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**Upstart Puzzles**

**Finding October**

**(figure)** Small cute/cartoon-like red submarine in the ocean. Two probes one at distance 15 from the sub that says “Yes” and one at distance 25 that says “No”.

Two teams Blue and Red are playing a game. Red has a submarine which we'll call October (the setting for this puzzle comes from Tom Clancy's novel The Hunt for Red October). The Blue Admiral is trying to locate the submarine using probes. A probe will detect the submarine if the probe's position is within some distance d of the submarine, returning either "detected within d" or "not detected within d".

The Blue Admiral's goal is to know the position of the submarine within a distance x throughout a time period T using as few probes as possible.

Knowledge becomes stale as time goes on, but the degree of staleness is bounded by the speed of the submarine. For example, suppose that the submarine must move on a line and can move at most one kilometer in one minute. If Blue Admiral knew October's position at minute t to be no more than distance e with respect to some position p and e < x, then October's position is within distance x of point p for at least x - e more minutes.

Warm-Up: If a probe can detect that October's position is within a distance of 20 kilometers from the probe or not, can Blue use several probes to determine October's position between say kilometer 40 and kilometer 45 along a particular line segment?

Solution: Yes. For example, suppose Blue drops one probe A at 60 and another B at 65+epsilon for some tiny epsilon. If probe A detects the submarine but B does not, then the submarine is in the interval [40..45].

Challenge: Suppose the submarine can move only on a line segment of length 100 kilometers and d is 20 kilometers. At every moment in time up to time 600 minutes from the starting time 0, the admiral like to know the location of the submarine within a distance of 20 kilometers. Assume the submarine can move one kilometer in one minute. Can the admiral do this with 65 probes or fewer? If so, show how.

Solution: Start by deploying probes at 20, 30, 40, 50, 60, and 70. This would be sufficient to know the submarine's location within an interval of size 10. So for example, the blue admiral could know that the submarine is within [0..9] if the probe at 20 detects the submarine but no other. The submarine is within [10..19] if the probes at 20 and 30 detect the submarine, but no others. The submarine is in [20..29] if the probes at 20, 30, and 40 detect the submarine, but no others. The submarine is in [30..39] if the probes at 20, 30, 40, and 50 detect the submarine, but no others. The submarine is in [40..49] if the probes at 30, 40, 50, and 60 detect the submarine (and possibly 20) but no higher ones. The submarine is in [50..60] if the probes at 40, 50, 60, and 70 detect the submarine (the probe at 30 may detect as well). The submarine is in [61..70] if the probes at 50, 60, and 70 detect the submarine. The submarine is in [71..80] if the probes at 60 and 70 detect the submarine. The submarine is in [81..90] if the probe at 70 detects the submarine. If no probes detect the submarine, then October is in [91..99]

At the end of the probing we know that the submarine is at some location [L..L+10]. We will make our point p be L+5 and know the submarine is within distance 5 of p.

Now the Blue Admiral waits 15 minutes at which point the submarine is in the interval [L-15..L+24]. Now the admiral puts probes at locations L-26, L+25, and L+35. The submarine is at [L-15..L-6] if only probe L-26 detects it. The submarine is at [L-5..L+4] if no probes detect it. The submarine is at [L+5..L+14] if L+25 detects it but L+35 does not. The submarine is at [L+15..L+24] if L+25 and L+35 both detect it. The admiral can do this probing every 15 minutes so we would need 9 probes initially and then 3 probes every 15 minutes. So the admiral would need 6 + 19 x 3 = 63 probes.

**Upstart 1.** On a line segment of length M, a detection radius d, and a time T, find an algorithm that uses the minimum number of probes so that at every moment of time up to time T, the admiral achieve a precision of x, i.e. at every moment in time, the admiral knows some position p such that the submarine is in the interval [p-x..p+x].

**Upstart 2.** Generalizing upstart 1 to two dimensions, consider an area of size M by M, detection radius d, and a time T find an algorithm that uses the minimum number of probes the admiral would need so that at every moment of time up to time T, the admiral achieve a precision of x, i.e. at every moment in time, the admiral knows some position p such that the submarine is in the circle of radius x around p.

**Upstart 3.** Generalize the previous upstarts to k dimensions so precision on a hypersphere of dimension k and radius x.

**Upstart 4.** How do the above upstarts change if you can specify for each probe its detection distance just before deploying it? For example, you can drop a first probe with detection radius 20 kilometers and then another probe with detection radius 8 kilometers.

All are invited to submit their solutions to [upstartpuzzles@cacm.acm.org](http://delivery.acm.org/10.1145/3050000/3040969/mailto%3Aupstartpuzzles%40cacm.acm.org); solutions to upstarts and discussion will be posted at <http://cs.nyu.edu/cs/faculty/shasha/papers/cacmpuzzles.html>

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