Problem Solving, practice exercise for post office

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Baskerhound at the Post Office.

"Imagine that a network works like the paper mail service (sometimes called snail mail)," said the chief detective. "Most letters arrive in one day. However, not all do. Some take many days, though they will arrive eventually. The letters may not arrive in order. That is, if A sends a letter to B on day 1 and then another letter to B on day 2, the first letter may arrive last.

"Now, there are 17 people, each of whom has a single message that he or she must send to the other 16 people. At the end, everyone should have the messages from themselves and everyone else. Each letter can contain only one message. All letters must go through a single central post office. Each person has 17 envelopes (one more than may seem strictly necessary up to this point).

"All would be well except that Baskerhound, our adversary, may try to read these messages. He may look at all the letters in the post office at one, perhaps two points of time that he can choose. He won't alter the letters, but he may read them.

"We want to avoid letting Baskerhound know the messages of all 17 people."

"So you have to make sure that copies of all the messages aren't in the mail at the same time?" Ecco asked.

"Exactly," responded the chief detective. "Here is how the solution might go if there were just two people, Bob and Alice and Baskerhound could just look once at the post office. Bob sends to Alice; when she receives, she sends her message to Bob."

"Can we assume that some participant, say number 1, starts the protocol?" asked Ecco.

"Yes, assume that 1 knows when the protocol is to begin," responded the chief detective.

"Finally," Ecco continued, "can a participant send any other message in a letter, e.g. 'I got a message from these participants'?"

"The only other message they can send is the simple phrase 'all clear,' " responded the chief detective.

Answer the following questions:

a. Baskerhound can look in the post office only once. We want a solution that will take two or fewer days if all letters arrive in a day, but which doesn't allow Baskerhound to see all the messages no matter how long any given letter takes. Remember that each participant has only 17 envelopes and one message is allowed per envelope.[1] (Actually it can be done with 16 envelopes.)

[1] So suppose that each letter arrives in a day, then the protocol should guarantee that everything finishes in two days. Suppose some letters take 0 days, others take 13 days, others take 9 days. Then there is no performance guarantee but Baskerhound should still not be able to see all messages.

b. Baskerhound can look twice. Try to find a solution that will take only five days if all letters arrive in a day, but which doesn't allow Baskerhound to see all the messages in his two looks no matter how long any given letter takes.

c. Baskerhound can look twice and only needs 10 distinct messages (any 10) to do severe damage. Each participant has 35 envelopes. Try to find a solution that takes eight days, if all letters arrive in a day, but which doesn't allow Baskerhound to see all the messages in his two looks no matter how long any given letter takes.