Dennis Shasha

Upstart Puzzles
Brighten Up

YOU ARE GIVEN two bags, each containing some number *NumPerBag* of flares. You know there are *NumBad* flares in one of the bags but not which bag. The other bag has all good flares. Each time you test a flare, you use it up.

Here is the first challenge: You want to take *NumBad*-1 flares with you and want to know all are good. Further, you want to use up as few flares as possible in the process. It is fine that when you are done, you may not know which of the unused flares you leave behind are good.

Warm-up. If all the flares in the bad bag are indeed bad, then how many flares would you need to test?

Solution to this warm-up. Test just one flare in one bag. After that you would have at least *NumPerBag*-1 = *NumBad*-1 good flares to take from the bag you know has only good flares.

For general values of *NumPerBag* and *NumBad*, consider two strategies:

Balanced. Take a flare from the first bag and test it, then one from the other bag and test it, and continue alternating until you find a bad one or you reach *NumBad*-1 in one of the bags, in which case you know the remaining ones in that bag are good; and

Unbalanced. Keep taking one flare from a single bag and testing it until you find a bad one or reach *NumBad*-1 in that bag.

Which strategy uses up fewer flares in the worst case?

The average case is worthy of an upstart challenge:

Upstart 1. Assuming *NumBad* = 3, are there values of *NumPerBag* for which the balanced strategy uses up fewer flares than the unbalanced strategy on average, assuming no particular ordering of the flares in either bag? And vice versa. Is there some mixed strategy that is better than either one alone?

2. Now imagine you want to take *NumBad* flares (not just *NumBad*-1) on your trip with the guarantee all are good. Which strategy would give you the best chance of achieving this? For this challenge, the number of flares you use up in testing is unimportant.

Upstart 2. Given *NumPerBag* and *NumBad*, suppose you want to take *d* more than *NumBad* with the guarantee all are good. What is the best strategy to use (it may be a hybrid), and what probability of success as a function of *d* can you achieve?

Solution to 1, or taking *NumBad* − 1 good flares on the trip. Unbalanced, because it will use up 1 + *NumPerBag* − *NumBad* flares in the worst case, whereas balanced may use up 1 + 2*(*NumPerBag* − *NumBad*) in the worst case.

Solution to 2, or taking NumBad good flares on the trip. With the unbalanced strategy, if you are lucky enough to start testing flares with the bad bag, you will be able to take *NumBad* good flares for sure. If you start testing flares with the good bag, then you will test 1 + *NumPerBag* − *NumBad* flares from that good bag, leaving you *NumBad* − 1 good flares. But you can get one more good flare from the bad bag by testing flares from that bag until you discover all *NumBad* flares and seeing whether you have any more flares left in that bag. With the balanced strategy, if you discover no bad flare by the time both bags have *NumBad* flares left and the next flare tested is also good, then the strategy fails to deliver *NumBad* flares that are all good. However, that is the only case in which the balanced strategy loses. Balanced is thus better. [C]

Dennis Shasha (dennisshasha@yahoo.com) is a professor of computer science in the Computer Science Department of the Courant Institute at New York University, New York, as well as the chronicler of his good friend the omniheurist Dr. Ecco.
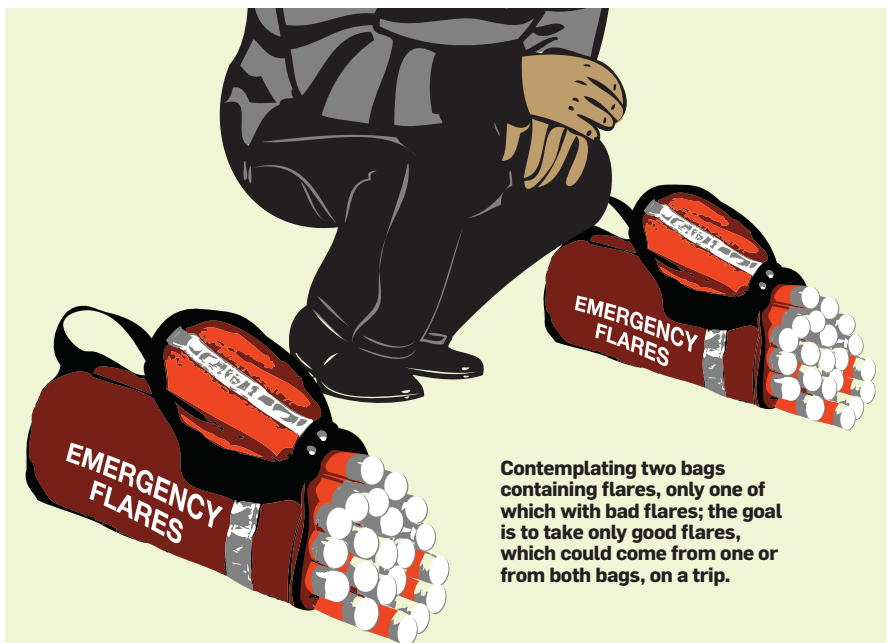
Contemplating two bags containing flares, only one of which with bad flares; the goal is to take only good flares, which could come from one or from both bags, on a trip.

EMERGENCY FLARES

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COLLAGE BY ANDRIJ BORYS ASSOCIATES/SHUTTERSTOCK