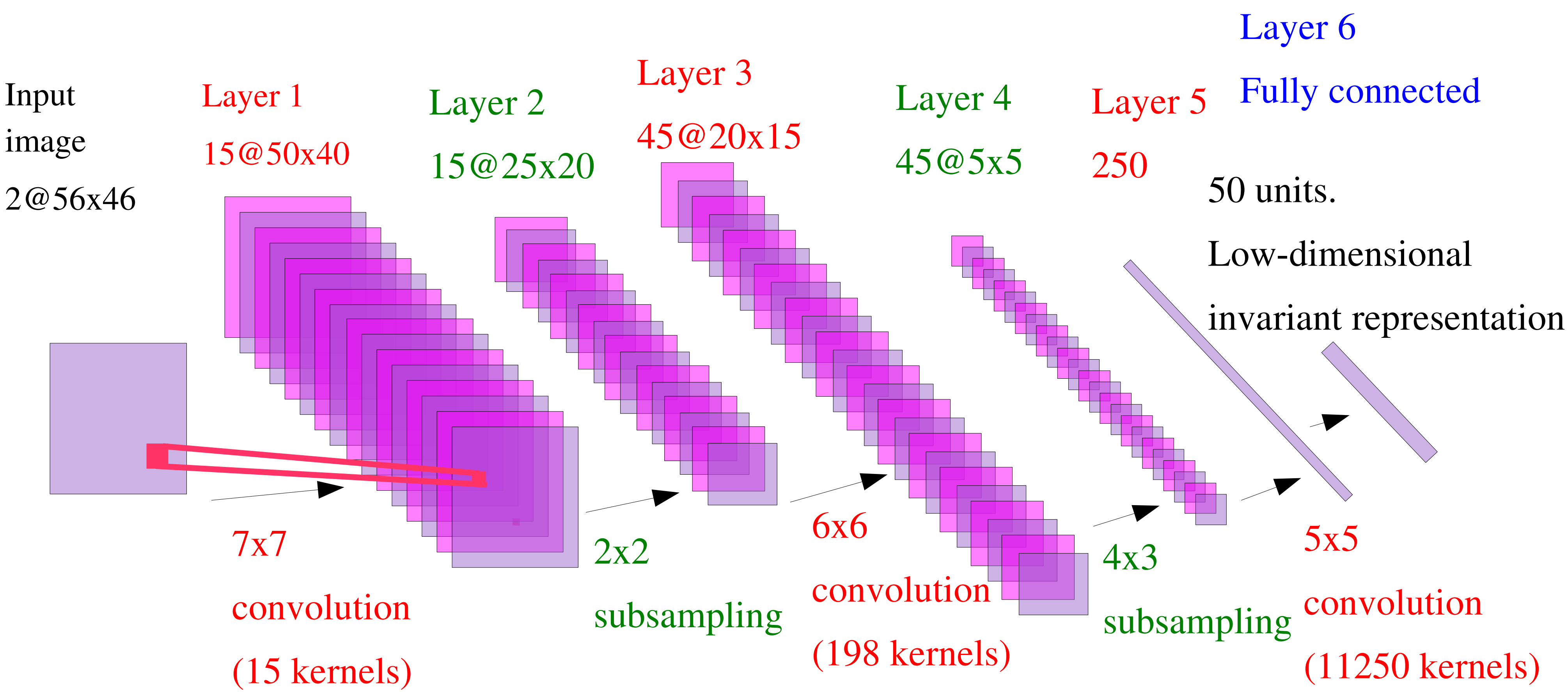
Face Verification datasets: AT&T, FERET, and AR/Purdue

- The FERET dataset. part of the dataset was used only for training.
- Total subjects: **96**. Images per subject: **6**. Total images: **1122**.
- Images had high degree of variation in pose, lighting, expression and head position.
- The images were used for training only.
- http://www.itl.nist.gov/iad/humanid/feret/

FERET Dataset

Architecture for the Mapping Function Gw(X)

Convolutional net



Internal state for genuine and impostor pairs

Dataset for Verification

- tested on AT&T and AR/Purdue
- AT&T dataset

Number of subjects:	5
Images/subject:	10
Images/Model:	5
Total test size:	5000
Number of Genuine:	500
Number of Impostors:	4500

- Purdue/AR dataset

Number of subjects:	40
Images/subject:	26
Images/Model:	13
Total test size:	5000
Number of Genuine:	500
Number of Impostors:	4500

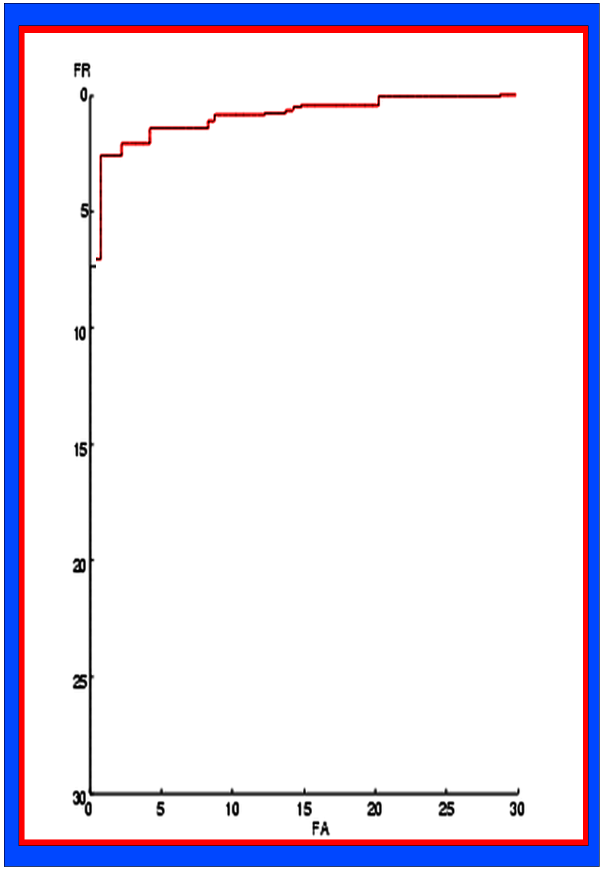
Verification Results

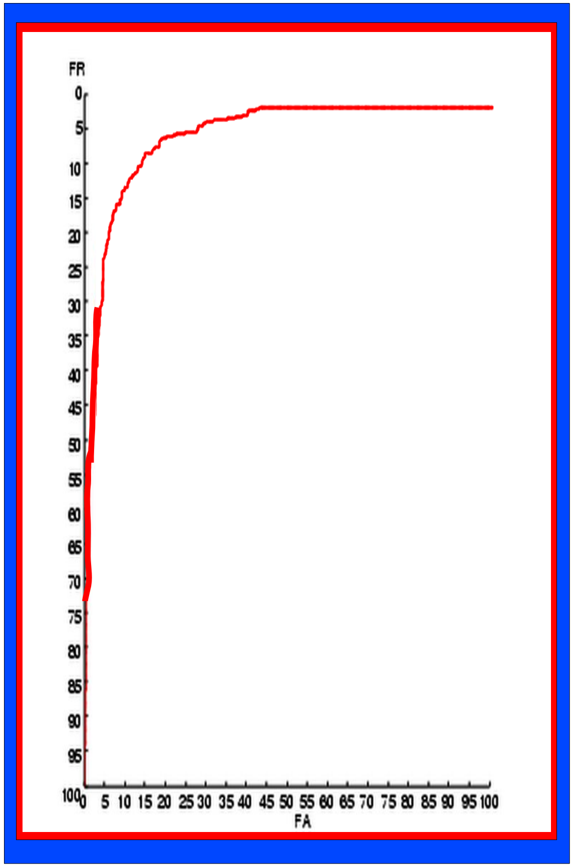
- The AT&T dataset

False Accept	False Reject
10.00%	0.00%
7.50%	1.00%
5.00%	1.00%

- The AR/Purdue dataset

False Accept	False Reject
10.00%	11.00%
7.50%	14.60%
5.00%	19.00%





Classification Examples

Example: Correctly classified genuine pairs

energy: 0.3159

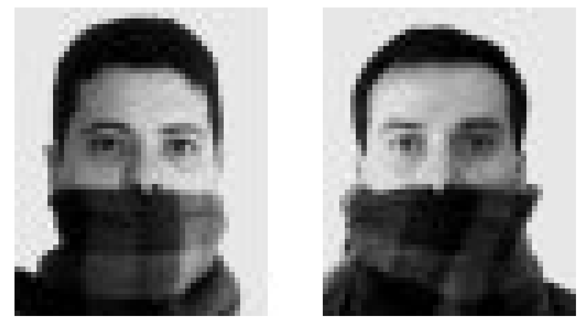
energy: 0.0043

energy: 0.0046

Example: Correctly classified impostor pairs

energy: 20.1259

energy: 32.7897



energy: 5.7186

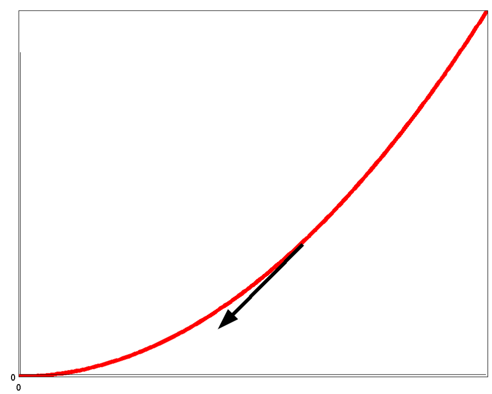
Example: Mis-classified pairs

energy: 10.3209

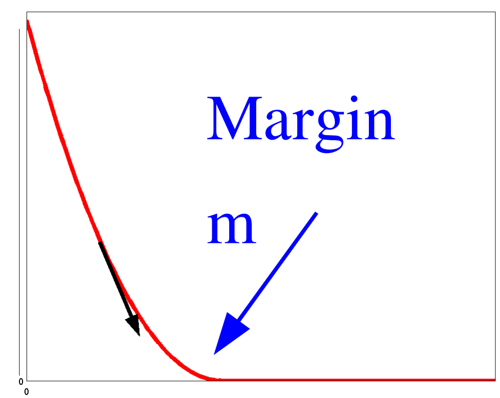
energy: 2.8243

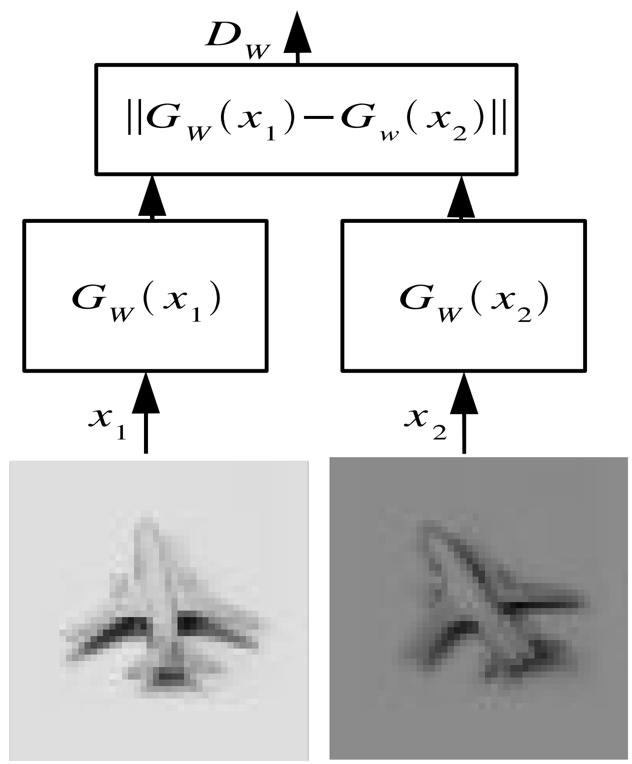
A similar idea for Learning a Manifold with Invariance Properties

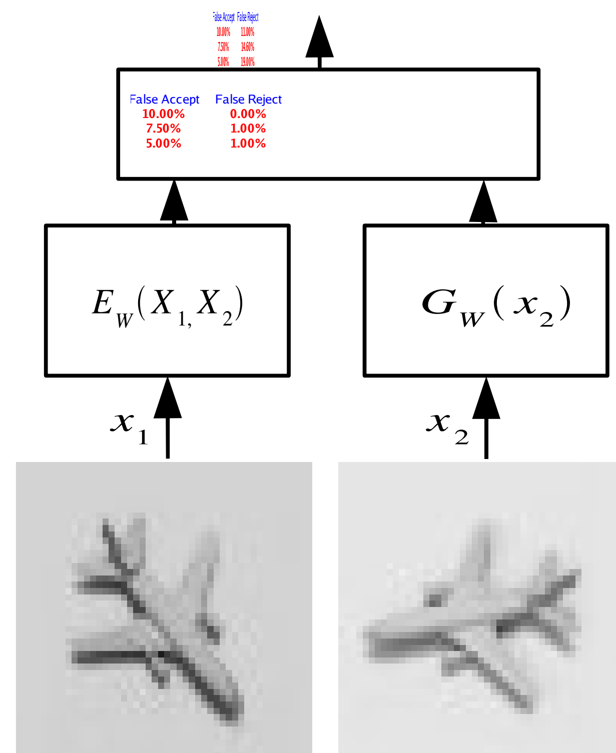
$$L_{similar} = \frac{1}{2} D_W^2$$



$$L_{dissimilar} = \frac{1}{2} \{ max(0, m - D_W) \}^2$$

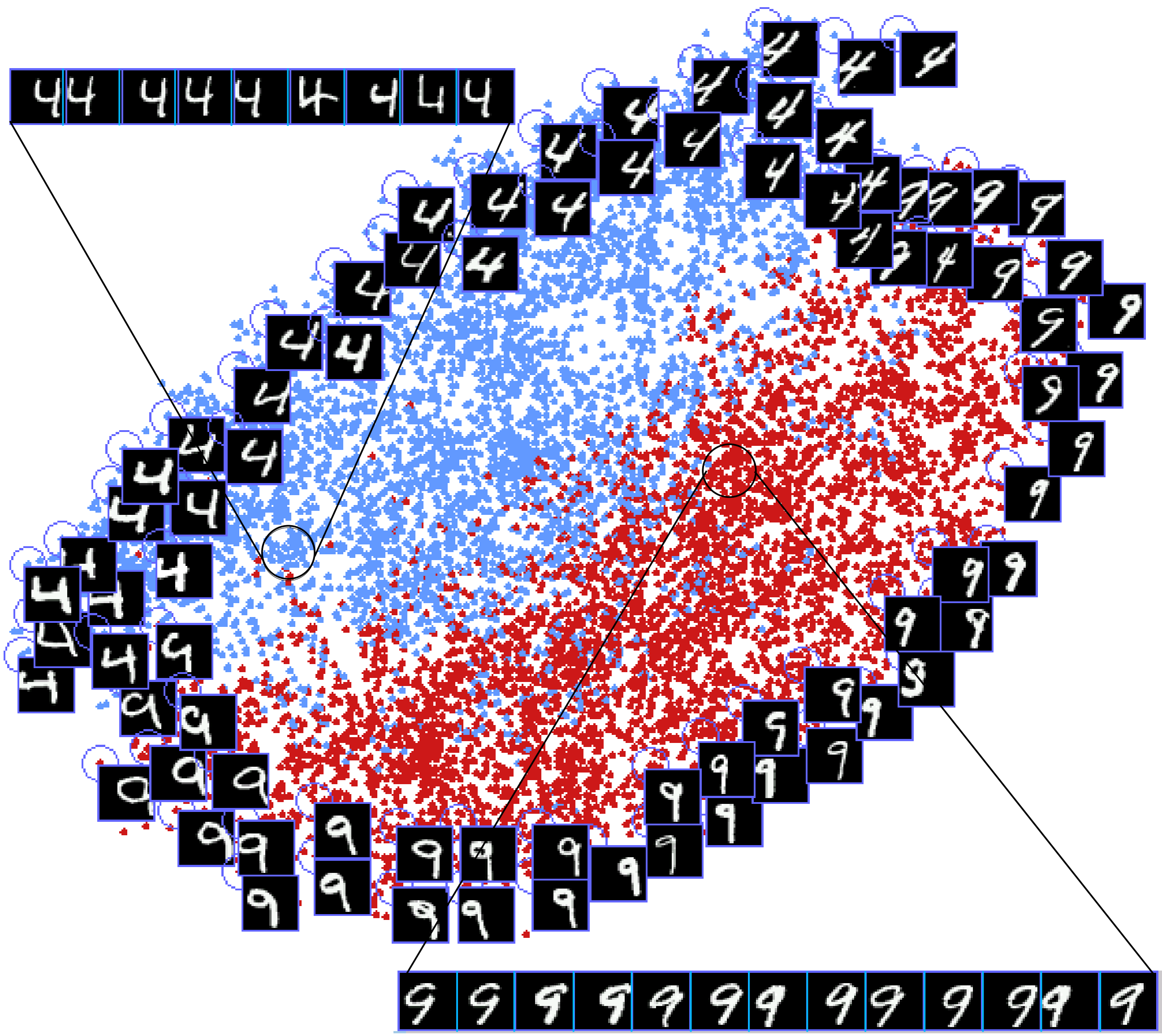






- **Loss function:**
  - Pay quadratically for making outputs of neighbors far apart
  - Pay quadratically for making outputs of non-neighbors smaller than a **margin** m

A Manifold with Invariance to Shifts



- Training set: 3000 “4” and 3000 “9” from MNIST. **Each digit is shifted horizontally by -6, -3, 3, and 6 pixels**
- Neighborhood graph: 5 nearest neighbors in Euclidean distance, **and shifted versions of self and nearest neighbors**
- Output Dimension: 2
- Test set (shown) 1000 “4” and 1000 “9”

Automatic Discovery of the Viewpoint Manifold with Invariant to Illumination