Dennis Shasha

Upstart Puzzles

Seesaw Games

[**Figure:** Seesaw of six meters with single support attached at the middle, leftmost point meter has no weight, next meter has one kilogram, next meter has no weight, fulcrum point, one kilogram, no weight no weight.

(caption) If torque is more on the keft side than on the right, all weights will move one space to the left. If a weight is at the leftmost position and the seesaw tilts left, the weight falls off the seesaw. At that point, of course, that weight no longer exercises torque. All this works symmetrically on the right side. How does the situation in this figure evolve?

Consider a seesaw of length six meters. The fulcrum is at the three meter mark. There is a weight of one at the one meter mark and one at the four meter mark. We represent this as follows _1_f1__ where the "_" represents no weight, the "1" represents a weight of one, and the "f" represents the fulcrum which is just a point. (See the figure on top.)

Warm-up: Once we let the seesaw tilt, what will happen?

Solution to warm-up:
Because the torque on the left side is greater than the torque on the right side, the seesaw tilts towards the left. The configuration will evolve as follows:
_1_f1__ initial configuration
1_1f___ all weights move one to the left (f takes no space)
_1_f___ the leftmost weight falls off the edge
1__f___
___f___ the second weight falls off and seesaw regains balance

We can ask many questions about such a system.

Question: Can you create a seesaw with twice the weight on the right side than on the left but where all the weights fall to the left?

Solution: Here is one of many possibilities:
1__f2__ initial configuration more torque on the left
__2f___ left weight falls off but right weight goes to left side

Question:
Can you find a starting configuration of length six meters that will start by tilting one way and then tilt the other before losing all the weights?

Solution: Here is one solution.

1__f_1_

which evolves as follows
1__f_1_ initial
___f1__ tilts to the left but loses that first weight
___f_1_ tilts to the right
___f__1 keeps tilting
___f___ the second weight falls off and seesaw regains balance

Question: On a seesaw of length 14, create a configuration such the seesaw tilts to the left, then to the right, then to the left again.

Solution:
1___1__f_____1_ initial
___1___f____1__ left weight falls off
____1__f_____1_ moves towards the right
_____1_f______1 moves towards the right
______1f_______ right weight falls off, so now seesaw will tilt to left

Now let's consider a game. One player is called Right and one is called Left. Each player wants as much weight as possible to fall to his or her side. Seesaw is fixed at some size. Left places weights and then Right places weights.

Question: Suppose Left has weights 3, 2, 1 in this configuration 321_f____ Where can Right put weights in order to make as much weight as possible to fall to the right?

Solution:
321_f_132 initial
21__f132_ move to the left, 3 falls to the left, torque is 11 on left (8+3) and 13 on right
_21_f_132 move to the right
__21f__13 move to the right
___2f1__1 move to the right
____f21__ move to the right

The weights that fall to the right are all the initial rightside weights and the 2 and 1 from the left side.
End of solution

Question: Is there a way for Left to guarantee that Right won't get any weights if Left has weights 3, 3, 3 while Right has weights 1, 2, 3 and the seesaw is of length 8?

Solution:
333_f_123 moves left and leftmost 3 falls off
33__f123_ left torque is 12 + 9 = 21 and right torque is 9 + 4 + 1 = 14
3__1f23__ left torque is 12 + 1 = 13 and right torque is 6 + 2 = 8
__12f3___ left torque is 2 + 2 = 4 and right torque is 3.

We call this a *shutout*.
End of solution.

Question: Would the strategy of putting all the 3s on the far left work to achieve a shutout if the seesaw were of length 10?

Solution:
No, here is why:
333__f__123 moves left and leftmost 3 falls off
33___f_123_ left torque is 15 + 12 = 27 and right torque is 12+6+2 = 20
3____f123__ left torque is 15 and right torque is 9 + 4 + 1 = 14
____1f23___ now the right torque wins and all the weights will fall to the right

We call the version of the game we've been playing so far in which Left places its weights before Right places any weights, Left-first.

Upstart 1: In the Left-first game, given a board length, weights for Left, and weights for Right, try to find an algorithm that determines a winning configuration for Left (meaning Right never gets more weight in total than Left) if there is one regardless of what Right does. If there is none, show how Right can win no matter how Left places its pieces. Do the same for shutouts.

We call the variation of the game in which Left places the first weight, then Right places a weight and so on until both sides run out Alternating.

Upstart 2: Answer the previous Upstart for the Alternating configuration.

All are invited to submit their solutions to upstartpuzzles@cacm.acm.org; solutions to upstarts and discussion will be posted at http://cs.nyu.edu/cs/faculty/shasha/papers/cacmpuzzles.html

Dennis Shasha (dennisshasha@yahoo.com) is a professor of computer science in the Computer Science Department of the Courant Institute at New York University, New York, USA, as well as the chronicler of his good friend the omniheurist Dr. Ecco.