Basic Algorithms, Problem Set 9

Due by 8 a.m. Wednesdy, April 7.
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The cautious seldom err. – Confucius

1. Let $a(x) = \sum_{j<n} a_j x^j$ be a polynomial of degree less than n. Find $a(0)$ as a simple expression of $a(1), a(\epsilon), a(\epsilon^2), \ldots, a(\epsilon^{n-1})$ where $\epsilon = e^{2\pi i/n} = \cos(2\pi/n) + i\sin(2\pi/n)$. (Idea: Inverse DFT)

2. Consider the undirected graph with vertices $1, 2, 3, 4, 5$ and adjacency lists (arrows omitted) $1 : 25$, $2 : 1534$, $3 : 24$, $4 : 253$, $5 : 412$. Show the d and π values that result from running BFS, using 3 as a source. Nice picture, please!

3. Show the d and π values that result from running BFS on the undirected graph of Figure A[1], using vertex u as the source.

4. We are given a set V of boxers. Between any two pairs of boxers there may or may not be a rivalry. Assume the rivalries form a graph G which is given by an adjacency list representation, that is, $Adj[v]$ is a list of the rivals of v. Let n be the number of boxers (or nodes) and r the number of rivalries (or edges). Give a $O(n + r)$ time algorithm that determines whether it is possible to designate some of boxers as `GOOD` and the others as `BAD` such that each rivalry is between a `GOOD` boxers and a `BAD` boxer. If it is possible to perform such a designation your algorithm should produce it.

   Here is the approach: Create a new field `TYPE`$[v]$ with the values `GOOD` and `BAD`. Assume that the boxers are in a list L so that you can program: For all $v \in L$. The idea will be to apply `BFS[v]` – when you hit a new vertex its value will be determined. A cautionary note: `BFS[v]` might not hit all the vertices so, just like we had `DFS` and `DFS-VISIT` you should have an overall `BFS-MASTER` (that will run through the list L) and, when appropriate, call `BFS[v]`.

   `Note:` The cognescenti will recognize that we are determining if a graph is bipartite!

5. Show how DFS works on Figure B. All lists are alphabetical except we put R before Q so it is the first letter. Show the discovery and finishing time for each vertex.

[1]Figures are on the website for Assignment 9

6. Show the ordering of the vertices produced by `TOP-SORT` when it is run on Figure C, with all lists alphabetical.
7. Let $S(n)$ satisfy initial condition $S(1) = 4$ and recursion $S(n) = S(n/7) + 11$ Assume n is a power of 7. Give a *precise* formula for $S(n)$.
8. **Not to be Submitted!** If one person is purple on December 10, 2020 and the number of purple people doubles every five days, at what day does the number of purple people reach $7 \cdot 10^9$?

What is night for all beings is the time of waking for the disciplined soul. Bhavagad Gita, II.69