Automatic Verification of Avionic Synchronous Safety Critical Embedded Software

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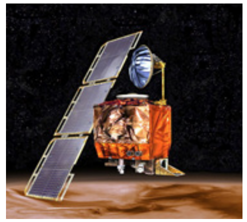
Talk Outline

Motivation

All Computer Scientists Have Experienced Bugs


Ariane 5.01 failure (overflow)


Patriot failure (float rounding)


Mars orbiter loss (unit error)

It is preferable to verify that mission/safety-critical programs do not go wrong before running them.

Static Analysis by Abstract Interpretation

Static analysis: analyse the program at compile-time to verify a program runtime property (e.g. the absence of some categories of bugs)

Undecidability $\longrightarrow$

Abstract interpretation: effectively compute an abstraction/ sound approximation of the program semantics,

– which is precise enough to imply the desired property, and

– coarse enough to be efficiently computable.

Abstract Interpretation, Reminder

Reference

[POPL '77] P. Cousot and R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In 4^{th} *ACM POPL*.

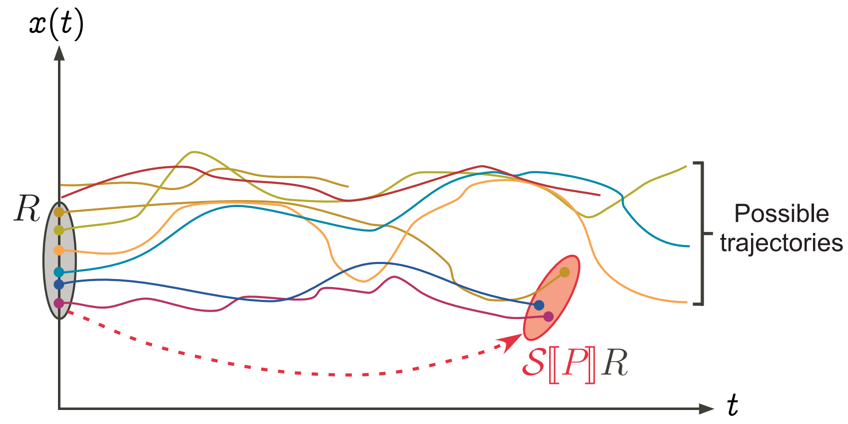
[Thesis '78] P. Cousot. Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique de programmes. Thèse ès sci. math. Grenoble, march 1978.

[POPL '79] P. Cousot & R. Cousot. Systematic design of program analysis frameworks. In 6^{th} *ACM POPL*.

Syntax of programs

X	variables $X \in \mathbb{X}$
T	types $T \in \mathbb{T}$
E	arithmetic expressions $E \in \mathbb{E}$
B	boolean expressions $B \in \mathbb{B}$
$D ::= T\ X\,;$	
$\mid\ T\ X\ ;\ D'$	
$C ::= X\ \texttt{=}\ E;$	commands $C \in \mathbb{C}$
$\mid\ \texttt{while}\ B\ C'$	
$\mid\ \texttt{if}\ B\ C'\ \texttt{else}\ C''$	
$\mid\ \texttt{\{}\ C_1 \ldots C_n\ \texttt{\}}, (n \geq 0)$	
$P ::= D\ C$	program $P \in \mathbb{P}$

Postcondition semantics



States

Values of given type:

$$\mathcal{V}[\![T]\!] \quad : \quad \text{values of type } T \in \mathbb{T}$$

$$\mathcal{V}[\![\texttt{int}]\!] \stackrel{\text{def}}{=} \{z \in \mathbb{Z} \mid \texttt{min\_int} \leq z \leq \texttt{max\_int}\}$$

Program states $\Sigma[\![P]\!]$ [1]:

$$\Sigma[\![D\ C]\!] \stackrel{\text{def}}{=} \Sigma[\![D]\!]$$
$$\Sigma[\![T\ X\,;]\!] \stackrel{\text{def}}{=} \{X\} \mapsto \mathcal{V}[\![T]\!]$$
$$\Sigma[\![T\ X\,;\ D]\!] \stackrel{\text{def}}{=} (\{X\} \mapsto \mathcal{V}[\![T]\!]) \cup \Sigma[\![D]\!]$$

[1] States $\rho \in \Sigma[\![P]\!]$ of a program P map program variables X to their values $\rho(X)$

Concrete Semantic Domain of Programs

Concrete semantic domain for reachability properties:

$$\mathcal{D}[\![P]\!] \stackrel{\text{def}}{=} \wp(\Sigma[\![P]\!]) \qquad \text{sets of states}$$

i.e. program properties where $\subseteq$ is implication, $\emptyset$ is false, $\cup$ is disjunction.

Concrete Reachability Semantics of Programs

$$\mathcal{S}[\![X = E;]\!]R \stackrel{\text{def}}{=} \{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in R \cap \text{dom}(E)\}$$
$$\rho[X \leftarrow v](X) \stackrel{\text{def}}{=} v, \qquad \rho[X \leftarrow v](Y) \stackrel{\text{def}}{=} \rho(Y)$$
$$\mathcal{S}[\![\texttt{if}\ B\ C']\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]R) \cup \mathcal{B}[\![\neg B]\!]R$$
$$\mathcal{B}[\![B]\!]R \stackrel{\text{def}}{=} \{\rho \in R \cap \text{dom}(B) \mid B \text{ holds in } \rho\}$$
$$\mathcal{S}[\![\texttt{if}\ B\ C'\ \texttt{else}\ C'']\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]R) \cup \mathcal{S}[\![C'']\!](\mathcal{B}[\![\neg B]\!]R)$$
$$\mathcal{S}[\![\texttt{while}\ B\ C']\!]R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \textsf{lfp}^{\subseteq}_{\emptyset}\ \lambda \mathcal{X} \cdot R \cup \mathcal{S}[\![C']\!](\mathcal{B}[\![B]\!]\mathcal{X}) \text{ in } (\mathcal{B}[\![\neg B]\!]\mathcal{W})$$
$$\mathcal{S}[\![\{\}]\!]R \stackrel{\text{def}}{=} R$$
$$\mathcal{S}[\![\{C_1 \ldots C_n\}]\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C_n]\!] \circ \ldots \circ \mathcal{S}[\![C_1]\!] \quad n > 0$$
$$\mathcal{S}[\![D\ C]\!]R \stackrel{\text{def}}{=} \mathcal{S}[\![C]\!](\Sigma[\![D]\!]) \quad \text{(uninitialized variables)}$$

Not computable (undecidability).

Abstract Semantic Domain of Programs

$$\langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq,\ \perp,\ \sqcup \rangle$$

such that:

$$\langle \mathcal{D},\ \subseteq \rangle \underset{\alpha}{\overset{\gamma}{\longleftrightarrow}} \langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq \rangle$$

hence $\langle \mathcal{D}^\sharp[\![P]\!],\ \sqsubseteq,\ \perp,\ \sqcup \rangle$ is a complete lattice such that $\perp = \alpha(\emptyset)$ and $\sqcup X = \alpha(\cup\, \gamma(X))$

Reduced Product of Abstract Domains

To combine abstractions

$$\langle \mathcal{D},\ \subseteq\rangle \xleftrightarrow[\alpha_1]{\gamma_1} \langle \mathcal{D}^\sharp_1,\ \sqsubseteq_1\rangle \text{ and } \langle \mathcal{D},\ \subseteq\rangle \xleftrightarrow[\alpha_2]{\gamma_2} \langle \mathcal{D}^\sharp_2,\ \sqsubseteq_2\rangle$$

the reduced product is

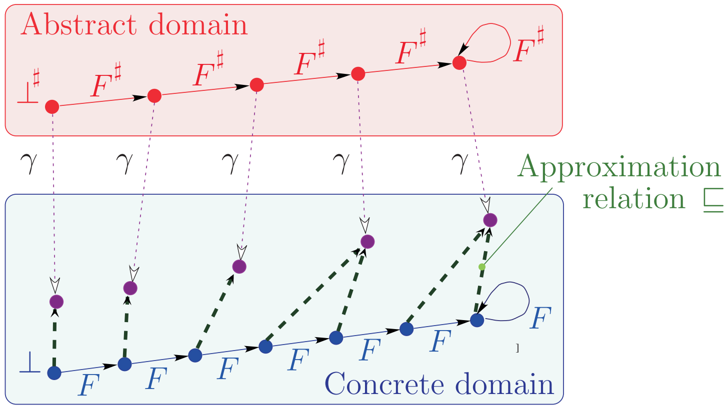
$$\alpha(X) \stackrel{\text{def}}{=} \sqcap\{\langle x,\ y\rangle \mid X \subseteq \gamma_1(X) \wedge X \subseteq \gamma_2(X)\}$$

such that $\sqsubseteq \stackrel{\text{def}}{=} \sqsubseteq_1 \times \sqsubseteq_2$ and

$$\langle \mathcal{D},\ \subseteq\rangle \xleftrightarrow[\alpha]{\gamma_1\times\gamma_2} \langle \alpha(\mathcal{D}),\ \sqsubseteq\rangle$$

Example: $\mathtt{x} \in [1, 9] \wedge \mathtt{x} \bmod 2 = 0$ reduces to $\mathtt{x} \in [2, 8] \wedge \mathtt{x} \bmod 2 = 0$

Approximate Fixpoint Abstraction

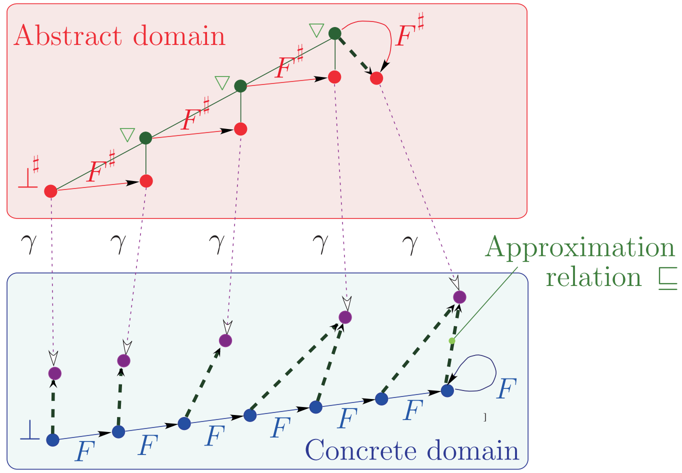


$$F \circ \gamma \sqsubseteq \gamma \circ F^\sharp \Rightarrow \mathsf{lfp}\ F \sqsubseteq \gamma(\mathsf{lfp}\ F^\sharp)$$

Abstract Reachability Semantics of Programs

$$\begin{aligned}
\mathcal{S}^\sharp[\![X = E;]\!]R &\stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in \gamma(R) \cap \mathrm{dom}(E)\})\\
\mathcal{S}^\sharp[\![\mathtt{if}\ B\ C']\!]R &\stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{B}^\sharp[\![\neg B]\!]R\\
\mathcal{B}^\sharp[\![B]\!]R &\stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \mathrm{dom}(B) \mid B \text{ holds in } \rho\})\\
\mathcal{S}^\sharp[\![\mathtt{if}\ B\ C'\ \mathtt{else}\ C'']\!]R &\stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{S}^\sharp[\![C'']\!](\mathcal{B}^\sharp[\![\neg B]\!]R)\\
\mathcal{S}^\sharp[\![\mathtt{while}\ B\ C']\!]R &\stackrel{\text{def}}{=} \text{let } \mathcal{W} = \mathsf{lfp}^{\sqsubseteq}_{\bot}\ \lambda\mathcal{X}\cdot R \sqcup \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]\mathcal{X})\\
&\qquad \text{in } (\mathcal{B}^\sharp[\![\neg B]\!]\mathcal{W})\\
\mathcal{S}^\sharp[\![\{\}]\!]R &\stackrel{\text{def}}{=} R\\
\mathcal{S}^\sharp[\![\{C_1 \ldots C_n\}]\!]R &\stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C_n]\!] \circ \ldots \circ \mathcal{S}^\sharp[\![C_1]\!] \quad n > 0\\
\mathcal{S}^\sharp[\![D\ C]\!]R &\stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C]\!](\top) \quad \text{(uninitialized variables)}
\end{aligned}$$

Convergence Acceleration with Widening



Abstract Semantics with Convergence Acceleration [2]

$$\mathcal{S}^\sharp[\![X = E;]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[\![E]\!]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\})$$
$$\mathcal{S}^\sharp[\![\texttt{if } B\ C']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{B}^\sharp[\![\neg B]\!]R$$
$$\mathcal{B}^\sharp[\![B]\!]R \stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\})$$
$$\mathcal{S}^\sharp[\![\texttt{if } B\ C' \texttt{ else } C'']\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]R) \sqcup \mathcal{S}^\sharp[\![C'']\!](\mathcal{B}^\sharp[\![\neg B]\!]R)$$
$$\mathcal{S}^\sharp[\![\texttt{while } B\ C']\!]R \stackrel{\text{def}}{=} \text{let } \mathcal{F}^\sharp = \lambda \mathcal{X} \cdot \text{let } \mathcal{Y} = R \sqcup \mathcal{S}^\sharp[\![C']\!](\mathcal{B}^\sharp[\![B]\!]\mathcal{X})$$
$$\text{in if } \mathcal{Y} \sqsubseteq \mathcal{X} \text{ then } \mathcal{X} \text{ else } \mathcal{X} \mathbin{\triangledown} \mathcal{Y}$$
$$\text{and } \mathcal{W} = \mathsf{lfp}_{\bot}^{\sqsubseteq} \mathcal{F}^\sharp \quad \text{in } (\mathcal{B}^\sharp[\![\neg B]\!]\mathcal{W})$$
$$\mathcal{S}^\sharp[\![\{\}]\!]R \stackrel{\text{def}}{=} R$$
$$\mathcal{S}^\sharp[\![\{C_1 \ldots C_n\}]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C_n]\!] \circ \ldots \circ \mathcal{S}^\sharp[\![C_1]\!] \quad n > 0$$
$$\mathcal{S}^\sharp[\![D\ C]\!]R \stackrel{\text{def}}{=} \mathcal{S}^\sharp[\![C]\!](\top) \quad \text{(uninitialized variables)}$$

[2] Note: $\mathcal{F}^\sharp$ not monotonic!

Applications of Abstract Interpretation

- **Static Program Analysis** [POPL '77], [POPL '78], [POPL '79] including **Dataflow Analysis** [POPL '79], [POPL '00], **Set-based Analysis** [FPCA '95], **Predicate Abstraction** [Manna's festschrift '03], ...
- **Syntax Analysis** [TCS 290(1) 2002]
- **Hierarchies of Semantics (including Proofs)** [POPL '92], [TCS 277(1–2) 2002]
- **Typing & Type Inference** [POPL '97]

Applications of Abstract Interpretation

Applications of Abstract Interpretation (Cont'd)

- **(Abstract) Model Checking** [POPL '00]
- **Program Transformation** [POPL '02]
- **Software Watermarking** [POPL '04]
- **Bisimulations** [RT-ESOP '04]

All these techniques involve sound approximations that can be formalized by abstract interpretation

A Practical Application of Abstract Interpretation to the Verification of Safety Critical Embedded Control-Command Software

Reference

[1] http://www.astree.ens.fr/

ASTRÉE: A Sound, Automatic, Specializable, Domain-Aware, Parametric, Modular, Efficient and Precise Static Program Analyzer

www.astree.ens.fr

– C programs:
  - with
    - pointers (including on functions), structures and arrays
    - floating point computations
    - tests, loops and function calls
    - limited branching (forward `goto`, `break`, `continue`)

  - without
    - `union`
    - dynamic memory allocation
    - recursive function calls
    - backward branching
    - conflict side effects
    - C libraries

– Application Domain: safety critical embedded real-time synchronous software for non-linear control of very complex control/command systems.

Concrete Operational Semantics

– International norm of C (ISO/IEC 9899:1999)

– *restricted by* implementation-specific behaviors depending upon the machine and compiler (e.g. representation and size of integers, IEEE 754-1985 norm for floats and doubles)

– *restricted by* user-defined programming guidelines (such as no modular arithmetic for signed integers, even though this might be the hardware choice)

– *restricted by* program specific user requirements (e.g. `assert`)

Abstract Semantics

- Reachable states for the concrete trace operational semantics
- Volatile environment is specified by a *trusted* configuration file.

Requirements:

- Soundness: absolutely essential
- Precision: few or no false alarm (full certification)
- Efficiency: rapid analyses and fixes during development

Implicit Specification: Absence of Runtime Errors

- No violation of the norm of C (e.g. array index out of bounds, division by zero)
- No implementation-specific undefined behaviors (e.g. maximum short integer is 32767, NaN)
- No violation of the programming guidelines (e.g. static variables cannot be assumed to be initialized to 0)
- No violation of the programmer assertions (must all be statically verified).

Example application

- Primary flight control software of the Airbus A340 family/A380 fly-by-wire system

- C program, automatically generated from a proprietary high-level specification (à la Simulink/Scade)
- A340 family: 132,000 lines, 75,000 LOCs after preprocessing, 10,000 global variables, over 21,000 after expansion of small arrays
- A380: $\times$ 3

The Class of Considered Periodic Synchronous Programs

```
declare volatile input, state and output variables;
initialize state and output variables;
loop forever
   - read volatile input variables,
   - compute output and state variables,
   - write to volatile output variables;
  wait_for_clock ();
end loop
```

- Requirements: the only interrupts are clock ticks;
- Execution time of loop body less than a clock tick [EMSOFT '01].

Challenging aspects

- Size: $>$ 100 kLOC, $>$ 10 000 variables
- Floating point computations
  including filtering, non linear control with feedback, interpolations...
- Interdependencies among variables:
  - Stability of computations should be established
  - Complex relations should be inferred among numerical and boolean data
  - Very long data paths from input to outputs

Characteristics of the ASTRÉE Analyzer

Static: compile time analysis ($\neq$ run time analysis Rational Purify, Parasoft Insure++)

Program Analyzer: analyzes programs not micromodels of programs ($\neq$ PROMELA in SPIN or Alloy in the Alloy Analyzer)

Automatic: no end-user intervention needed ($\neq$ ESC Java, ESC Java 2)

Sound: covers the whole state space ($\neq$ MAGIC, CBMC) so never omit potential errors ($\neq$ UNO, CMC from coverity.com) or sort most probable ones ($\neq$ Splint)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Multiabstraction: uses many numerical/symbolic abstract domains ($\neq$ symbolic constraints in Bane or the canonical abstraction of TVLA)

Infinitary: all abstractions use infinite abstract domains with widening/narrowing ($\neq$ model checking based analyzers such as VeriSoft, Bandera, Java PathFinder)

Efficient: always terminate ($\neq$ counterexample-driven automatic abstraction refinement BLAST, SLAM)

Characteristics of the ASTRÉE Analyzer (Cont'd)

Specializable: can easily incorporate new abstractions (and reduction with already existing abstract domains) ($\neq$ general-purpose analyzers PolySpace Verifier)

Domain-Aware: knows about control/command (e.g. digital filters) (as opposed to specialization to a mere programming style in C Global Surveyor)

Parametric: the precision/cost can be tailored to user needs by options and directives in the code

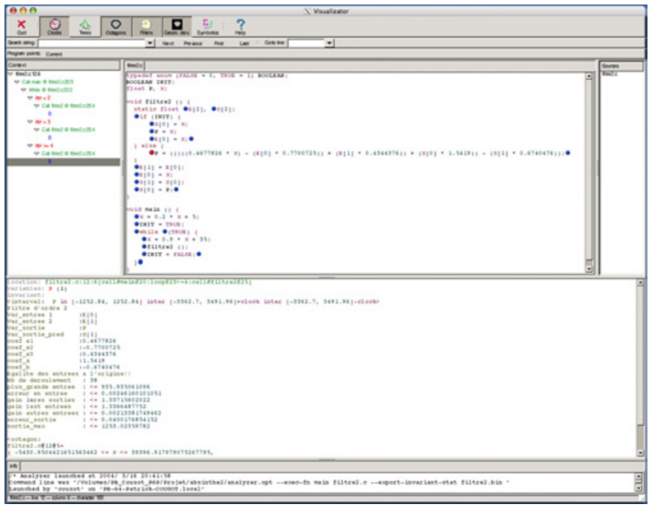
Characteristics of the ASTRÉE Analyzer (Cont'd)

Automatic Parametrization: the generation of parametric directives in the code can be programmed (to be specialized for a specific application domain)

Modular: an analyzer instance is built by selection of O-CAML modules from a collection each implementing an abstract domain

Precise: very few or no false alarm when adapted to an application domain ⟶ it is a VERIFIER!

Example of Analysis Session



Benchmarks (Airbus A340 Primary Flight Control Software)

- 132,000 lines, 75,000 LOCs after preprocessing
- Comparative results (commercial software):
  4,200 (false?) alarms,
  3.5 days;
- Our results:
  0 alarms,
  40mn on 2.8 GHz PC,
  300 Megabytes
  ⟶ A world première!

(Airbus A380 Primary Flight Control Software)

- 450,000 lines
- 0 alarms (Nov. 2004),
  7h [3] on 2.8 GHz a PC,
  1 Gigabyte
  ⟶ A world grand première!

[3] It would be possible to favour computation costs rather than precision, and this should go down. For example, the A340 analysis went up to 5 h, before being reduced by requiring less precision while still getting no false alarm.

Examples of Abstractions

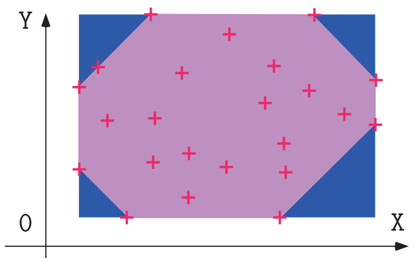
Floating-Point Computations

```
/* float-error.c */
int main () {
  float x, y, z, r;
  x = 1.000000019e+38;
  y = x + 1.0e21;
  z = x - 1.0e21;
  r = y - z;
  printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
double x; float y, z, r;
/* x = ldexp(1.,50)+ldexp(1.,26); */
x = 1125899973951488.0;
y = x + 1;
z = x - 1;
r = y - z;
printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
134217728.000000
```

$$(x + a) - (x - a) \neq 2a$$

General-Purpose Abstract Domains: Intervals and Octagons



Intervals:

$$\begin{cases} 1 \leq x \leq 9 \\ 1 \leq y \leq 20 \end{cases}$$

Octagons [10]:

$$\begin{cases} 1 \leq x \leq 9 \\ x + y \leq 77 \\ 1 \leq y \leq 20 \\ x - y \leq 04 \end{cases}$$

Difficulties: many global variables, arrays (smashed or not), IEEE 754 floating-point arithmetic (in program <u>and</u> analyzer) [10, 11]

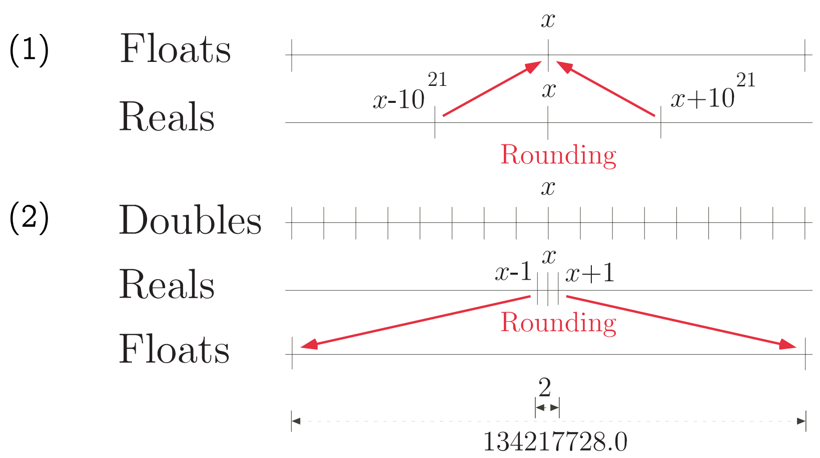
Floating-Point Computations

```
/* float-error.c */
int main () {
  float x, y, z, r;
  x = 1.000000019e+38;
  y = x + 1.0e21;
  z = x - 1.0e21;
  r = y - z;
  printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
double x; float y, z, r;
/* x = ldexp(1.,50)+ldexp(1.,26); */
x = 1125899973951487.0;
y = x + 1;
z = x - 1;
r = y - z;
printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
0.000000
```

$$(x + a) - (x - a) \neq 2a$$

Explanation of the huge rounding error



Symbolic abstract domain

- Interval analysis: if $x \in [a, b]$ and $y \in [c, d]$ then $x - y \in [a - d, b - c]$ so if $x \in [0, 100]$ then $x - x \in [-100, 100]$!!!
- The symbolic abstract domain propagates the symbolic values of variables and performs simplifications;
- Must maintain the maximal possible rounding error for float computations (overestimated with intervals);

```
% cat -n x-x.c
     1  void main () { int X, Y;
     2          __ASTREE_known_fact(((0 <= X) && (X <= 100)));
     3          Y = (X - X);
     4          __ASTREE_log_vars((Y));
     5  }
```

```
astree -exec-fn main -no-relational x-x.c
Call main@x-x.c:1:5-x-x.c:1:9:
<interval: Y in [-100, 100]>
```

```
astree -exec-fn main x-x.c
Call main@x-x.c:1:5-x-x.c:1:9:
<interval: Y in {0}> <symbolic: Y = (X -i X)>
```


Clock Abstract Domain for Counters

- Code Sample:

```
R = 0;
while (1) {
  if (I)
    { R = R+1; }
  else
    { R = 0; }
  T = (R>=n);
  wait_for_clock ();
}
```

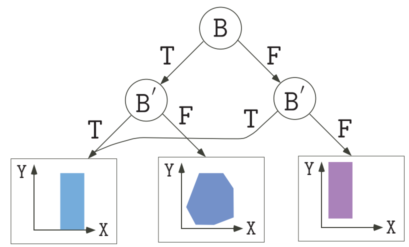
- Output `T` is true iff the volatile input `I` has been true for the last `n` clock ticks.
- The clock ticks every `s` seconds for at most `h` hours, thus `R` is bounded.
- To prove that `R` cannot overflow, we must prove that `R` cannot exceed the elapsed clock ticks (*impossible using only intervals*).

- Solution:
  - We add a phantom variable `clock` in the concrete user semantics to track elapsed clock ticks.
  - For each variable `X`, we abstract three intervals: `X`, `X+clock`, and `X-clock`.
  - If `X+clock` or `X-clock` is bounded, so is `X`.

Boolean Relations for Boolean Control

- Code Sample:

```
/* boolean.c */
typedef enum {F=0,T=1} BOOL;
BOOL B;
void main () {
  unsigned int X, Y;
  while (1) {
    ...
    B = (X == 0);
    ...
    if (!B) {
      Y = 1 / X;
    }
    ...
  }
}
```



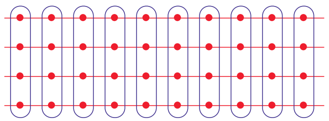
The boolean relation abstract domain is parameterized by the height of the decision tree (an analyzer option) and the abstract domain at the leafs

Control Partitionning for Case Analysis

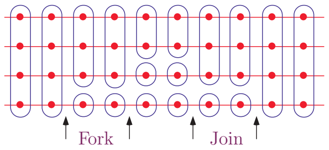
– Code Sample:

```
/* trace_partitionning.c */
void main() {
  float t[5] = {-10.0, -10.0, 0.0, 10.0, 10.0};
  float c[4] = {0.0, 2.0, 2.0, 0.0};
  float d[4] = {-20.0, -20.0, 0.0, 20.0};
  float x, r;
  int i = 0;

  ... found invariant −100 ≤ x ≤ 100 ...

  while ((i < 3) && (x >= t[i+1])) {
    i = i + 1;
  }
  r = (x - t[i]) * c[i] + d[i];
}
```

Control point partitionning:



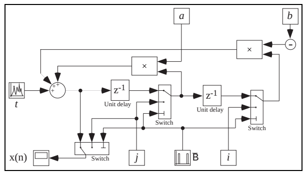
Trace partitionning:



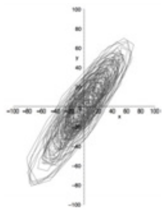
Delaying abstract unions in tests and loops is more precise for non-distributive abstract domains (and much less expensive than disjunctive completion).

Ellipsoid Abstract Domain for Filters

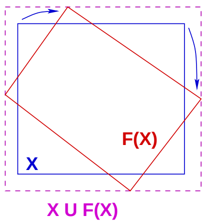
2^d Order Digital Filter:



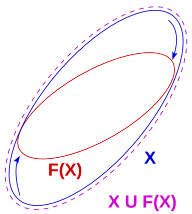
- Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$
- The concrete computation is bounded, which must be proved in the abstract.
- There is no stable interval or octagon.
- The simplest stable surface is an ellipsoid.



execution trace



unstable interval



stable ellipsoid

Filter Example [7]

```
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;
void filter () {
  static float E[2], S[2];
  if (INIT) { S[0] = X; P = X; E[0] = X; }
  else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
              + (S[0] * 1.5)) - (S[1] * 0.7)); }
  E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
  /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}
void main () { X = 0.2 * X + 5; INIT = TRUE;
  while (1) {
    X = 0.9 * X + 35; /* simulated filter input */
    filter (); INIT = FALSE; }
}
```


Arithmetic-geometric progressions

```
% cat retro.c
typedef enum {FALSE=0, TRUE=1} BOOL;
BOOL FIRST;
volatile BOOL SWITCH;
volatile float E;
float P, X, A, B;

void dev( )
{ X=E;
  if (FIRST) { P = X; }
  else
    { P = (P - ((((2.0 * P) - A) - B)
            * 4.491048e-03)); };
  B = A;
  if (SWITCH) {A = P;}
  else {A = X;}
}
```

```
void main()
{ FIRST = TRUE;
  while (TRUE) {
    dev( );
    FIRST = FALSE;
    __ASTREE_wait_for_clock(());
  }}
% cat retro.config
__ASTREE_volatile_input((E [-15.0, 15.0]));
__ASTREE_volatile_input((SWITCH [0,1]));
__ASTREE_max_clock((3600000));
|P| <= (15. + 5.87747175411e-39
/ 1.19209290217e-07) * (1 +
1.19209290217e-07)^clock -
5.87747175411e-39 / 1.19209290217e-07
<= 23.0393526881
```

(Automatic) Parameterization

– All abstract domains of ASTRÉE are parameterized, e.g.
  – variable packing for octagones and decision trees,
  – partition/merge program points,
  – loop unrollings,
  – thresholds in widenings, ...;
– End-users can either parameterize by hand (analyzer options, directives in the code), or
– choose the automatic parameterization (default options, directives for pattern-matched predefined program schemata).

Possible origins of imprecision and how to fix it

In case of false alarm, the imprecision can come from:

– Abstract transformers (not best possible) $\longrightarrow$ improve algorithm;
– Automatized parametrization (e.g. variable packing) $\longrightarrow$ improve pattern-matched program schemata;
– Iteration strategy for fixpoints $\longrightarrow$ fix widening [4];
– Inexpressivity i.e. indispensable local inductive invariant are inexpressible in the abstract $\longrightarrow$ add a new abstract domain to the reduced product (e.g. filters).

[4] This can be very hard since at the limit only a precise infinite iteration might be able to compute the proper abstract invariant. In that case, it might be better to design a more refined abstract domain.

The main loop invariant for the A340

A textual file over 4.5 Mb with

– 6,900 boolean interval assertions ($x \in [0; 1]$)
– 9,600 interval assertions ($x \in [a; b]$)
– 25,400 clock assertions ($x + \mathrm{clk} \in [a; b] \wedge x - \mathrm{clk} \in [a; b]$)
– 19,100 additive octagonal assertions ($a \leq x + y \leq b$)
– 19,200 subtractive octagonal assertions ($a \leq x - y \leq b$)
– 100 decision trees
– 60 ellipse invariants, etc ...

involving over 16,000 floating point constants (only 550 appearing in the program text) $\times$ 75,000 LOCs.

Conclusion

Conclusion

– Most applications of abstract interpretation tolerate a small rate (typically 5 to 15%) of false alarms:
  - Program transformation → do not optimize,
  - Typing → reject some correct programs, etc,
  - WCET analysis → overestimate;
– Some applications require no false alarm at all:
  - Program verification.
– Theoretically possible [SARA '00], practically feasible [PLDI '03]

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The Future & Grand Challenges

Forthcoming (1 year):

– More general memory model (`union`)

Future (5 years):

– Asynchronous concurrency (for less critical software)

– Functional properties (reactivity)

– Industrialization

Grand challenge:

– Verification from specifications to machine code (verifying compiler)

– Verification of systems (quasi-synchrony, distribution)

THE END, THANK YOU

More references at URL `www.di.ens.fr/~cousot`
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