Basic Algorithms, Problem Set 3

Due by 8 a.m. Wednesdy, February 17.
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> Well, you see, Haresh Chacha, its like this. First you have ten, that's just ten, that is, ten to the first power. Then you have a hundred, which is ten times ten, which makes it ten to the second power. Then you have a thousand which is ten to the third power. Then you have ten thousand, which is ten to the fourth power - but this is where the problem begins, don't you see? We don't have a special word for that, and we really should. ... But you know, said Haresh, I think there is a special word for ten thousand. The Chinese tanners of Calcutta once told me that they used the number ten-thousand as a standard unit of counting. What they call it I can't remember ... Bhaskar was electrified. But Haresh Chacha you must find that number for me, he said. You must find out what they call it. I have to know, he said, his eyes burning with mystical fire and his small frog-like features taking on an astonishing radiance.
> – from A Suitable Boy by Vikran Seth

1. Write each of the following functions as $\Theta(g(n))$ where $g(n)$ is one of the standard forms: $2n^4 - 11n + 98$; $6n + 43n \lg n$; $63n^2 + 14n \lg^5 n$; $3 + \frac{5}{n}$

2. Illustrate the operation of `RADIX-SORT` on the list: COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX following the Figure in the Radix-Sort section. (Use alphabetical order and sort one letter at a time.)

3. Given $A[1 \cdots N]$ with $0 \leq A[I] < N^N$ for all I.

   (a) How long will `COUNTING-SORT` take?

   (b) How long will `RADIX-SORT` take using base N?

4. `Just for Fun:`[1] Your instructor shares his family name (no relation, unfortunately!) with one of the most famous women of the twentieth century. Who was that woman?

[1] Just for Fun problems are not graded!

5. Prof. Squander decides to do Bucket Sort on n items with n^2 buckets while his student Ima Hogg decides to do Bucket Sort on n items with $n^{1/2}$ buckets. Assume that the items are indeed uniformly distributed. Assume that Ima's algorithm for sorting inside a bucket takes time $O(m^2)$ when the bucket has m items.

   (a) Argue that Prof. Squander has made a poor choice of the number of buckets by analyzing the time of Bucket Sort in his case.

   (b) Argue that Ima has made a poor choice of the number of buckets by analyzing the time of Bucket Sort in her case.

   (c) (Thanks, Dianjing!) Compare the *space* usage of the above two solutions and of using n buckets. Which solutions (if any) use substantially more or less space than which other solutions?

6. Analyze the time for the following algorithm. Your answer should be $\Theta(g(n))$ for some nice function $g(n)$ (* is product, ++ is increment 1)
   FOR $I = 1$ TO N
   $J = 1$
   WHILE $J * J < I$
   $J{+}{+}$
   END WHILE
   END FOR

A good traveler has no fixed plans and is not intent on arriving.
A good artist lets his intuition lead him wherever it wants.
A good scientist has freed himself of concepts and keeps his mind open to what is.
Laozi, Philosopher and Writer, 6th century B.C.