Regularization of Neural Networks using DropConnect

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Introduction

- Neural Networks are good at classifying large labeled datasets
- Large capacity is essential: more layers and more units
- But without regularization, model with millions or billions of parameters can easily overfit
- Existing regularization methods:
  - ℓ_1 or ℓ_2 penalty
  - Bayesian methods
  - Early stopping of training
  - DropOut network [Hinton et al. 2012]

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- Existing regularization methods:
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    - Bayesian methods
    - Early stopping of training
    - DropOut network [Hinton et al. 2012]
- We introduce a new form of regularization: DropConnect

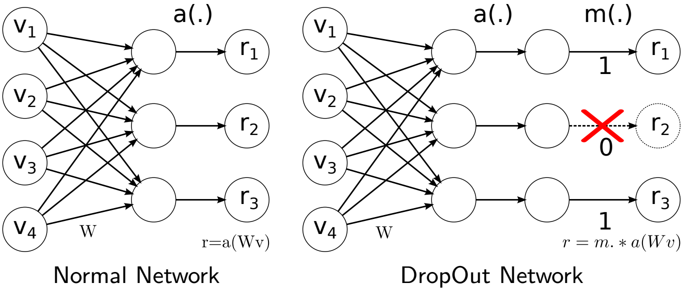
Overview

Review of DropOut Network [Hinton et al. 2012]

- Stochastic dropping of units
- Each element of a layer's output is kept with probability p, otherwise being set to 0 with probability $(1 - p)$
- Input v, weights W, activation function $a(.)$, output r and DropOut mask m:

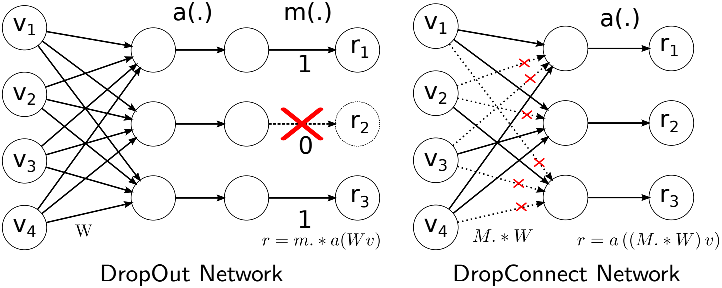
$$r = m \,.\!*\, a(Wv)$$

- For *every* training example at *every* epoch has different mask m



DropConnect Network

- Applies only to fully-connected layers
- Randomly drop *connections* in network, with probability $1 - p$
- Generalization of Dropout: $r = a((M .* W)v)$ (for $a(0) = 0$, e.g. *relu*)



DropConnect Network: Training and Inference

Training

- For *every* training example at *every* epoch has different binary mask matrix M
- Backward-prop gradient uses the same M as forward-prop, for each example
- Use SGD with mini-batch
- Efficient implementation requires care

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Inference

- Exact solution is intractable
- Approximate neuron activation by Gaussian distribution

DropConnect Network: Inference

DropConnect Network Inference

Exact solution requires sum over all possible $2^{|M|}$ masks M:

$$r = \mathbf{E}_M[a((M .* W)v)] = \sum_M p(M)a((M .* W)v)$$

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Inference Approximation

A single neuron u_i before activation function: $u_i = \sum_j (W_{ij}v_j)M_{ij}$. Approximate u_i by a Gaussian distribution via moment matching.

$$u \sim \mathcal{N}(pWv, p(1-p)(W .* W)(v .* v))$$

where $1 - p$ is drop connect rate

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Inference Algorithm

- Compute layer response at test time: $r \approx \mathbf{E}_u[a(u)]$ by sampling or numerical integration
- Each neuron activation sampled independently, thus very efficient

Inference comparison: DropOut v.s. DropConnect

DropOut Network Inference[Hinton et al. 2012]

Approximate by changing the order of expectation and neuron activation:

$$\mathbf{E}_M[a((M .* W)v] \approx a(\mathbf{E}_M(M .* W)v) = a(pWv)$$

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Failure Example

$u \sim \mathcal{N}(0,1)$ with $a(u) = max(u,0)$. $a(\mathbf{E}_M(u)) = 0$ while $\mathbf{E}_u(a(u)) = 1/\sqrt{2\pi} \approx 0.4$

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DropConnect Network Inference

Approximate by Gaussian moment matching:

$$\mathbf{E}_M[a((M .* W)v] \approx \mathbf{E}_u\left[a(u)\right]$$

gives the right answer for the above example

Why DropConnect Regualize Network

DropConnect Network Model

Model averaging interpretation: a mixture model of $2^{|M|}$ classifiers

$$f(x;\theta) = \mathbf{E}_M\left[f_M(x;\theta,M)\right] = \sum_M p(M) f_M(x;\theta,M)$$

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Rademacher Complexity of Model

Let W_s be the weights of soft-max layer and W be the weights of DropConnect layer, where $max|W_s| \leq B_s$, $max|W| \leq B$. Define k is the number of classes, $\hat{R}_\ell(\mathcal{G})$ is the Rademacher complexity of the feature extractor (layers before DropConnect layer), n and d are the dimensionality of the input and output of the DropConnect layer respectively.

$$\hat{R}_\ell(\mathcal{F}) \leq p\left(2\sqrt{k}dB_s n\sqrt{d}B_h\right)\hat{R}_\ell(\mathcal{G})$$

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Special Cases of p

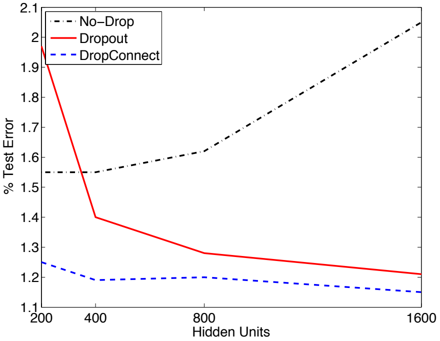
1. $p = 0$: the model complexity is zero, since the input has no influence on the output.
2. $p = 1$: it returns to the complexity of a standard model.
3. $p = 1/2$: all sub-models have equal preference.

Experiments overview

- DataSet: MNIST, CIFAR-10, and SVNH
- Kept Rate $p = 0.5$ for both DropOut and DropcConnect network (except where explicitly stated)
- Normal Network, DropOut Network and DropConnect Network have:
  - exactly the same architecture
  - exactly the same model/training parameters
  - exactly the same data augmentation algorithm
- We use:
  - Data Augmentation: Translation, Rotation and Scaling
  - Aggregating multiple models: 5 or more

Varying Size of Network

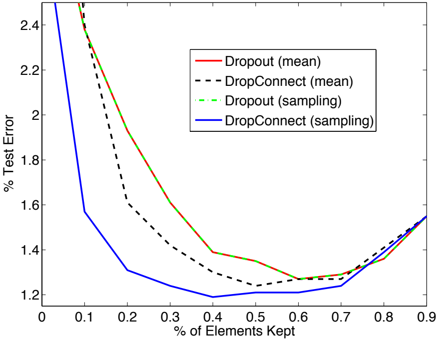
MNIST test error with two hidden layer network ($p = 0.5$)



- No-Drop Network overfits with a large number of neurons
- Both DropConnect and DropOut regularize model nicely
- DropConnect performance better than DropOut and No-Drop

Varying Kept Rate p

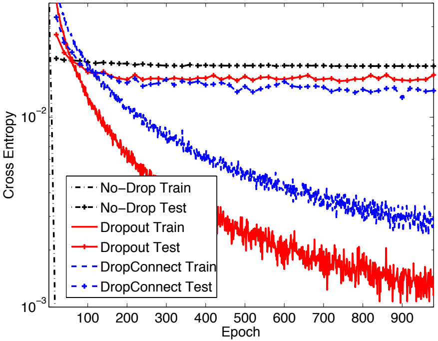
MNIST test error with two hidden layer network with 400 neurons each



- Optimal value of p is roughly at 0.5 for both DropOut and DropConnect
- DropConnect: Sampling inference works better than mean inference

Comparison of Convergence Rates

MNIST test error with two hidden layer network with 400 neurons each



- Cross Entropy, continuous measure of error
- DropConnect slower than DropOut but better test error

MNIST Results(1)

MNIST 784-800-800-10 network classification error rate without data augmentation:

neuron	model	error(%) 5 network	voting error(%)
relu	No-Drop	1.62 ± 0.037	1.40
	Dropout	1.28 ± 0.040	1.20
	DropConnect	1.20 ± 0.034	1.12
sigmoid	No-Drop	1.78 ± 0.037	1.74
	Dropout	1.38 ± 0.039	1.36
	DropConnect	1.55 ± 0.046	1.48
tanh	No-Drop	1.65 ± 0.026	1.49
	Dropout	1.58 ± 0.053	1.55
	DropConnect	1.36 ± 0.054	1.35

- *relu* neuron is always performs better than other neuron
- Error Rate: DropConnect $<$ DropOut $<$ No-Drop for *relu* and *tanh*
- DropOut works best for *sigmod* which does not have $a(0) = 0$.

MNIST Results(2)

MNIST classification error

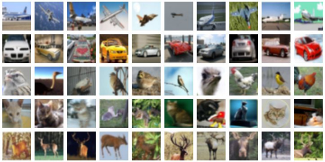
crop	rotation scaling	model	error(%) 5 network	voting error(%)
no	no	No-Drop	0.77 ± 0.051	0.67
		Dropout	0.59 ± 0.039	0.52
		DropConnect	0.63 ± 0.035	0.57
yes	no	No-Drop	0.50 ± 0.098	0.38
		Dropout	0.39 ± 0.039	0.35
		DropConnect	0.39 ± 0.047	0.32
yes	yes	No-Drop	0.30 ± 0.035	0.21
		Dropout	0.28 ± 0.016	0.27
		DropConnect	0.28 ± 0.032	0.21

0.21% is the new state-of-the-art

Previous state-of-the-art is:

- *0.45*% for a single model without elastic distortions [Goodfellow et al. 2013]
- *0.23*% with elastic distortions and voting [Ciresan et al. 2012]

CIFAR-10 Results



- 10 classes of 32 $\times$ 32 colored images
- 50K train/class
- 10K test/class

CIFAR-10 classification error

model	error(%) 5 network	voting error(%)
No-Drop	11.18± 0.13	10.22
Dropout	11.52± 0.18	9.83
DropConnect	11.10± 0.13	9.41

Voting with 12 DropConnect networks produces a new state-of-the-art of 9.32%

Previous state-of-the-art is:

- *11.21*% [Ciresan et al. 2012]
- *9.5*% [Snoek et al. 2012]
- *9.38*% [Goodfellow et al. 2013]

SVHN Results

- 10 classes of 32 × 32 colored image
- 604,388 training images (both training set and extra set)
- 26,032 testing images
- large variety of colors and brightness

SVHN classification error

model	error(%) 5 network	voting error(%)
No-Drop	2.26 ± 0.072	1.94
Dropout	2.25 ± 0.034	1.96
DropConnect	2.23 ± 0.039	1.94

1.94% is the new state-of-the-art

Previous state-of-the-art is:

- *2.8*% stochastic pooling[Zeiler et al. 2013]
- *2.47*% maxout network[Goodfellow et al. 2013]

How-to Implement DropConnect Layer

Implementation	Mask Weight	Time(ms)				Speedup
		fprop	bprop acts	bprop weights	total	
CPU	float	480.2	1228.6	1692.8	3401.6	1.0 ×
CPU	bit	392.3	679.1	759.7	1831.1	1.9 ×
GPU	float(global memory)	21.6	6.2	7.2	35.0	97.2 ×
GPU	float(tex1D memory)	15.1	6.1	6.0	27.2	126.0 ×
GPU	bit(tex2D aligned memory)	2.4	2.7	3.1	8.2	414.8 ×

- NVidia GTX580 GPU relative to a 2.67Ghz Intel Xeon (compiled with `-O3` flag).
- Input and output dimension: 1024 and mini-batch size: 128
- Tricks:
  - encode connection information with bits
  - bind mask weight matrix to 2D texture memory
- CUDA code avaiable at `http://cs.nyu.edu/~wanli/dropc/`

Conclusion

We introduced DropConnect Network

- A simple stochastic regularization algorithm for neural network
- Generalization of DropOut
- Only effective on fully-connected layers
- Set new state-of-the-art on three popular data sets