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Rationale

Making the Best of Complexity

The concept of 'rational' decision making breaks down when one considers the randomness, uncertainty, and incomplete information common to the agents of complex systems. The application of traditional mathematical modeling theory to such systems is reminiscent of searching for Euclid's geometry in the natural world--the cross-section of a tree trunk may be approximately a circle, but the novel perspective of Mandelbrot's fractal geometry far more closely resembles the forms found in nature.

Modeling complex systems, and knowledge extraction therefrom, often requires both numerical simulation/quantitative analysis, as well as a qualitative approach informed by systems dynamics. Throughout my studies at Gallatin, I have focused on developing a technical and philosophical toolkit to augment one's understanding of the world, and thus inform decision making--particularly when faced with uncertainty. The tools include applied mathematical modeling and complexity theory, complemented by computer science. This rationale explores when and why such techniques are applicable and necessary for survival and success.

Simplifying Complexity

The ancients abstained from the study of complexity due to the widespread belief that cosmic order was maintained by divine intervention. Their mathematics were concerned with purity and their behavior was concerned with following their respective divine commandments to attain harmony with divine Order.[1]

However, Pythagoras and Hippasus can teach us a lesson about how much of existence is hidden on the other side of our subjective ideas about Order. Hippasus proved the existence of irrational numbers, dispelling the Pythagorean fiction of a divinely ordered universe which was governed by ratios. Pythagoras, the leader of the cult which bore his name, sentenced Hippasus to death by drowning for doing so. Today, we not only recognize the existence of irrational numbers, but that there are **far** more irrationals than rationals; there are countably infinite rational numbers, while the set of irrationals is considered uncountable (there are infinitely many more irrationals).[2]

When we open our minds to the idea of studying complexity, we find we are immersed within it--but we first need to understand why traditional mathematical modeling does not suffice in capturing complexity. Mathematics, from number theory through differential equation theory, forms a strong foundation for the quantification and applied analysis of our surroundings. Generations of mathematicians and scientists have built and continue to build on top of each others' work, producing proven statements about the nature of numbers and abstract relationships, relating their discoveries to the world we live in.

Before the age of computation, the works of Euclid, al-Khwarizmi, Descartes, and Newton were combined to create the most powerful tools for describing systems. While scholars prior to Newton feared infinity, the essence of calculus lies in approaching the infinite and the

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[1] "Chaos and Order." Encyclopædia Britannica, Encyclopædia Britannica, Inc., https://www.britannica.com/topic/chaos-and-order.
[2] Seife, 10-13

infinitesimal and conquering them in the limit.[3] The power of infinity is baked into calculus; differentiation and integration leverage a few steps of symbolic manipulation to perform an infinite number of calculations in the blink of an eye. When modeling systems using differential equation theory, we express dynamics as equations. 'Success' lies in the ability to integrate these equations, with the analytic solution containing the full history and future results of the interactions in question.

The results of the models we create by implementing these mathematical tools are only as valid as the assumptions behind them. In reality, there are very few systems that behave as smoothly and continuously as the equations we use to model them. Stability and convergence to analytic solutions are primarily found far above and below human scale (e.g. planetary motion, chemical reactions), where small modeling errors remain small, or may scale linearly. This is the regime of Order: perturbations are often damped in time; approximations are useful; equilibrium and stability are useful concepts when applied to the real world. Engineering is an example of applied modeling and analysis in an orderly regime--or rather, approximately orderly, as a structure's mere presence on Earth exposes it to uncertainty from other sources (e.g. tsunamis, asteroids, or even war).

It might seem absurd to question the usefulness of physics and engineering in light of their reliance on approximations (e.g. small-angle); however, economics is a perfect example of over-extending the power of traditional modeling techniques. Heterogeneity and nonlinearity in the components (people) and interactions (transactions) of an economy result in the ability to solve only trivial/special cases, often only after a plethora of other simplifying and

[3] Seife, 93

approximating techniques are applied. Knowledge validation is one of the driving themes of the scientific revolution and the cornerstone of Descartes' *Meditations on First Philosophy*--however, model validation in economics has by-and-large been disregarded (e.g. Cobb-Douglas utility functions). In such systems, information which determines its state and direction often lies in the frictions that are ignored in traditional economic modeling theory (e.g. homogeneity, Cobb-Douglas smoothness, and linearization to ignore higher-order effects).

Many systems around the human scale are characterized by or exposed to chaotic storms of volatility that cannot be predicted or avoided, but at best weathered. Examples include social networks (Facebook, capital markets, cultures, terrorist cells) which are at danger to contagion, herding or groupthink; the weather, manifesting natural disasters; ecologies which may be prone to collapse (wildfires); the economy, fluctuating between boom and bust; even city streets, prone to gridlock or rioting.

Nonlinearity and coupling in the relationships between variables in such systems often result in partial differential equations which remain unsolvable, and it has been shown that linearization is inappropriate for use in such cases: Newton used calculus to solve the two-body problem in 1687, yet the three-body problems still eludes us; Edward Lorenz's simple weather model showed the 'sensitive dependence on initial conditions' we have come to expect from chaotic and dynamic systems--so goes the saying, 'if a butterfly flaps its wings in New Mexico, it can cause a hurricane in China.'

Yet, in the age of computation, the Navier-Stokes equations aren't as frightening; while we can't predict the formation and course of a hurricane, we can continuously track it, simulating its most probable next few steps, and adaptively form evacuation policies to save many lives.

The solution to extract useful information from complex systems lies in changing our motives, goals, and perspective. Rather than focus on false predictive power from first-order approximations, we can leverage analysis (from the perspective of systems theory) and simulation to develop heuristics and optimal strategies.

Identifying Complexity

How do we determine whether a system is complex? There are two groups in which we can sort systems, mechanical and dynamical. "Mechanical systems can be *complicated* but not *complex*. (Taleb 2012, 56)." This idea is built into the very purpose of technology, tools, and machines; technology is intentionally designed and engineered to function in a predictable way, and when our devices don't work as we expect them to, we generally consider them broken. Mechanical systems experience material fatigue from consistent exposure to low-level stressors ('wear and tear')--as opposed to atrophy from the absence of low-level stressors ('use it or lose it').[4]

Dynamical systems, on the other hand, incorporate a plethora of methods to respond to changes in their environment. These adaptive systems are composed of a number of parts whose interaction with each other and their environment result in behavior, patterns, or structures to emerge at the system scale. From this perspective, biology, psychology, and sociology are studies of the emergent processes of chemistry and physics; 'rules' and patterns often can be identified and analyzed at every scale--all the way down and all the way up.

[4] Taleb 2012, 60

Almost all complex systems are organic[5], and even for those that are not (a rarity), they are generally designed to implement ideas of complexity and biomimicry. These systems self-optimize, primarily in the form of adaptation and adjustment to an ever changing environment. They implement feedback mechanisms, filtering and selection processes (taking advantage of the weakness of constituents) to improve the well being of the system. As Taleb points out, selection processes benefit from random variations in two ways: randomness in mutations and randomness in the environment, where the latter tests the strength of the former.[6]

By bringing mechanisms that are ingrained in our biology to our conscious mind and utilizing them in our strategic thinking, we stand to benefit from a body of knowledge that nature has formed over millions of years of 'research and development'. We will explore a powerful framework for doing so in antifragility.

Antifragility

The ideal strategies for survival in a complex environment[7] are well-adjusted (i.e. properly suited to the degree of randomness/uncertainty in the system) and antifragile. Nassim Taleb coined the term *antifragile* to describe that which benefits from volatility. He demonstrates the idea of shipping an item to a friend.[8] If the item is fragile, like fancy china, you would write "FRAGILE: DO NOT SHAKE OR DROP" on the side, whereas if the item is robust, like a block of iron, you forgo any writing. The antifragile item is one which would prompt writing "PLEASE MISHANDLE" on the box. The fragile is that which has only

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[5] In the context of this rationale, organic refers to adaptive or evolving, similar to living organisms.
[6] Taleb 2012, 71
[7] Complex environments refer to complex systems from the perspective of the component/
[8] Taleb 2012, 39

negative effects from volatility, the robust is that which it resistant to volatility, and the antifragile is that which benefits from volatility. To explain this concept further, consider your own bicep muscle; muscles grow due to small fiber tears created during exercise, and consistent exposure to such volatility increases the muscle's strength and performance.

The antifragile is organic (or at least responds to stimulus organically) becoming stronger through exposure to low-intensity stressors and experiencing atrophy due to a lack of exposure to such stressors. While it may be difficult to think of an antifragile physical item, which actually could be packed into a box, Taleb points out that there exist organisms, systems, ideas, and strategies which are, or can be tailored to be antifragile. Again, antifragility applies as much to muscles as to economic theories and philosophical concepts; it can be abstracted to apply to any domain. It must be noted that antifragility is bounded, in that volatility and stressors are beneficial only to a point and often require a recovery period. Compare consistent small muscle fiber tears from exercise to an injurious muscle tear.

It seems that modernity and the future are associated with increasing volatility. As our financial markets become increasingly connected through globalization, they become more fragile. In place of localized crashes, interdependence of investments leads to an increased risk of global financial meltdowns. Climate scientists predict that high intensity weather patterns and natural disasters will occur with increasing frequency. By seeking antifragility in decisions ranging from where to live to how to invest, one can position oneself better for the risks ahead.

Survival and Strategy

Survival (and success) in complex adaptive systems requires high adaptability, with insight into the mechanisms that drive important system phenomena; however, this does not mean that one's strategy need be complex or complicated. In a lecture where Professor Charles Peskin reduced the dynamics of the aortic valve to LaPlace's equation[9], he exclaimed: "Nature has a small bag of tricks! Or maybe this one trick is just very useful." As evident in Peskin's comment, and validated by Mandelbrot's Fractal Geometry, nature loves simplicity, which evolves into complexity through recursion.

For an agent in a system, its success is purely dependent on its decisions. As agents ourselves in systems from financial markets to the climate to society, we benefit from properly considered decisions. Some domain independent tips and lessons for strategic thinking inspired by complexity and antifragility are as follows:

- Seek strategies with payouts which are a convex function of volatility
  - Returns increase nonlinearly to increases in volatility
  - Opens exposure to the positive Black Swan[10]
- KISS thinking - keep it simple and stupid
  - Lowers possibility of hidden risks, allows more accurate modeling
  - The more simple and independent your strategy is, the easier and more accurately you can characterize fragilities, less vulnerable to shocks
- Omission versus prescription
  - When attempting to alter strategy, seek omission of behaviors/rules over creating new behaviors/rules
  - Complex systems respond nonlinearly and unpredictably to new behaviors/rules
- Form and implement heuristics in place of linear modeling and prediction
- Seek domain independent knowledge and apply it to new domains

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[9] LaPlace's equation ($\Delta f = 0$) has appeared in many disparate modeling applications.

[10] A "Black Swan" is defined as an event characterized [p. xviii] by rarity, extreme impact, and retrospective (though not prospective) predictability (e.g. market crashes, September 11th, Fukushima Nuclear reactor meltdown). [definition from https://www.stat.berkeley.edu/~aldous/157/Books/taleb.html]

- Attempt to characterize non-equilibrium[11] or identify strange attractors[12]
- Notice herding (or groupthink) and react accordingly
- Maintain optionality and thus adaptability
  - Cash in on free options
- Learn from your own and others' mistakes
  - Non-terminal mistakes are a datapoint
  - Others' mistakes are free knowledge
- Change is often more powerful and significant on the smaller scale
- Innovation is born out of necessity
- Over-insure when insurance is cheap
  - This strategy will result in a large payout during a squeeze
  - Redundancy leads to longer survival under selection
- Is *fitness* being suited to the past history of a specific environment or extrapolating to an environment with stressors of a higher intensity?
  - Current stress testing methods often involve backtesting to highest intensity historical data points (ignores the concept of Black Swans)
  - In contrast, nature overreacts in recovery seeking to become stronger than the next, (even) higher intensity stressor—this post-traumatic growth is observed in muscle strengthening and bacterial resistance
  - Strive to be prepared for way worse than the worst thing you've experienced

Alternative methods for informed decision making rely on modeling and simulation of a system and implementing qualitative and quantitative analysis of the results. In doing so we search for strange attractors, thresholds for phase transitions[13], and characterizing the sources and degree of variability in a system's multiple regimes.

Order and Chaos

Two fundamental regimes of systems are Order and Chaos. Order is the realm of the synthetic, mechanical, linear, synchronized, predictable; to reiterate, perturbations are damped in

[11] Non-equilibrium is a concept of Complexity Economics that takes into account that the economy is never truly in a 'steady state'.
[12] Strange attractors are geometric patterns in the paths through a system's phase space.
[13] Phase transitions are changes between different states of organization.

time, approximations are useful, equilibrium and stability are useful real-world concepts. Chaos is the default state (i.e. entropy), the ultimate point of uncertainty, discontinuous and nonlinear, where positive feedback loops, power laws, and causal ambiguity dominate. Living on the border between the two are the organic, natural, living, adaptive—organisms, mechanisms and ideas inspired by Chaos, variation and volatility which are applied to restore Order.

The ideas of Order and Chaos are not only useful for decision making as an agent within our universe, but seem to be inherent to its existence and our perception of it. In 12 Rules for Life, Jordan Peterson claims that these ideas are built into our perception and understanding, manifested within the left (Order) and right (Chaos) sides of the brain. He also connects the Daoist ideas of Yin (femininity) and Yang (masculinity) to Chaos and Order, respectively. Beyond the dichotomy represented in the Yin and Yang are the spots of the opposing force found within each side.[14] There is Order within Chaos when a population of fireflies spontaneously synchronize their flashing, when a system flows along a strange attractor, or even when chaos is controlled and applied as in chaotic cryptography (see Robert Matthews). Chaos can also manifest within Order: when the mixture of two chemicals stable on their own results in an explosive release of energy; when a California wildfire ignites without a lighter in a 10 mile radius and sets swaths of forest ablaze; or in general, when Black Swans and tail events shock a seemingly ‘tame‘ system into dysfunction.

Peterson further points to the Daoist and Kabbalist mystic belief that the universe emerged from a primordial soup of Chaos. To the Kabbalists, the World of Nothing (more accurately the World of No Thing) is the realm where borders and distinctions are absent, and all

[14] Peterson, 48:00-50:00

is one. It is the ‘matter’ of this World, combined with borders and distinctions, that results in the physical universe (“Yesh mi Ayin” or creation ex-nihilo).

The generality and ubiquitous presence of the concepts Order and Chaos make them particularly interesting for interdisciplinary study. Their abstract nature lends to their ability to be applied to a great number of contexts, allowing us to discover previously unexplored connections between disparate domains--they can be analyzed in contexts from mathematics to sociology, finance to biology, psychology to evolutionary theory, metaphysics, religion, and beyond.

Infinity and Conclusion

I stress that mathematics is a versatile and exact language for expressing systems and relationships in fundamental and domain independent ways. This has been one of the driving themes of my undergraduate study. However, in Gödel Escher Bach, Douglas Hofstader shows the ambiguity of the symbolic form of mathematics with the creation of “Typographical Number Theory” to implement the rules of mathematics in a different form. Math feels like so much more than a language—it feels closer to magic. By expressing abstract relationships, we can ‘massage’ equations into revealing esoteric information about a system. However, in studying the real world, valid results often requires elements of the alternate perspective detailed above. In doing so, we find a philosophy that is as applicable to everyday decision-making as to research.

The toolkit I have developed at Gallatin provides a language and a cognitive framework for modeling the world, validating these models, and developing a more profound understanding of my environment. This in turn allows me to tailor my behavior and actions to the outcomes I seek with increased clarity and intention. When implemented as a research methodology, it lends the benefits of mathematical analysis to research areas that have been difficult to quantify, and have alluded scholars throughout the ages.

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