The Giant Component

Here we do a computer simulation for a great discovery of Paul Erdős and Alfred Rényi in 1960 called "the giant component."

Start with the empty graph on n vertices. Add randomly chosen edges to the graph. Use Union-Find to keep track of the components. In particular, set MAX equal the maximal size of any of the components and update MAX each time you add an edge.

Letting E be the count of the number of edges added and set $MAX(E)$ equal the value of MAX at that time. Run the program for $0 \leq E \leq 3n$. Display the graph of $MAX(E)$ for various values of n. (Suggestion: $n = 10^3, 10^4, 10^5, 10^6$.) Find a good scaling so that the graphs look the same.

To generate the edges pick x, y random in $\{1, \ldots, n\}$, reject if $x = y$ else add edge $\{x, y\}$. (Occasionally you may get an edge that you have already selected – this turns out to have a negligible effect so don't worry about it.)

Finally – Surprise your instructor! Come up with some other analyses.

Most of all, have fun – explore – take to heart the words of the founder of Theoretical Computer Science, Don Knuth:

> ...pleasure has probably been the main goal all along. But I hesitate to admit it, because computer scientists want to mantain their image as hard-working individuals who deserve high salaries. Sooner or later society will realise that certain kinds of hard work are in fact admirable even though they are more fun than just about anything else.