Basic Algorithms, Problem Set 1
NOT TO BE SUBMITTED

Computer science is no more about computers than astronomy is about telescopes. – Edsger Dijkstra

1. Let A is a max-heap with heapsize fifty million, being used as a priority queue. Suppose `HEAP-INCREASE-KEY(A,300,key)` is called. What is the maximum number of exchanges that can take place. What is the minimal number of exchanges that can take place.

2. When A is a array with length fifty million and `MAX-HEAPIFY(A,300)` is called. What is the maximum number of exchanges that can take place. What is the minimiml number of exchanges that can take place.

3. Consider a min-heap H with length 1023. [1] Assume the elements of the array are distinct. Let x be the third smallest element in the array. What are the possible positions for x. Let $y = H[700]$. Can y be the largest element in the array? Can y be the smallest element in the array? Give all i for which it is possible that y is the i-th smallest element of the array.

4. Using the figures in the text as a model, illustrate the operation of `BUILD-MAX-HEAP` on the array $A = (5, 3, 17, 10, 84, 19, 6, 22, 9)$

5. The operation `HEAP-DELETE(A,t)` deletes the item in node t from heap A. Give an implementation of `HEAP-DELETE` that runs in $O(\lg n)$ time for an n-element max-heap.

6. Let A be an array of length 127 in which the values are distinct and in increasing order. In the procedure `BUILD-MAX-HEAP(A)` precisely how many times will two elements of the array be exchanged? Now suppose the values are distinct and in decreasing order. Again, in the procedure `BUILD-MAX-HEAP(A)` precisely how many times will two elements of the array be exchanged?

The opposite of Donald Trump is an Asian man who likes math.
Andrew Yang (candidate, New York City major)

---

[1] Did you recognize 1023 as a special number? Its one less than $1024 = 2^{10}$. The binary tree with that many nodes just fills out a row!