Email (.jpg, .jpeg, or .pdf) exam to Jingshuai at jj2903@nyu.edu

BASIC ALGORITHMS MIDTERM

Originality and a feeling of one's own dignity are achieved only through work and struggle. – Dostoyevsky

Open book. Open Notes. NO Websearch. Maximal Score: 80.

1. (20) In a BST *assume* z has parent p, left child w, no right child, and that z is the right child of p. Consider the operation $DELETE[z]$.
   (a) (5) Give $DELETE[z]$ in this case.
   (b) (5) Draw two nice pictures illustrating the changing parts of the tree before and after the $DELETE[z]$ operation.
   (c) (10) Further assume BST has $desc[v]$ which tells, for each vertex v, the number of descendents (counting v itself) of v. Assume z has p, w as above. Extend $DELETE[z]$ so that it *updates* all $desc[v]$ that change value. Your updating should take time $O(H)$.

2. (20) Let $A[1 \cdots N]$ be an unsorted array. Consider the following algorithm for creating a maxheap B with this data. Initialize by setting $length[B] = n$, $heapsize[B] = 1$ and $B[1] = 1$. Then
   FOR I = 2 TO N
   INSERT[B,A[I]]; $heapsize[B]++$; ENDFOR
   (a) (5) How long (in Θ-land) does the INSERT step take as a function of I. Brief reason, please!
   (b) (10) Give the total time for the algorithm as $\Theta(g(n))$ for a nice $g(n)$. Give full arguments for both upper and lower bounds.
   (c) (5) Is this a good method to create a maxheap? Brief reason please.

3. (10) Dr. Stingy creates a Hash Table of size n (initially empty) with doubly linked lists to register vaccine applicants. n^3 people register.
   (a) (5) How much time (in Θ-land) does the registration of the n^3 people take. Brief reason please.
   (b) (5) William Gates arrives to get his vaccine, but he hasn't registered. How long will it take (on average) to determine that he hasn't registered. Brief reason please.

4. (10) *Assume* the existence of an algorithm $QT[A, p, r]$ which produces an i, $p \leq i \leq r$, such that $A[i]$ is precisely the first quartile of the values $A[p \cdots r]$. (That is, a quarter of the $A(j)$ are $\leq A[i]$, the rest are $> A[i]$.

   (a) (5) Write a variant $VQ[A, p, r]$ of quicksort that uses QT and sorts $A[p \cdots r]$.

   (b) (5) Further, assume QT takes $4n$ comparisons when applied to n data points. Let $T(n)$ be the total number of comparisons for your $VQ[A, p, r]$. Give a recursion (don't worry about initial values) for $T(n)$. (Note: Do *not* attempt to solve the recursion!)

5. (10) Suppose that in implementing the Huffman code we weren't so clever as to use Min-Heaps. Rather, at each step we found the two letters of minimal frequency and replaced them by a new letter with frequency their sum. (That is, use the "standard" method to find the minimum of a set of numbers and apply it twice.) How long (reasons, please!) would that algorithm take, in Θ-land, as a function of the initial number of letters n.

6. (10) Let two strings X, Y in the English alphabet both begin with q. Give a *logical argument* why there is a longest common subsequence of X, Y which uses the first q in both sequences. (An example will help!)

I force myself to contradict myself in order to avoid conforming
to my own taste. - Marcel Duchamp