Strategic Bullying

BY DENNIS E. SHASHA

In schoolyards, the world of crime, and international affairs, certain entities try to take valuables from others by force. They attack most happily when they have overwhelming power and the target is wealthy. Sometimes individuals or groups join forces, forming a temporary coalition to gain wealth or deter an attack.

Let's assume the following rules hold. An entity's power is represented by a number; if the entity is a coalition, the coalition's power is equal to the sum of its members' powers; wealth is proportional to power. Entity A will attack B only if A is sure of winning (if A's power is greater than B's). When an entity is beaten, the winners gain its wealth but not its power. Every entity plays selfishly, striving to acquire riches yet avoid certain destruction. A configuration of power values is said to be stable if no fighting will occur—as will happen if a coalition can form that is strong enough to prevent fighting and if being loyal to the coalition is in every member's best interests.

Suppose the power values are 4, 2, 1. The configuration is unstable because 4 will fight 2 and 1 and beat them whether they form a coalition or not. In contrast, if the power values are 4, 2, 1, 1, the configuration is stable: 2, 1 and 1 can form a coalition to prevent 4 from attacking. Moreover, if any coalition members were destroyed, the remaining ones would be, too, so each coalition member would be strongly motivated to stay in the coalition.

To warm up, consider the configuration 5, 4, 4. Is it stable? What about 5, 4, 3? The 5, 4, 4 configuration is unstable because the two 4s will certainly gang up on 5. After splitting 5's wealth, they will separate into a stable (in fact invincible) 4, 4 configuration. But 5, 4, 3 is stable because no one can be assured of winning with impunity (*below*).

Your problem: What is the largest set of distinct power values (positive integers all different from one another) that is stable when the highest value is 21? What if the values need not be distinct?

Here's another challenge. Certain entities can have a "bee-sting" capability: defenses able to destroy even a superior attacking power. Those using the sting, though, die in the battle along with the enemy. For example, if a 5 attacks a 4 having a bee sting, both the 5 and the 4 will perish. So, 5 won't attack. A bee-sting capability may seem inherently stabilizing because it should prevent attacks. But might the acquisition of this capacity by an entity render a configuration unstable?

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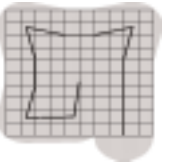
Answer to Last Month's Puzzle

To find the missing hiker, begin two miles west of the square's southeastern corner and follow the route shown in the illustration below. The total length of the route is about 29.31 miles, so the search party will detect the hiker's signal within about 293 minutes.



Web Solution

For a fuller answer to last month's problem and a peek at this month's solution, visit www.sciam.com

CORNELIA BLIK

WARM-UP SOLUTION A 5, 4, 3 configuration is stable because peace is in each group's best interests.