OLD FINAL EXAM

Warning: Topics vary somewhat from year to year. Also, each exam will only cover *some* of the topics given in class. Also, this exam was no notes, no text – our exam will allow notes and text.

1. (20) Describe the algorithm `TOPDOLLAR(`$PRICE$`,`N`)`. The input is an array of nonnegative real numbers $PRICE[1 \cdots N]$ where $PRICE[I]$ represents the price for a rod of length I. Your output should be $VALUE$, which should be the optimal total price among all cuttings of the rod. You may, and should, use an auxilliary array. You may write in pseudocode or actual code but in either case you **must** give clear comments describing what you are doing and *why* it works. Analyze (just the answer will not suffice!), in Θ-land, the time for your algorithm.

2. (5) For n large which is faster: $\Theta(\lg^3 n)$ or $\Theta(\sqrt{n})$ algorithm?

3. (15) Given $A[1 \cdots N]$ with $0 \leq A[I] < N^N$ for all I.

   (a) How long will `COUNTING-SORT` take?

   (b) How long will `RADIX-SORT` take using base N?

   (c) Give an algorithm (no proof required!) that does better than both of the above, and does so in worst case.

4. (5) In a max Heap what is the property of the value at the root? (That is, how does it compare to the values of the other nodes.)

5. (20) Let G be a connected graph on vertex set V. Let a nonnegative weight $w(e)$ be assigned to every edge e. Let $S \subset V$ with $S \neq \emptyset$ and $S \neq V$.

   (a) Prove (yes, prove!) that the Minimal Spanning Tree necessarily contains that edge $e = \{x, y\}$ with $x \in S$ and $y \notin S$ which has minimal weight. A good picture won't hurt. Assume no two edges have the same weight. (Warning: Saying that an algorithm tells you to pick this edge e is **not** a proof!)

   (b) In what algorithm is the above result used.

6. (10) Consider the recursion $T(2n) = 3T(n) + n + 1$ with initial value $T(1) = 5$.

(a) What is $T(4)$?

(b) What it $T(n)$ in Θ-land.

7. (20) If multiplication mod p takes 1 second then show, using Island-Hopping, how to calculate x^{1023} quickly. How many seconds did it take? (Hint: What is special about 1023?)

8. (20) Find the Huffman code on letters a, b, c, d, e, f with frequencies $.13, .16, .09, .45, .05, .12$ respectively. Give pictures and words clearly indicating the intermediate steps in finding the code.

9. (5) State the Chinese Remainder Theorem.

10. (15) We call a positive integer n `SQUAREFREE` if there is no integer $d > 1$ with d^2 dividing n. We call n `NOTSQUAREFREE` if n is not `SQUAREFREE`.

    (a) Argue `NOTSQUAREFREE` is in `NP`.

    (b) (This is harder!) Argue `SQUAREFREE` is in `NP`.

11. (5) What was the breakthrough of Agrawal, Kayal and Saxena?

12. (20) In Depth-First Search each vertex v gets a discovery time $d[v]$ and a finishing time $f[v]$. Let G be a graph and v, w two distinct vertices for which $w \in Adj[v]$.

    (a) Assume $d[v] < d[w]$. Argue that $f[w] < f[v]$.

    (b) Now further assume that G is a DAG (a Directed Acyclic Graph.) Assume $d[w] < d[v]$. Argue that $f[w] < f[v]$.

13. (5) What is the difference, if any, between a $\Theta(\lg n)$ algorithm and a $\Theta(\log_{10} n)$ algorithm? (*Short* reason please.)