ABSTRACT

This lecture describes the author's approach to the representation of color spaces and their use for color image processing. The lecture starts with a precise formulation of the space of physical stimuli (light). The model includes both continuous spectra and monochromatic spectra in the form of Dirac deltas. The spectral densities are considered to be functions of a continuous wavelength variable.

This leads into the formulation of color space as a three-dimensional vector space, with all the associated structure. The approach is to start with the axioms of color matching for normal human viewers, often called Grassmann's laws, and developing the resulting vector space formulation. However, once the essential defining element of this vector space is identified, it can be extended to other color spaces, perhaps for different creatures and devices, and dimensions other than three. The CIE spaces are presented as main examples of color spaces. Many properties of the color space are examined.

Once the vector space formulation is established, various useful decompositions of the space can be established. The first such decomposition is based on luminance, a measure of the relative brightness of a color. This leads to a direct-sum decomposition of color space where a two-dimensional subspace identifies the chromatic attribute, and a third coordinate provides the luminance. A different decomposition involving a projective space of chromaticity classes is then presented. Finally, it is shown how the three types of color deficiencies present in some groups of humans leads to a direct-sum decomposition of three one-dimensional subspaces that are associated with the three types of cone photoreceptors in the human retina. Next, a few specific linear and nonlinear color representations are presented. The color spaces of two digital cameras are also described. Then the issue of transformations between *different* color spaces is addressed.

Finally, these ideas are applied to signal and system theory for color images. This is done using a vector signal approach where a general linear system is represented by a three-by-three system matrix. The formulation is applied to both continuous and discrete space images, and specific problems in color filter array sampling and displays are presented for illustration.

The book is mainly targeted to researchers and graduate students in fields of signal processing related to any aspect of color imaging.

KEYWORDS

color, color space, color image, colorimetry, color Fourier transform, color filter array, color mosaic display, color image filtering, color image sampling

To my colorful grandchildren

Kiernan, Quinn, and Juliette

Contents

Preface

Color is everywhere and electronic color imaging is now pervasive. Color is an aspect of human perception and the attempt to describe it mathematically has been going on for centuries, with important contributions from great scientists like Newton, Maxwell, Helmholz, Schrödinger and many, many more [32]. There are numerous excellent, comprehensive and beautiful treatises on color theory, color science, colorimetry and so on ([61], [21], [28], [65] to name just a few). However, it is my belief that none of these adequately present the algebraic theory of color spaces and their applications in color imaging in a suitably comprehensive fashion for signal processing theorists. This lecture describes my approach to the representation of color spaces and their use for color image processing. It is mainly targeted to researchers and graduate students in fields of signal processing related to any aspect of color imaging. However, I will be happy if it finds use in other aspects of color science in general. I have attempted to establish a complete and consistent notation to deal with all aspects of this subject. Since there is no single universally adopted notation for many of the concepts used here, I had to introduce new notation for many aspects. However, I have tried to remain consistent with all standard nomenclature such as color matching functions, chromaticity, etc. All the notation has been summarized in the Notation section, along with a synthetic diagram illustrating the main sets, spaces and transformations that have been introduced.

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Eric Dubois
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