Mathemagical

Take a handful of number-crunchers, a sprinkling of puzzlers, add a few magicians and stir for four days. **Alex Bellos** joined the mix, along with photographer **Kendrick Brinson**

GARY FOSHEE, a collector and designer of puzzles from Issaquah near Seattle walked to the lectern to present his talk. It consisted of the following three sentences: "I have two children. One is a boy born on a Tuesday. What is the probability I have two boys?"

The event was the Gathering for Gardner earlier this year, a convention held every two years in Atlanta, Georgia, uniting mathematicians, magicians and puzzle enthusiasts. The audience was silent as they pondered the question.

"The first thing you think is 'What has Tuesday got to do with it?'" said Foshee, deadpan. "Well, it has everything to do with it." And then he stepped down from the stage.

The gathering is the world's premier celebration of recreational mathematics. Foshee's "boy born on a Tuesday" problem is a gem of the genre: easy to state, understandable to the layperson, yet with a completely counter-intuitive answer that can leave you with a smile on your face for days. If you have two children, and one is a boy, then the probability of having two boys is significantly different if you supply the extra information that the boy was born on a Tuesday. Don't believe me? We'll get to the answer later.

As a melting pot of outside-the-box abstract thinking, this gathering is one of a kind. Attendees were invited to make the world's first snub dodecahedron out of balloons, shown how to solve the Rubik's cube while blindfolded and given tips on how to place a lemon under a handkerchief without anyone knowing. The 300 guests included magicians, origamists, artists, maze designers, puzzle writers, toy inventors ›

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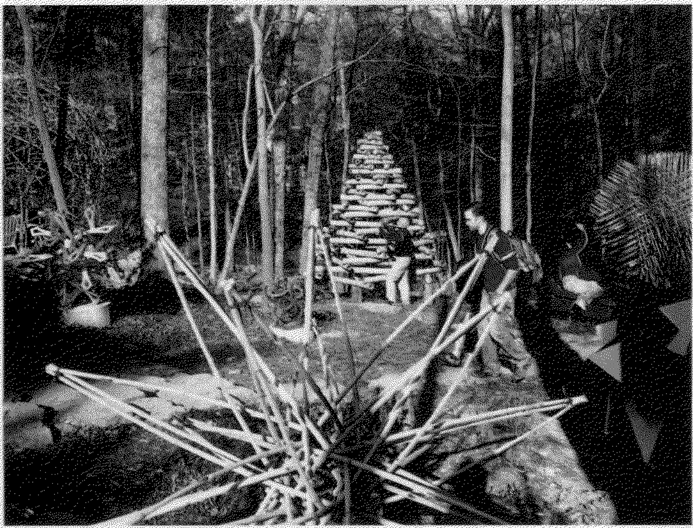
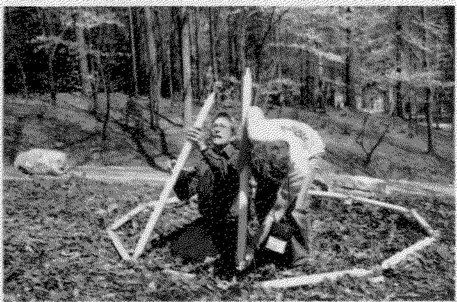


Mathematical sculptures fill the organiser's garden



Work begins (left) on constructing a hyperbolic paraboloid

and cognitive psychologists, as well as some of the world's most gifted mathematicians.

Erik Demaine, for example, is a former winner of the MacArthur fellowship, aka the "genius award", and the youngest person in recent years to be made a professor at the Massachusetts Institute of Technology. Now aged 29, he presented to the gathering some typefaces that he had invented as a result of his academic work in computational geometry. The "hinged dissection font" is a font in which each letter can be made from the same linked chain of 128 identical isosceles triangles. (A hinged dissection is a technique in which a large shape is divided up into smaller shapes, linked together with "hinges", and then refolded into another large shape.)

The letters looked distinctive, if a little clunky. Demaine reckoned the typeface had another more serious problem: "It's way too easy to read!" So he revealed a brilliantly incomprehensible font based on his work in origami: the "origami maze font", for which each letter of the alphabet is the origami fold-pattern that, once folded, would make that letter protrude from the page (bit.ly/az64ky).

When he started to write in this font, and the screen at the front of the hall filled with an impenetrable grid of red and blue lines – red lines are the "mountain" folds and the blue ones the "valley" folds – the audience clapped and guffawed.

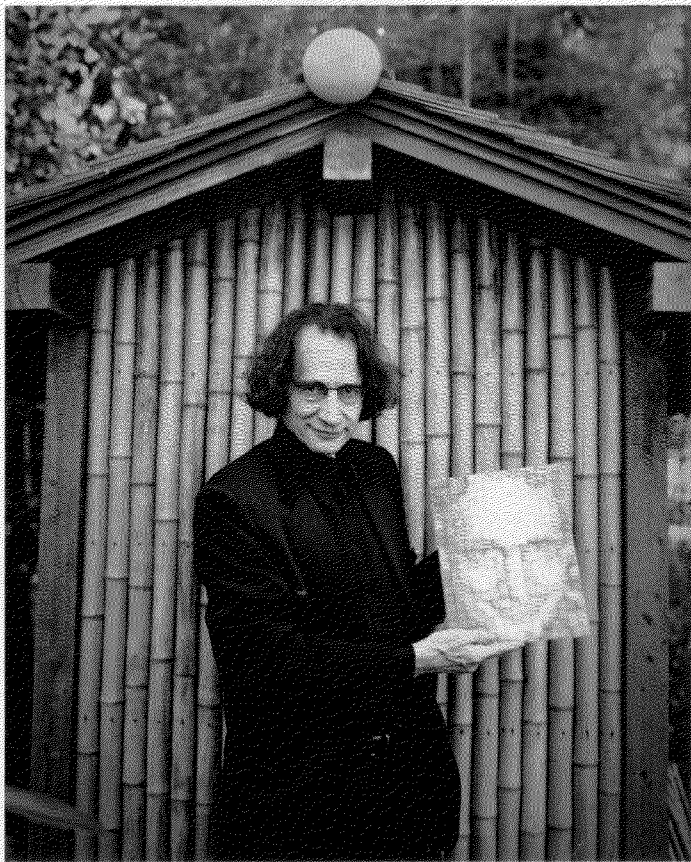
Of tiles and tetrominoes

Recreational mathematics may be the "mathematics of fun" but it often inspires serious science. Hinged dissections were invented by the British puzzle creator Henry Dudeney a century ago. Though Demaine became interested in them and in origami for purely playful purposes, they have resulted in some of his most important academic research. Hinged dissections form the basis of his designs for reconfigurable robots, in which blocks hinged together in a chain can be made to fold into any three-dimensional shape.

The pop-up lettering technique behind his origami font could, he says, be used with paper that can pucker up to form tactile shapes: to make, for example, a map you can read in the dark. "There is no way to predict from the recreational side to the product side," Demaine says.

The four-day Gathering for Gardner, or G4G, owes its name to the journalist Martin Gardner, who died as this article was going to press, aged 95. Between 1957 and 1981 Gardner wrote the monthly "Mathematical games" column in *Scientific American*, which inspired a cult following. A decade after he put down his pen, Atlanta businessman and puzzle collector Tom Rodgers decided to pay tribute to Gardner by organising a gathering in his honour.

This year the gathering mostly took place in the windowless conference room of an Atlanta hotel, apart from one afternoon when everyone relocated to Rodgers's residence on the outskirts of Atlanta. It's a modern house surrounded by a Japanese garden, containing geometrical sculptures and a pathway based on the Fibonacci sequence. Here the guests are free to indulge in all manner of mathematical recreations: University of Arkansas mathematician Chaim Goodman-Strauss led a group in building a Hilbert-space-filling



Mark Setteducati shows off his customisable jigsaw puzzle Jigazo

curve out of 320 kilograms of steel; performance artist Caspar Schwabe constructed a "kinetic bamboo hyperboloid"; and composer Vi Hart demonstrated how to make polyhedra out of balloons.

Of the three main professions represented – mathematicians, magicians and "puzzle people" – the magicians were the easiest to spot: they tended to wear black jackets. Mark

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Setteducati was a prime example, and he also wore a dandyish black shirt. He was displaying a copy of a puzzle he had invented called Jigazo, currently the top-selling puzzle in Japan.

Setteducati is a performing magician who curates the magic side of the G4G. Magic was chosen as a theme because Gardner himself was an amateur magician and his first foray into mathematics writing in the 1950s was a book on the mathematics of magic. The two disciplines are happy bedfellows: both exhibit a wonder at the structure of the world, and seek playful ways to subvert it.

The G4G facilitates the free exchange of ideas between the magical and mathematical communities. The Jigazo – which will be released in Europe and the US later this year – was a product of this traffic. At a previous G4G, Setteducati met the computer graphics pioneer Ken Knowlton, and the puzzle is a collaboration between them.

The Jigazo is a jigsaw puzzle that can be customised to display any picture you want. It contains 300 identically shaped pieces that are all a different shade of blue, and all have a unique symbol on the back. If you take a portrait of yourself and email the image to the Jigazo website, you will receive an email back with a map of the arrangement of the symbols such that when the pieces are assembled in this way the jigsaw shows your portrait.

"What I love about this is the magic behind it," says Setteducati. "With 300 pieces you can make any face in the world." In this context, "magic" means a good deal of fiendishly clever number-crunching.

Setteducati gave me his business card, which is written in backwards-script and incorporates a mirror to read it the right way round. Judged on the originality of its guests' business cards, the G4G is a pretty special event: several were origami models or unusual shapes. The card of Pablos Holman, futurist and security expert, was completely black, his details only visible in UV light.

Another charming aspect of the gathering is the age range of those present. Many of Gardner's contemporaries still come, including 93-year-old mathematician Richard Guy and Solomon Golomb, aged 88, whose work on polyominoes – dominoes with more than two squares – gained prominence thanks to a Gardner column in the late 1950s. (And, many years later, inspired the computer game Tetris, in which falling tetrominoes must be stacked together.) At the opposite end of the age spectrum was Neil Bickford, aged 12, a mathematics blogger from San Jose, California.

In 1964, Golomb coined the term "reptile" – short for repetitive tiling – to mean any tile that can reproduce a larger copy of itself. ›

An example of this is the L-shaped tile. Carolyn Yackel, who is best known for her book *Making Mathematics with Needlework*, demonstrated that the number of L-tiles you need to make into a bigger L is always a square: 4, 9, 16 and so on. And she showed a table made of ceramic reptiles that she made with her father.

Tiling is a major theme in recreational mathematics – possibly because as well as being conceptually rich it is also aesthetically gratifying. For example, computer scientist Craig Kaplan from the University of Waterloo in Ontario, Canada, spoke of different ways to design parquet tiles that morph from one shape to another as they cross the floor. The evolution of tiles, he said, suggested "an inexhaustible source of inspiration for art".

Mathematicians are especially fascinated by sets of tiles that can only produce a pattern that never repeats. Such sets are called "aperiodic" and were discovered in the 1960s and later popularised by Roger Penrose in the 1970s. Aperiodicity is interesting because it is so counter-intuitive, and because if patterns never repeat it means that every tile in an arrangement somehow influences the position of every other tile.

Until now, at least two different shapes of tiles were needed, such as Penrose's famous "kite" and "dart" tiles. Which is why a new design known as the "einstein" tile presented at the conference was such a significant

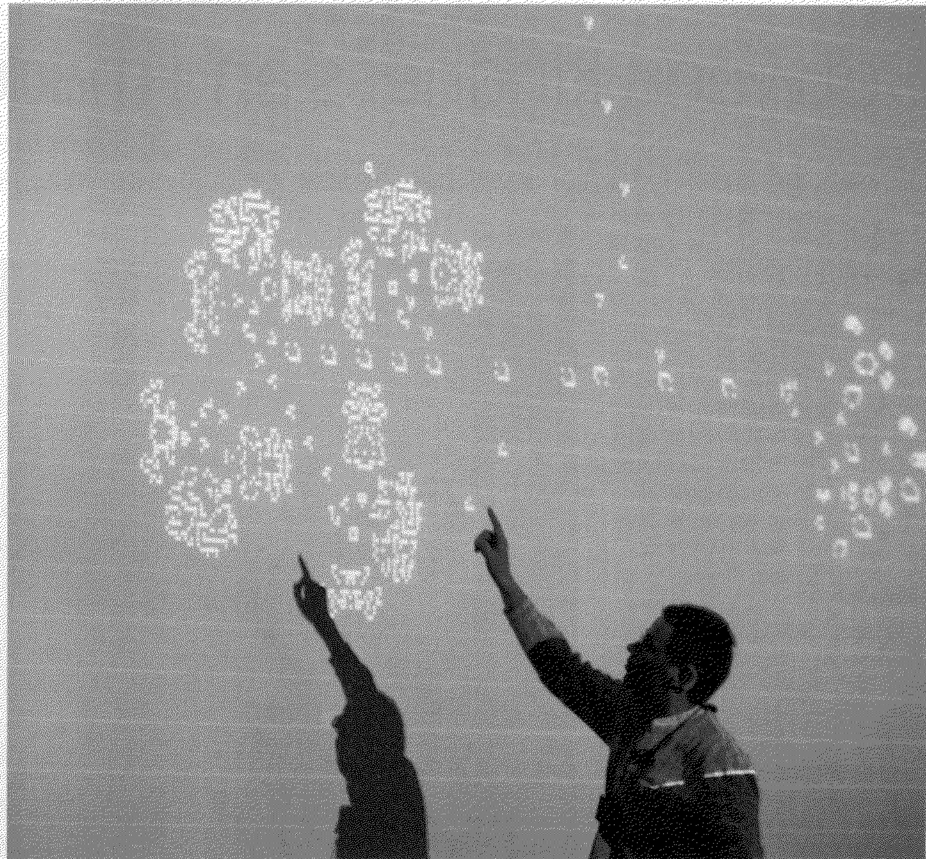
"Everyone finds probability questions mind-bending, even the world's best mathematicians"

mathematical breakthrough (arxiv.org/abs/1003.4279). It is the first aperiodic tile that works on its own, with no need for any other shape. Designed by Joshua Socolar, a physicist from Duke University in Durham, North Carolina, and Joan Taylor from Burnie, Tasmania, Australia, it looks like a cross between a snowflake and a small craft from an early *Star Wars* movie. Identical einsteins can only fit together without any gaps or overlaps in a non-periodic way.

Perhaps the mathematician whose career most embodies the spirit of recreational mathematics is G4G stalwart John Horton Conway of Princeton University in New Jersey. "I absolutely love this. It's just so interesting being here with such different people." Yet Conway's presence in Atlanta was not merely physical. In 1970 Gardner wrote one of his most widely read columns, on a Conway invention called the Game of Life. Conway's "Life" is an example of what is known as a cellular automaton, a grid in which the cells can be either alive (white) or dead (black) and where four simple rules determine what happens to the cells in successive generations. The "game" is to choose a pattern of live cells and see how they evolve. As well as providing something fun for early computer programmers, the wonder of seeing how complexity emerges from such simple beginnings captured many people's imaginations in the early 1970s.

Forget sausage dogs. Vi Hart makes an inflatable octahedron

Tom Rokicki points out the intricate patterns cellular automata make



The Game of Life seemed to be everywhere at the G4G. California-based mathematician Tom Rokicki showed some new patterns of mind-boggling ingenuity, including one which produced the image of the numeral 3, then the numeral 1, then 4 and continued writing out the digits of pi. William Gosper, who once held the record for discovering pi to the greatest number of decimal places and is considered one of the first computer hackers, was showing new Life forms on his laptop: "The ingenuity is staggering," he says.

Life appeared again in a talk by Stephen Wolfram, the mathematician and software entrepreneur behind the Wolfram Alpha computational engine and the blockbuster software Mathematica. He believes that cellular automata can recreate the universe and is trying to find the ones that do. Wolfram attended the G4G with his wife and two of his four children. Which brings us back to the opening question: "I have two children. One is a boy born on a Tuesday. What is the probability I have two boys?"

The first thing to remember about probability questions is that everyone finds them mind-bending, even mathematicians. The next step is to try to answer a similar but simpler question so that we can isolate what the question is really asking.

So, consider this preliminary question: "I have two children. One of them is a boy. What is the probability I have two boys?"

This is a much easier question, though a controversial one as I later discovered. After the gathering ended, Foshee's Tuesday boy problem became a hotly discussed topic on blogs around the world. The main bone of contention was how to properly interpret the question. The way Foshee meant it is, of all the families with one boy and exactly one other child, what proportion of those families have two boys?

To answer the question you need to first look at all the equally likely combinations of two children it is possible to have: BG, GB, BB or GG. The question states that one child is a boy. So we can eliminate the GG, leaving us with just three options: BG, GB and BB. One out of these three scenarios is BB, so the probability of the two boys is 1/3.

Now we can repeat this technique for the original question. Let's list the equally likely possibilities of children, together with the days of the week they are born in. Let's call a boy born on a Tuesday a BTu. Our possible situations are:

- When the first child is a BTu and the second is a girl born on any day of the week: there are seven different possibilities.
- When the first child is a girl born on any day of the week and the second is a BTu: again, there are seven different possibilities.
- When the first child is a BTu and the second is a boy born on any day of the week: again there are seven different possibilities.
- Finally, there is the situation in which the first child is a boy born on any day of the week and the second child is a BTu – and this is where it gets interesting. There are seven different possibilities here too, but one of them – when both boys are born on a Tuesday – has already been counted when we considered the first to be a BTu and the second on any day of the week. So, since we are counting equally likely possibilities, we can only find an extra six possibilities here.

"Judged on the originality of attendees' business cards, this is a pretty special event"

Summing up the totals, there are 7 + 7 + 7 + 6 = 27 different equally likely combinations of children with specified gender and birth day, and 13 of these combinations are two boys. So the answer is 13/27, which is very different from 1/3.

It seems remarkable that the probability of having two boys changes from 1/3 to 13/27 when the birth day of one boy is stated – yet it does, and it's quite a generous difference at that. In fact, if you repeat the question but specify a trait rarer than 1/7 (the chance of being born on a Tuesday), the closer the probability will approach 1/2.

Which is surprising, weird... and, to recreational mathematicians at least, delightfully entertaining. ■

Alex Bellos is a writer in London and the author of *Alex's Adventures in Numberland: Dispatches from the wonderful world of mathematics* (Bloomsbury). He blogs at alexbellos.com