Parallel Dynamic Tree Contraction

via Self-Adjusting Computation

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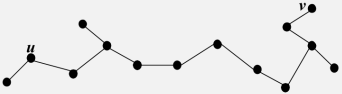
Introduction

Dynamic Algorithms

HeaviestEdgeBetween (



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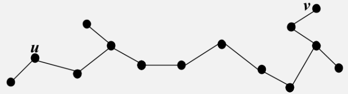
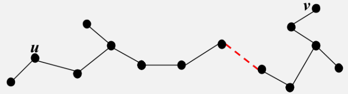
HeaviestEdgeBetween ()

modify input $\Downarrow$

HeaviestEdgeBetween ()

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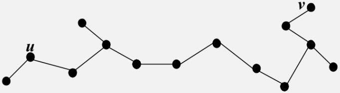
HeaviestEdgeBetween $\left(\right.$  $\left. \right)$

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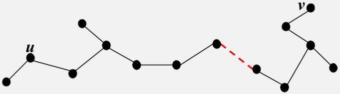
HeaviestEdgeBetween $\left(\right.$  $\left. \right)$

Efficiency?

- Lots of work on **sequential algorithms** with **unit changes**.
  - e.g., single edge insertion/deletion
- More efficient to apply **many changes** simultaneously.

Contributions

Question

In a forest of size n, how efficiently can we recompute some desired property after applying a batch of m changes (insertions/deletions of edges/vertices)?

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(optimal) (optimal)

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- Miller, Reif (1985)
- *rake* and *compress*
- $O(n)$ work
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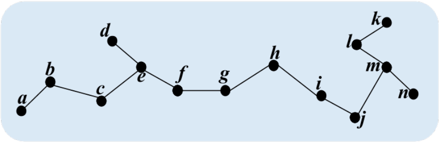

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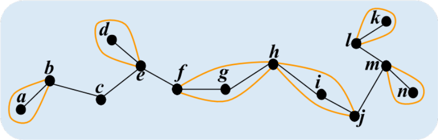

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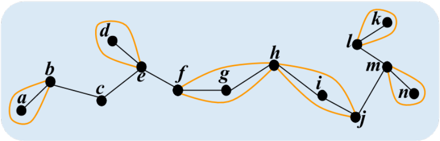

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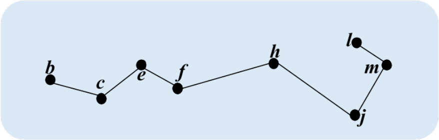

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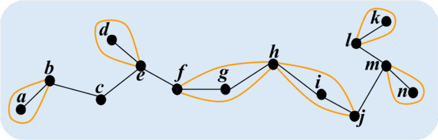

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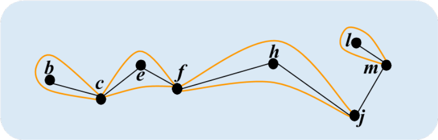

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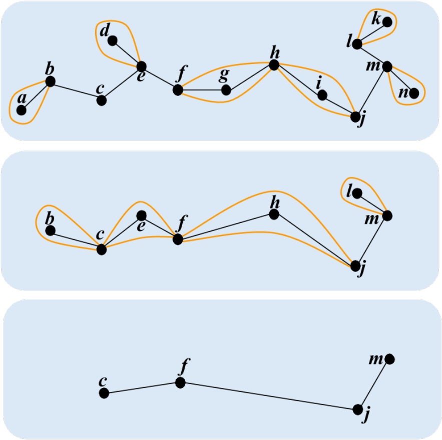
Round 2



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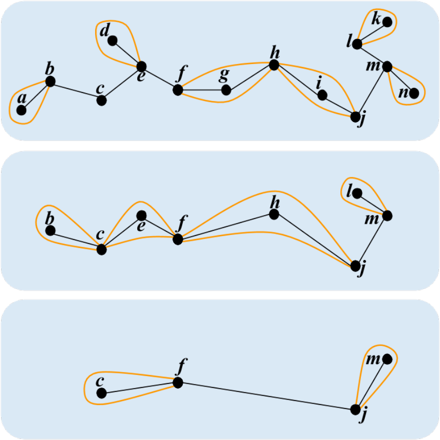

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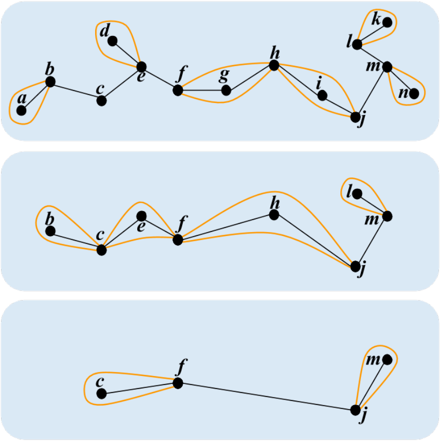

⋮

Dynamizing Parallel Tree Contraction

$$\text{Algorithm} \xrightarrow{\textit{Dynamization}} \text{Dynamic Algorithm}$$

Dynamizing Parallel Tree Contraction

Parallel Algorithm $\xrightarrow{\textit{Dynamization}}$ **Parallel ?** Dynamic Algorithm

Dynamizing Parallel Tree Contraction

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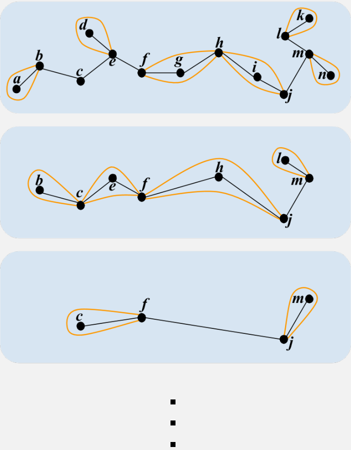
Dynamizing Parallel Tree Contraction

Construction



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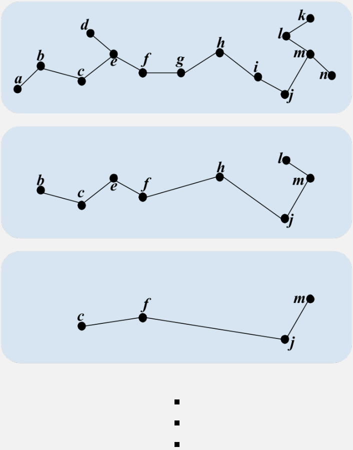

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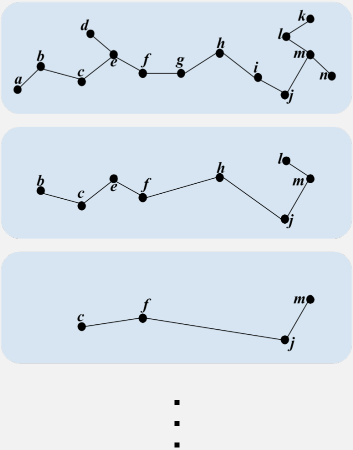
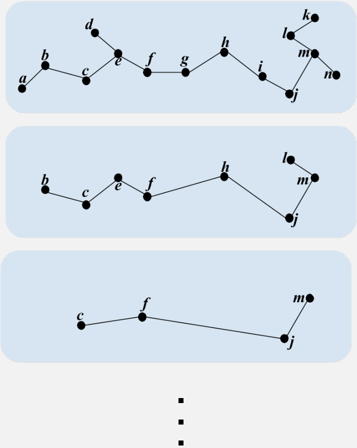


Update



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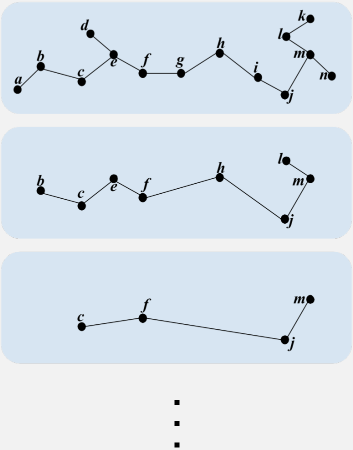

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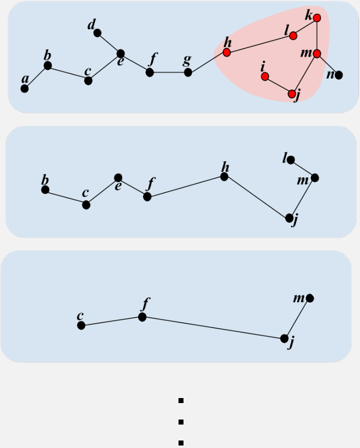

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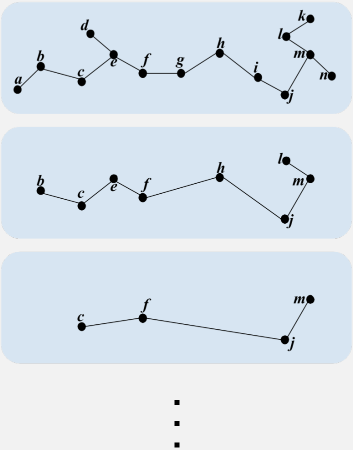

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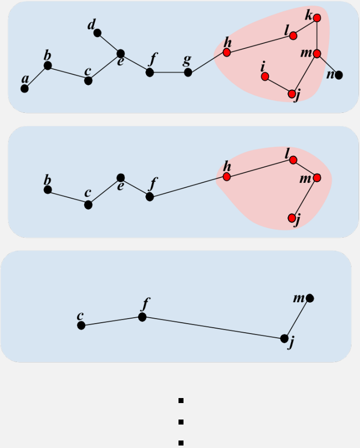

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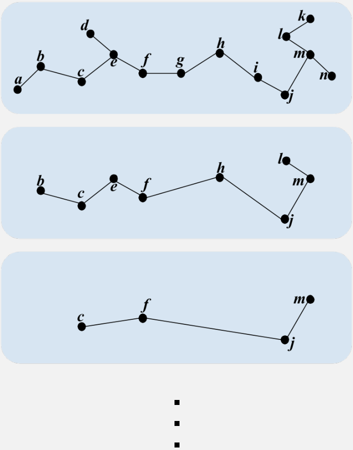

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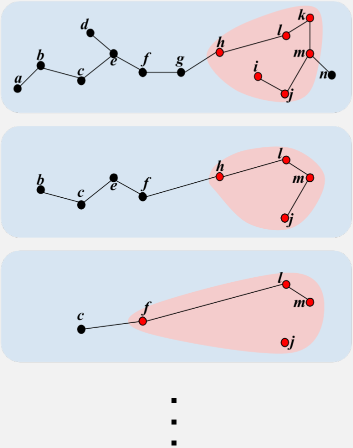

Measuring Performance

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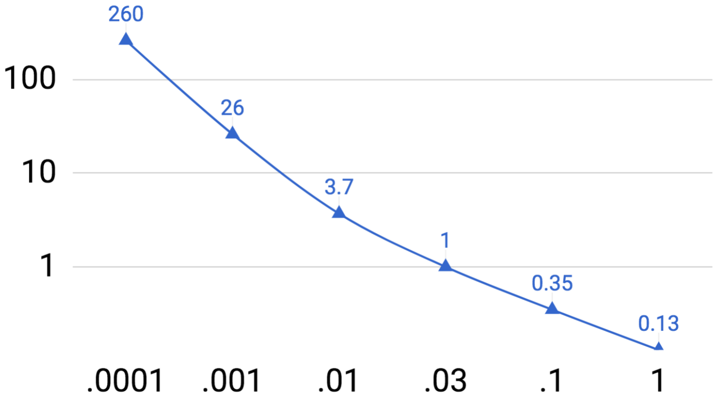
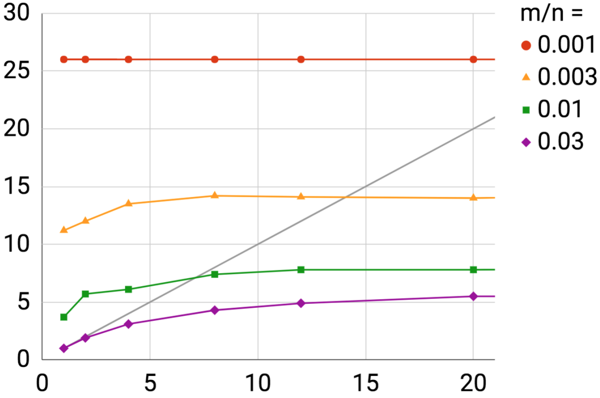
- Work Improvement: $\dfrac{T_{\text{static}}(\text{processors} = 1)}{T_{\text{update}}(\text{processors} = 1)}$
- Benefit of dynamism alone.



m/n (number of changes relative to forest size)

Measuring Performance

- Speedup: $\dfrac{T_{\text{static}}(\text{processors} = 1)}{T_{\text{update}}(\text{processors} = P)}$
- Combined benefit of dynamism and parallelism on P processors.



P (number of processors)

Conclusion

Summary

- We dynamized parallel tree contraction.
- The resulting algorithm is efficient both in theory and practice.

Closing Thoughts

- Some parallel algorithms are amenable to dynamization.
  - Take advantage of independent subproblems.
  - How many more examples are there?
- Are there general purpose techniques for *automatic* dynamization?

End

Thank You!

Questions?

Clustering Hierarchy