Here we give a corrected version of the result of April 6. We will show that the vertex v that finishes last in $DFS[G]$ must lie in a top scc (strongly connected component). Suppose, *reductio ad absurdum*, that $v \in C'$ and there is a scc C with $C \to C'$, so an $x \in C, y \in C'$ with $y \in Adj[x]$.

Consider $DFS[G]$. Let z be the first point in $C \cup C'$ that is discovered.

`Case I:` $z \in C$. When z is discovered all other points of $C \cup C'$ are white. By the white path theorem all of them (including v are found during $DFS-VISIT[z]$ and turn black before z turns black. Thus v did not finish last.

`Case II:` $z \in C'$. When z is discovered all other points of C' are white and so there is a white path from z to v. Thus v turns black by the end of $DFS - VIST[z]$. (Maybe $v = z$, even easier.) But there are no paths from z to any point in C so when z turns black the points of C are still white. That is, when v turns black the points of C are still white. Thus v did not finish last.

In class we made the *incorrect* statement that if there is a path from w to v but no path from v to w then w finishes after v. Here is a counterexample that shows some of the subtleties that may occur. Picture:

$$w \longleftrightarrow z \longrightarrow v$$

Let the vertices be zwv, $Adj[w] = z$, $Adj[z] = wv$, $Adj[v] = \emptyset$, in those orders, and apply DFS. First $DFS - VISIT[z]$, z turning grey. z finds w, start $DFS - VISIT[w]$, w grey. But now $Adj[w] = z$ and z *is already grey* so $DFS - VISIT[w]$ ends (before w finds v!) and w becomes black. Later v becomes black. Observe that if, instead, $Adj[z] = vw$ then v is found first and becomes black before w.