Fundamental Algorithms, Assignment 8
NOT TO BE SUBMITTED!!

The search for truth is more precious than its possession
– Albert Einstein

1. Set $W = \lfloor \sqrt{N} \rfloor$. We are *given* $PRICE[I]$, $1 \leq I \leq W$, the price of a rod of length I. (Pieces of length greater than W may not be sold.) Give a program that will output the optimal revenue for a rod of length N^2 and give the time, in Θ-land, of the algorithm. Use an auxilliary array $R[J]$, $0 \leq J \leq N^2$, where $R[J]$ will give the optimal revenue for a rod of length J. You may **not** use the term `MAX` nor `MIN` in your program Explain, in clear words, how your program is working. (You can and should use `MAX` in your explanations.)

2. Suppose that the Huffman Code for $\{v, w, x, y, z\}$ has 0 or 1 as the code word for z. *Prove* that the frequency for z cannot be less than $\frac{1}{3}$. (Hint: Consider the situation in implementing the Huffman code when there are three letters remaining.) *Give an example* where the frequency for z is 0.36 and z does get code word 0 or 1. (This is a tough problem – give it a try!)

3. Suppose, in the Activity Selector problem, we instead select the last activity to start that is compatible with all previously selected activities. Describe how this approach works, write a program for it (psuedocode allowed) and prove that it yields an optimal algorithm.

4. Students (professors too!) often come up with very clever ideas for optimization programs. The problem (often!) is that they (sometimes, but that is enough) give the wrong answer. Here are three approaches and *your* problem, in each case, is to give an example where it yields the wrong answer.

   (a) Pick the activity of the shortest duration from amongst those which do not overlap previously selected activities.

   (b) (*) Pick the activity which overlaps the fewest other remaining activities from amongst those which do not overlap previously selected activities.

   (c) Pick the activity with the earliest start time from amongst those which do not overlap previously selected activities.

5. (a) What is an optimal Huffman code for the following code when the frequencies are the first eight Fibonacci number?

$$a:1, b:1, c:2, d:3, e:5, f:8, g:13, h:21$$

(b) The Fibonacci sequence is defined by initial values $0, 1$ with each further term the sum of the previous two terms. Generalize the previous answer to find the optimal code when there are n letters with frequencies the first n (excluding the 0) Fibronacci numbers. Give a nice picture of the tree.

Dear Sir,
I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about 25 years of age. I have no University education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at Mathematics... I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as "startling"
– Ramanujan

The truth is that the theory of primes is full of pitfalls, to surmount which requires the fullest of trainings in modern rigorous methods. This you are naturally without. I hope you will not be discouraged by my criticisms. I think your argument a very remarkable and ingenious one. To have proved what you claimed to have proved would have been about the most remarkable mathematical feat in the whole history of mathematics.
G.H. Hardy, letter to Ramanujan, 1913 [after finding error in Ramanujan argument]