In this annex the experimental results will be commented more in details.

Randomly connected graphs

The behaviour of the selected algorithms is shown in three interesting cases.

In the first case, Fig.1a, the size of the attribute alphabet M is 75% of the number of nodes and the graph density η is 0.05. In this case McGregor performs better, because the other two algorithms face the problem using the association graph. The association graph is large and dense and so it not convenient to find out its maximum clique.

In the second case, Fig.1b, M = 50% and η= 0.1. This case is very interesting because the behaviour of the three algorithms is almost the same. The meaning is that the density of the two starting graphs is enough to make McGregor not the fastest and, the association graph is enough dense to make Durand Pasari not the fastest too. Finally the heuristic of Balas Yu, is too complex to elaborate, to make this algorithm winning on the others. The conclusion is that in this case there is not an algorithm definitely faster that the others.

In the third case, Fig.1c, M is 75% and η is 0.2. In this case the density of the two starting graphs becomes enough high to make convenient to face the problem using the association graph. Moreover, this density is not high enough to permit to the heuristic of Balas Yu (which is quite expensive to elaborate) a good pruning, thus also this algorithm is slow, and Durand Pasari is the fastest algorithm.

Fig.1a)

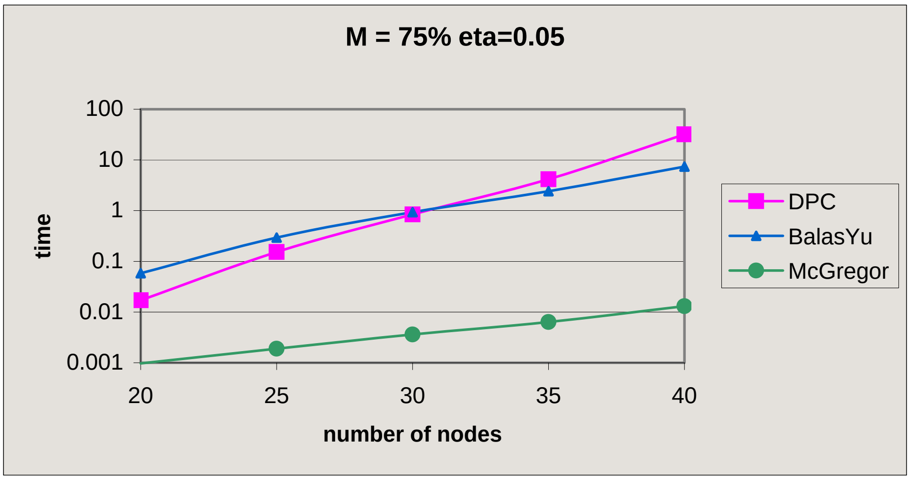


Fig.1b)

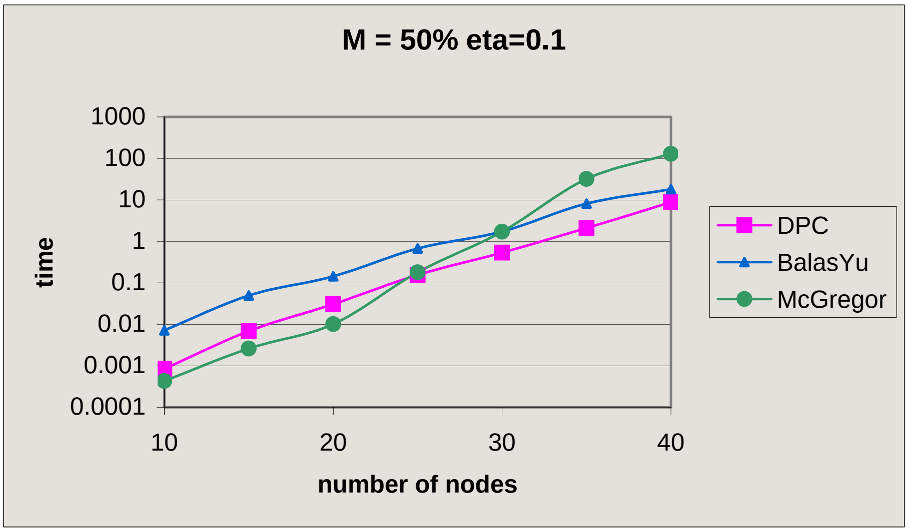


Fig.1c)

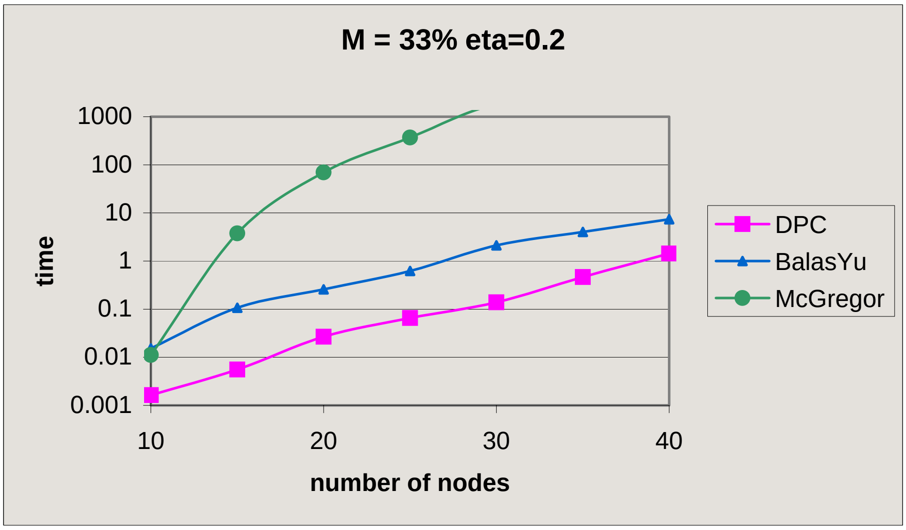
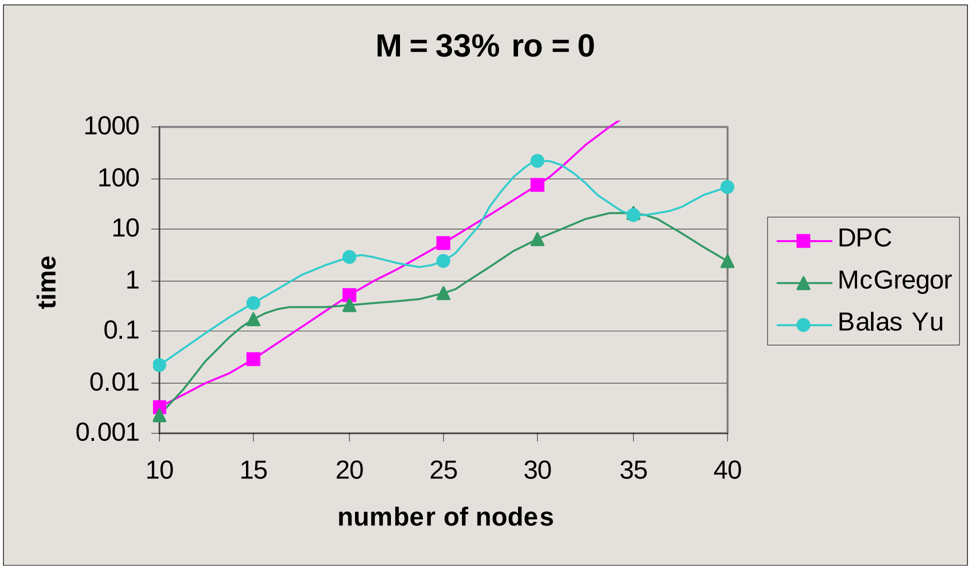
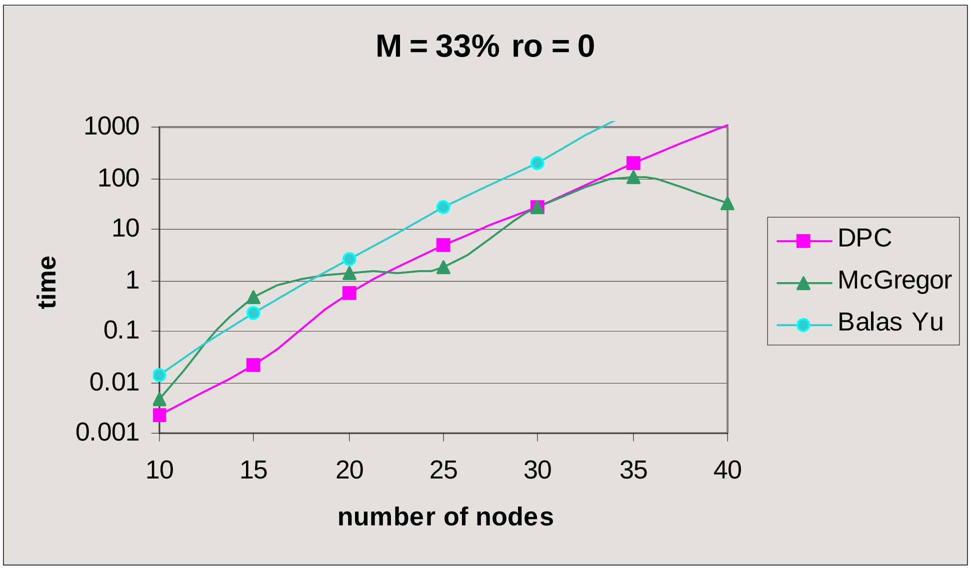


Fig.1: experimental results for random graphs. a) the size of the attribute alphabet is M=75% of the number of nodes, the density η is 0.05; b) M = 50%, η = 0.1: c) M = 33%, η = 0.2.

Regular Mesh Graphs

In the Fig.2a, 2b and, 2c the most interesting behaviours of the algorithms on regular meshes are shown. M is always fixed to 33%, the behaviour in the case of M = 50% and in case of 75% are very similar but faster, because of a better pruning due the larger alphabet of attributes. More in detail, in the first case, Fig.2a, the behaviour on bi-dimensional meshes is shown. All the three algorithms shows similar characteristics, but a tendency is clear: both McGregor and Balas Yu perform better than Durand Pasari. The main difference between Durand Pasari and Balas Yu is that the second algorithm provides a more refined heuristic function. In this case the role of a more refined heuristic on larger graphs is very clear. In the second case, Fig.2b, the behaviour on tri-dimensional meshes is shown. Here Balas Yu algorithm is slower because of the complex heuristic. McGregor and Durand Pasari show a similar performance. Finally in Fig.2c, the behaviour on quadri-dimensional meshes is shown. In this last case McGregor algorithm performs better for larger graphs.

a) 

b) 

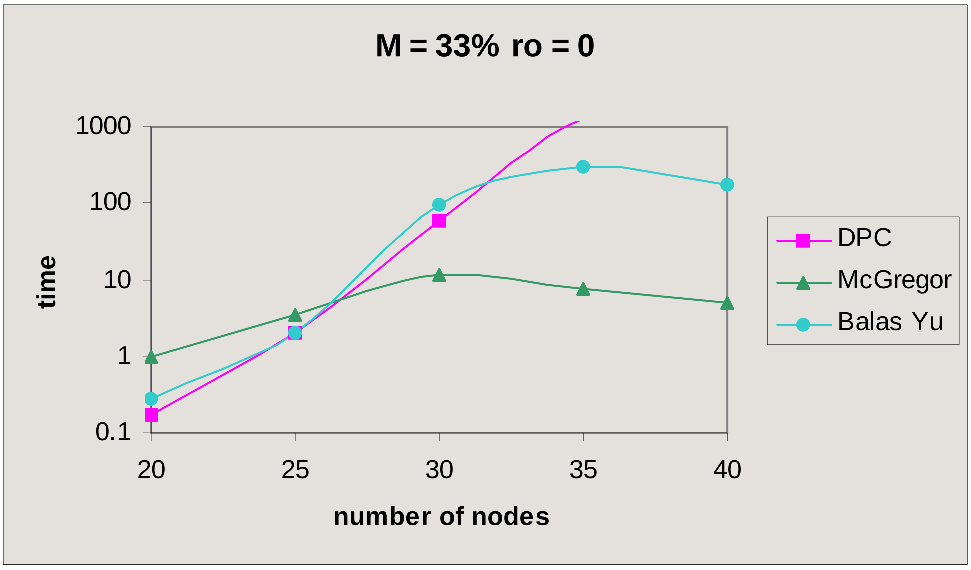
c) 

Fig.2: experimental results for regular meshes. The size of the attributes alphabet is always M=33% of the number of nodes. a) results for meshes 2D graphs; b) meshes 3D graphs; c) meshes 4D graphs.

Irregular Mesh Graphs

The behaviour of the three algorithms is shown in nine interesting cases.

In Fig.3a, 3b and, 3c the behaviour of the algorithms on irregular 2D meshes is shown. M is always fixed to 33%, the behaviour in the case of M = 50% and in case of 75% are very similar but faster, because of a better pruning of the candidate solutions due to the larger alphabet of attributes. More in detail, in the first case, Fig.3a, the behaviour for irregularity degree $\rho = 0.2$ is shown, then in Fig.3b the irregularity becomes $\rho = 0.4$ and, finally in Fig.3c $\rho = 0.6$. In Fig.3a the three algorithms show a similar behaviour, this is another case in which the graph morphology doesn't favours an algorithm respect to another. But when the mesh irregularity increases, in Fig.3b and Fig.3c, the tendencies of the algorithms become more clear. Firstly, it can be notices that McGregor algorithm becomes slower and slower. The reason is that when the irregularity increases the structure of the graph is going to become a little similar to the randomly connected graphs and in that case, it has already been shown that McGregor is not the best algorithm. Moreover, Durand Pasari algorithm is going to improve its behaviour for an increasing irregularity. The main reason is that the association graph is large and dense when the two starting graphs have a mesh structure and, the increasing irregularity of the graphs generates an association graph becoming less dense. Finally, Balas Yu algorithm is also going to improve its behaviour for ρ increasing, because it also uses the association graph and, for larger graphs there is an additional improvement because of the better heuristic.

In Fig.4a, 4b and, 4c the behaviour of the algorithms on irregular 3D meshes is shown. M is always fixed to 50%. More in detail, in the first case, Fig.4a, the behaviour for irregularity degree $\rho = 0.2$ is shown, then in Fig.4b the irregularity becomes $\rho = 0.4$ and, finally in Fig.4c $\rho = 0.6$. The main difference with the previous three cases is that the alphabet of attributes is larger and then there is a smaller number of candidate solution to examine. As a consequence the three algorithms are faster than the previous cases. In Fig.4b it is clear that McGregor is the best algorithm and it is also evident that this algorithm becomes slower for ρ increasing. The behaviour of Balas Yu and Durand Pasari is also similar to the cases of Fig.3a, 3b and, 3c, i.e. they improve for ρ increasing. Finally when $\rho = 0.6$, all the three algorithms have the same behaviour for small graphs, but McGregor has a better tendency for large graphs.

In Fig.5a, 5b and, 5c the behaviour of the algorithms on irregular 4D meshes is shown. M is always fixed to 75%. More in detail, in the first case, Fig.5a, the behaviour for irregularity degree $\rho = 0.2$ is shown, then in Fig.5b the irregularity becomes $\rho = 0.4$ and, finally in Fig.5c $\rho = 0.6$. In this three cases the alphabet of attributes is still larger and then there is a smaller number of candidate solution to examine. Also in these cases there is a tendency of McGregor to make worse when the irregularity increases, while the other two algorithms works better for a higher ρ. However McGregor is always the best algorithm because of the large alphabet of attributes: when M=75%, it is never convenient to use the association graph. It is also clear that the heuristic of Balas Yu makes this algorithm more efficient than Durand Pasari.

a)
b)
c)

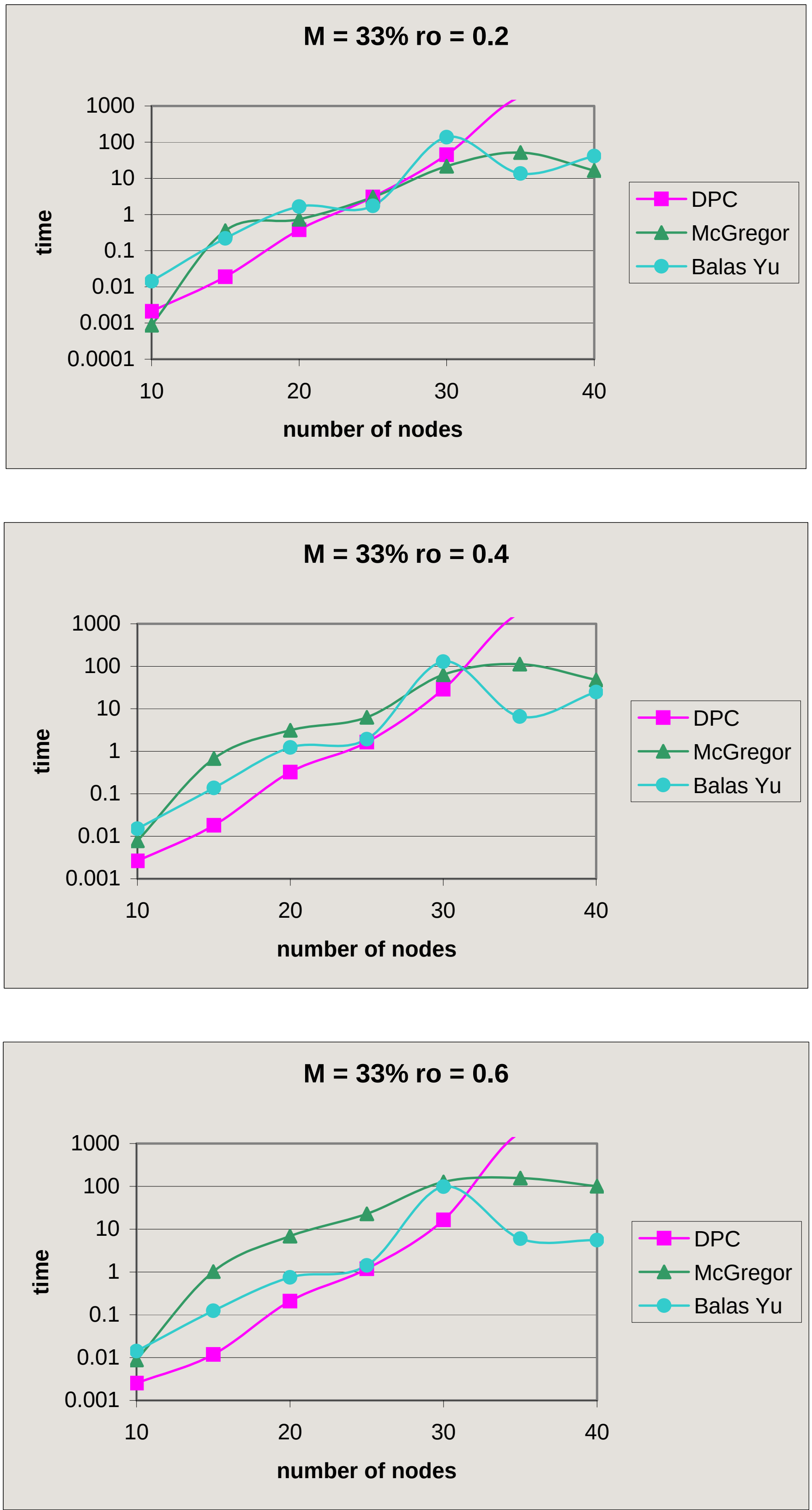
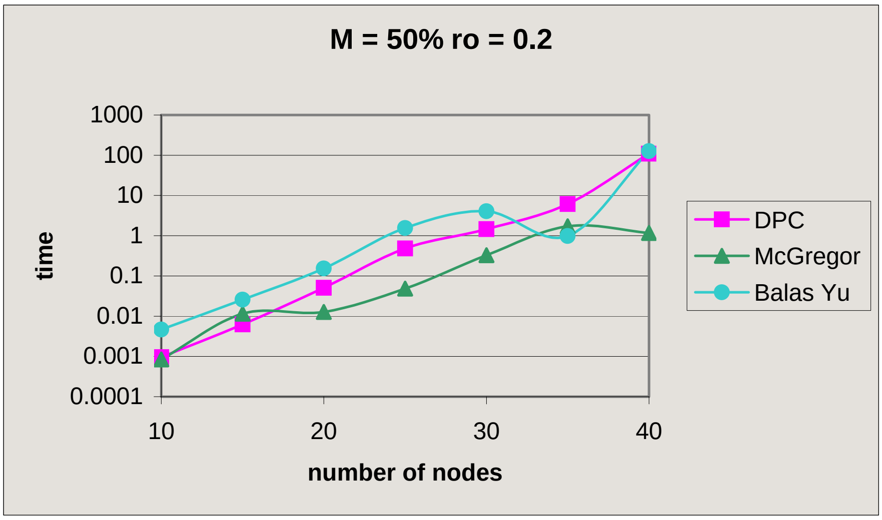
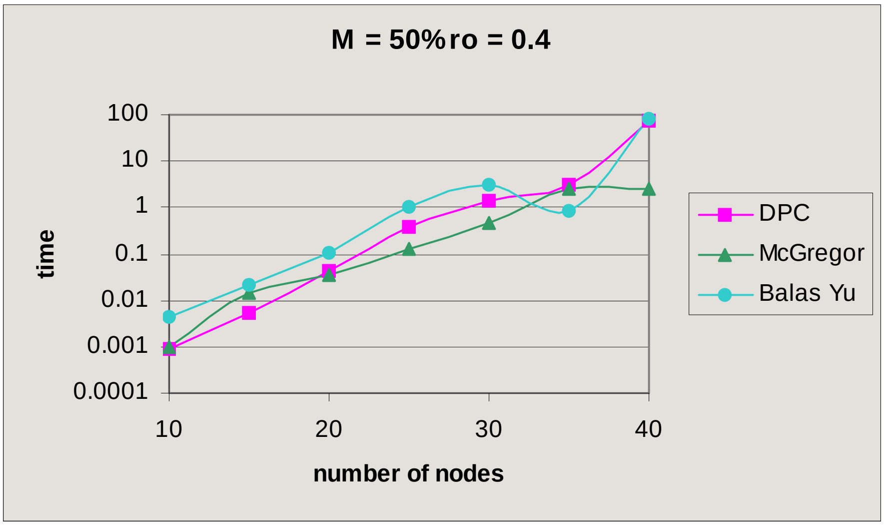


Fig.3: experimental results for irregular meshes 2D graphs. The size of the attributes alphabet is always M=33% of the number of nodes. a) the irregularity degree is ρ = 0.2; b) the irregularity degree is ρ = 0.4; c) the irregularity degree is ρ = 0.6.

a)



b)



c)

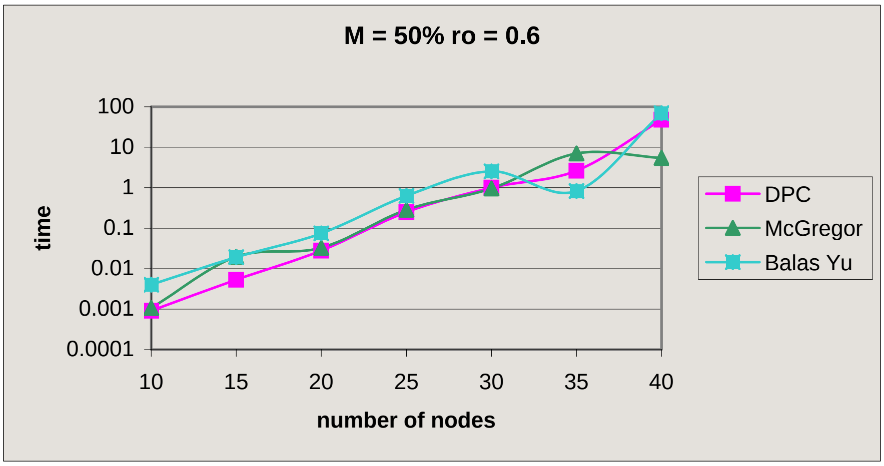


Fig.4: experimental results for irregular meshes 3D graphs. The size of the attributes alphabet is always M=50% of the number of nodes. a) the irregularity degree is $\rho = 0.2$; b) the irregularity degree is $\rho = 0.4$; c) the irregularity degree is $\rho = 0.6$.

a) b) c)

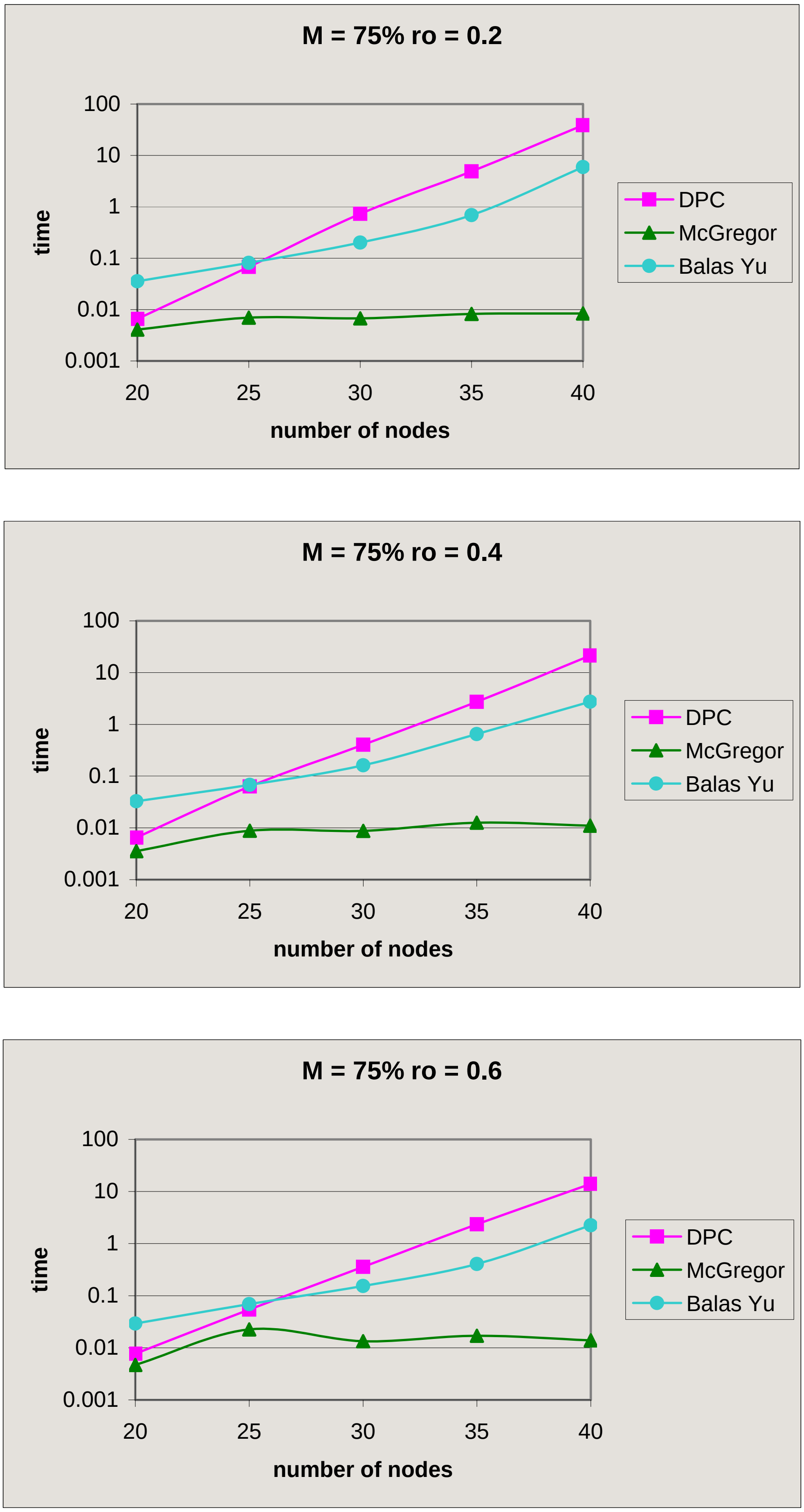


Fig.5: experimental results for irregular meshes 4D graphs. The size of the attributes alphabet is always M=75% of the number of nodes. a) the irregularity degree is ρ = 0.2; b) the irregularity degree is ρ = 0.4; c) the irregularity degree is ρ = 0.6.

Bounded Regular Graphs

In all the three selected cases the maximum connection degree is v =6.

In the first case, Fig.6a, the size of the attribute alphabet M is 33% of the graph size; in the second case, Fig.6b, M = 50% and, finally, in the third case, Fig.6c, M is 75%.

Firstly, it is interesting to notice that Balas Yu and Durand Pasari have a similar behaviour for all the three cases: for small graphs Durand Pasari is better, but for larger graphs Balas Yu become definitely better than the other one. As in precedent cases, the main reason is the more refined heuristic cutting more candidate solutions when the dimension of the staring graphs increases.

What is also interesting to notice is the behaviour of McGregor: this algorithm is incredibly sensible to the size of alphabet. Indeed when M = 33% McGregor is incredibly slower, when M=50% it is a little slower than the other algorithms, but for M=75% it becomes the fastest algorithm, for large graphs.

In this three graphs is really clear that the use of the association graph is convenient only when the size of the alphabet of attributes is enough small, otherwise it is better to use algorithms facing the problem without the association graph.

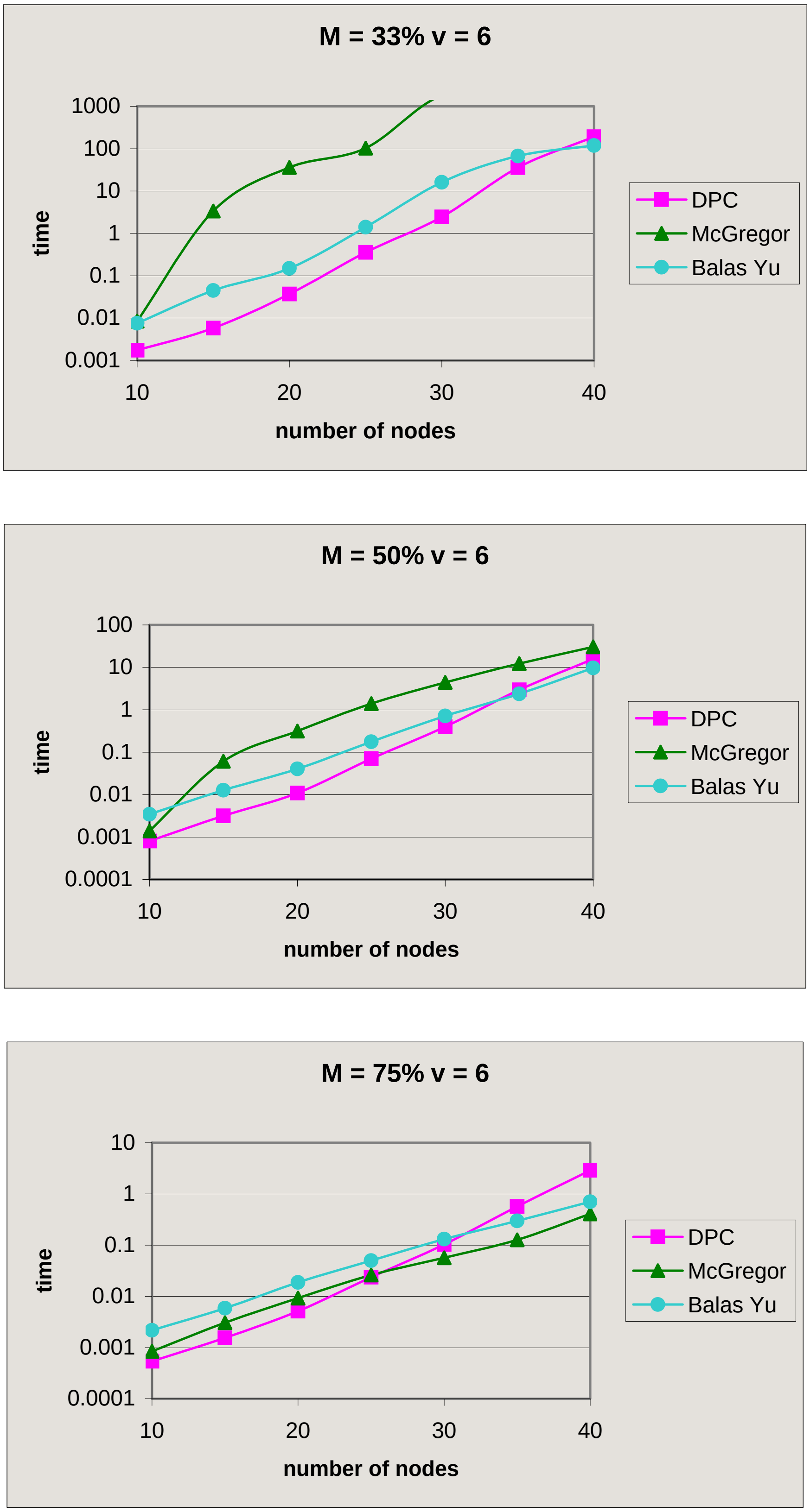


Fig.6: experimental results for regular bounded graphs. The maximum degree connection is always ν=6. a) The size of the attributes alphabet is M=33% of the number of nodes the; b) M=50%; c) M=75%.

Bounded Irregular Graphs

In the first case, Fig.7a, the size of the attribute alphabet M is 75% of the number of nodes and maximum connection degree is v = 3. In this case McGregor performs better; the number of edges per node and the number of attributes are low and then the association graph is large and dense. Then it is not convenient to find out its maximum clique.

In the second case, Fig.7b, M = 50% and v= 6. In this case, for small graphs Durand Pasari is the best algorithm, but it is interesting to notice that the tendency of all the three algorithms is the same for larger graphs. The meaning is that the density of the two starting graphs is enough to make McGregor not the fastest and, the association graph is enough dense to make Durand Pasari not the fastest. Finally the heuristic of Balas Yu, is too complex to elaborate, to make this algorithm winning on the others. The conclusion is that in this case there is not an algorithm definitely faster that the others.

Finally, in the third case, Fig.7c, M is 33% and v=9. In this case the density of the two starting graphs becomes enough high to make convenient to face the problem using the association graph. McGregor is not adapt to use for these graphs. Moreover, this density is not high enough to permit to the heuristic of Balas Yu (which is quite expensive to elaborate) a good pruning, thus this algorithm is not winning on Durand Pasari and performs very similarly but a little worse.

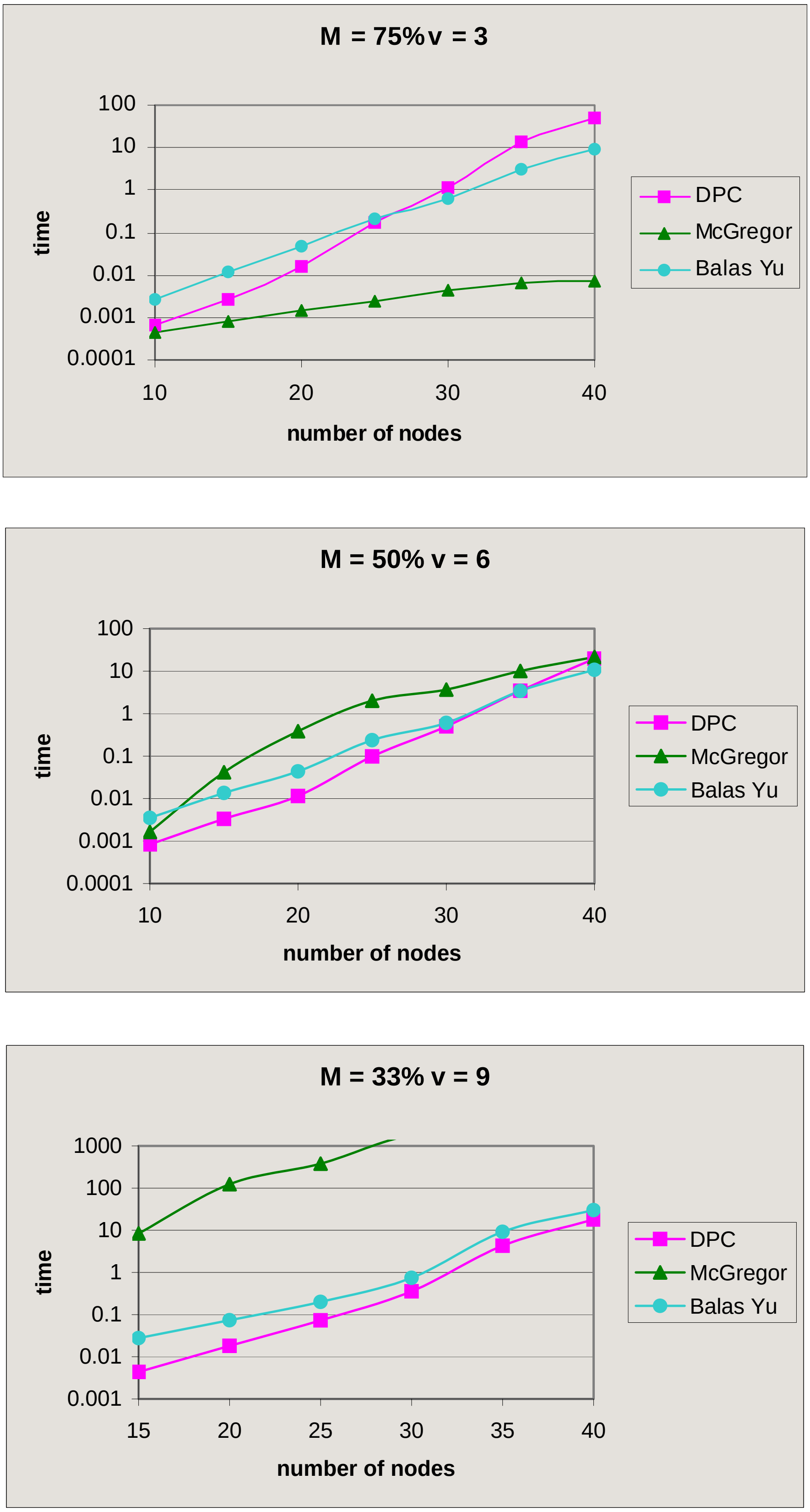


Fig.7: experimental results for irregular bounded graphs. a) The size of the attributes alphabet is M=33%, and the maximum connection degree is ν=3; b) M=50% and ν=6; c) M=75% and ν=9.