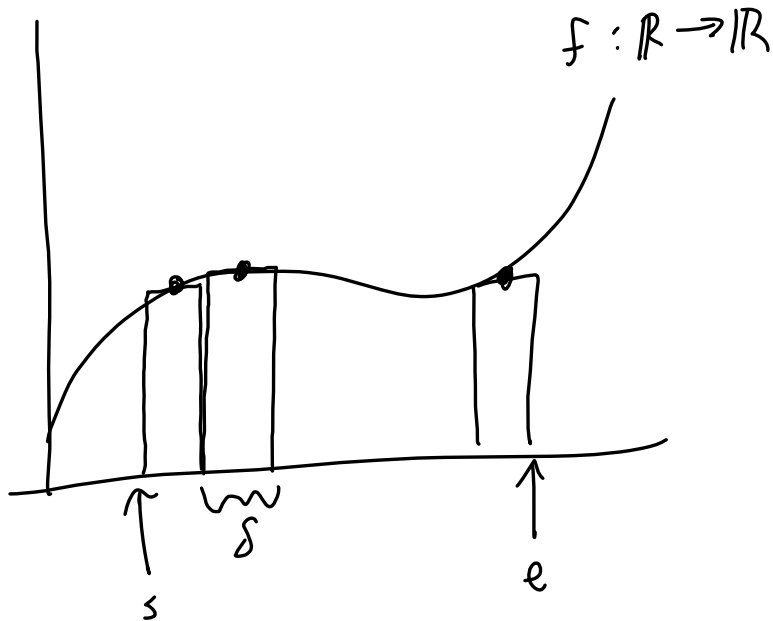


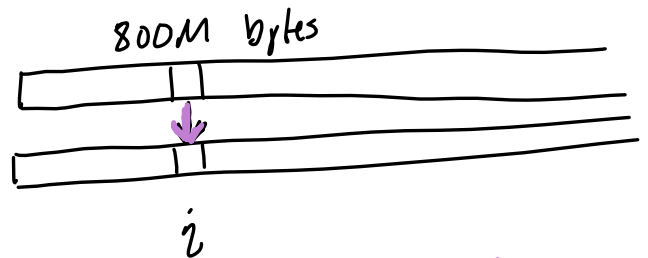
Numerical Integration



$n = 100\text{ M}$ sample points (rectangles)

64-bit reals (8 bytes)

rectangle-positions
rectangle-heights



$$f\left(s + \frac{\delta}{2} + i \cdot \delta\right)$$

S : delayed-req
 $S = \text{Delayed}(\dots, f)$
reduce g $\Leftarrow$ S

assume g is $O(1)$

$\nwarrow$
work? span?

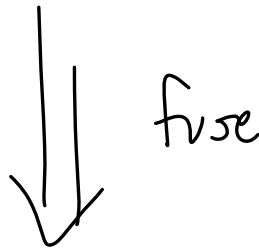
$$W(f(i))$$

$$\left[\begin{aligned} \text{work} &= \sum_{i=0}^{n-1} W(f(i)) \\ \text{span} &= \log(n) + \max_{i=0}^{n-1} S(f(i)) \end{aligned} \right]$$

"operator fusion"

$$s' = \text{map } s \ f$$

$$s'' = \text{map } s' \ g$$



$$s'' = \text{map } s \ (g \circ f)$$

map-tabulate fusion (map after tabulate)

can 1 fuse: $\text{scan } g \ z \ (\text{map } f \ s)$

$\text{map} (\text{scan } g \ z \ s)$

input



blocks



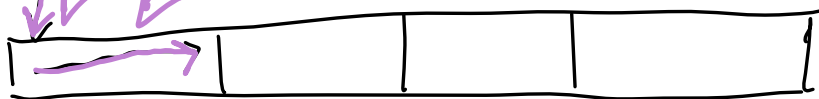
reduce
within
blocks



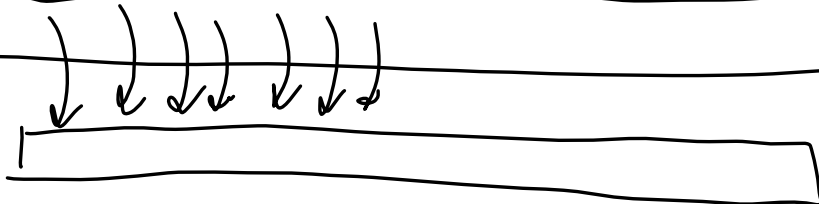
recursively
scan



expand
output



map



Linear Recurrence

input: (a_i, b_i) sequence

↳ an encoding of a recurrence:

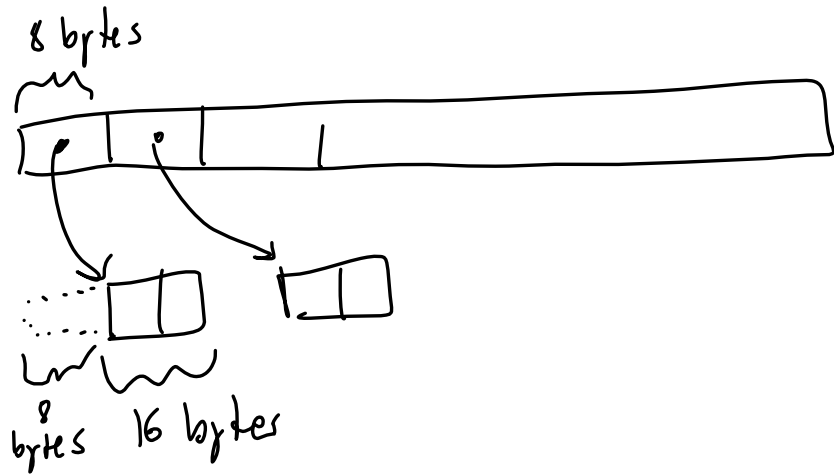
$$R_i = a_i(R_{i-1}) + b_i$$

Output: R_i (sequence)

$$[0, b_1, a_2 b_1 + b_2, a_3(a_2 b_1 + b_2) + b_3, \dots]$$

$a_3 a_2 b_1 + a_3 b_2 + b_3$

$$\text{fun combine}((x_1, y_1), (x_2, y_2)) =$$
$$(x_1 * x_2, x_2 * y_1 + y_2)$$



$$8 + 8 + 16 = 32 \text{ bytes}$$

+

10 M