The ASTRÉE Analyzer

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Certification of Critical Software

Goal of ASTRÉE:
Prove the absence of runtime errors in embedded, C programs

- Situation: bugs are no longer acceptable in such systems
  - May cause human casualties: transportation, energy
  - Prohibitive financial cost: Ariane 501 failure
- Automatic certification of the code is required
  - Soundness: absolutely essential
  - Strong efficiency requirements:
    - Keep analysis time low: analyses and fixes possible during development
    - Precision: number of alarms should be reasonable (e.g. < 10)
  - No alarm = full certification (If possible)

Specialization of the Analyzer

- Specialization to a family of embedded programs: Synchronous, real-time code:

```
declare and initialize state variables;
loop forever
  read volatile input variables,
  compute output and state variables,
  write to volatile output variables;
  wait for next clock tick
end loop
```

- Properties of interest: Absence of run-time errors
  - No Run-Time Error e.g. division by 0, NaN, out-of-bound array access
  - No integer / floating point overflow
  - User-defined properties, e.g. architecture dependent

Specialization of the Analyzer

- Simplifying aspects:
  - Not full C: no malloc, no pointer arithmetic
  - No recursion
  - Data mostly static
- Challenging aspects:
  - Size: > 100 kLOC, $> 10\,000$ variables
  - Floating point computations including filtering, non linear control with feed-back, interpolations...
  - Interdependencies among variables:
    - Stability of computations should be established
    - Complex relations should be inferred among numerical, boolean data
    - Very long data paths from input to outputs

Principle

Compute an over-approximation of the reachable states

- Model of the language: semantics
  $[\![P]\!]$ = set of runs (i.e. traces) of P
  - C 99 norm
  - IEEE 754-1985 Floating point norm
  - User/architecture defined assumptions:
    - Size of integer data types
    - Initialization of statically allocated variables
    - Range of inputs; maximum program run-time
- Abstraction = approximation relying on abstract domains
- Derivation of an automatic, sound static analyzer
- Certification of a program:
  - Fully automatic computation of an invariant
  - Verification of the absence of runtime errors

Outline

Development of ASTRÉE

- Fall 2001: demand for a high precision, fast analyzer
  ASTRÉE project started:
  - Scalability ensured first (algorithms and data-structures)
  - Simple, non relational domains
  - First refinements
  - Analysis of 10 kLOCs, low number of alarms
- Then, real applications considered:
  - Investigation of an alarm:
    $\Rightarrow$ ource of imprecision
    $\Rightarrow$ true alarm or need for a refinement?
  - Implementation of new domains,
    solve imprecisions and preserve scalability
  - Analysis of two families of real-world, large applications

Abstractions and Abstract Domains

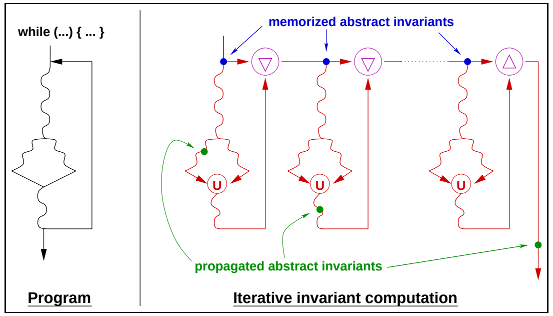
- What ASTRÉE computes:
  Invariant $I \in D^\sharp$: approximation for the set of traces $[\![P]\!]$
- Structure of the abstraction $I \in D^\sharp$:
  - For each control point l
  - For each execution context κ (e.g. calling stack)

  $\Rightarrow$ an approximation $I(l, \kappa) \in D^\sharp_{\mathtt{M}}$ for a set of memory states
- Layout of $D^\sharp_{\mathtt{M}}$:
  - Reduced product of a collection of abstract domains
  - Each domain:
    - Expresses a (generally infinite) family of predicates
    - Transfer functions: $\mathit{assign}, \mathit{guard}$, ... operators
    - Approximations for $\cup$: $\sqcup$ and ∇ (convergence acceleration)

The Abstract Interpreter

Principle: play all executions in a single, abstract computation

- Analysis of a basic statement $x = e$:
  - Transfer function $\mathit{assign}(x = e) : D^\sharp_{\mathtt{M}} \to D^\sharp_{\mathtt{M}}$
  - $D^\sharp_{\mathtt{M}}$: accounts for the new/removed constraints
- Analyzing compound programs, e.g. loops:



Outline

- The ASTRÉE Abstract Interpreter
- ✓ Abstract Domains
- Practical Use and Benchmarks
- Conclusions and Perspectives

A Simple Domain: Intervals

- Resolution of concrete cells:
  - 1 abstract cell $\equiv 1$ or more concrete cells (smashed arrays)
  - Simple points-to information
- Interval abstraction:
  - Constraints $a \leq x \leq b$ (x abstract cell)
  - Very common abstraction
  - Implementation: sound approximation for floating point (in all domains)
- Development of ASTRÉE: structure + intervals was the base:
  - Enough to express the absence of runtime-errors (array bounds, overflows)
  - Not enough to express its proof

Octagons

assume$(\mathtt{x} \in [-10, 10])$
if$(\mathtt{x} < 0)\{\mathtt{y} = -\mathtt{x};\}$
else$\{\mathtt{y} = \mathtt{x};\}$
① **if**$(y \leq 5)$
$\{$② **assert**$(-5 \leq x \leq 5);\}$

- Interval analysis:
  - At ①, $\mathtt{x} \in [-10, 10];\ \mathtt{y} \in [0, 10]$
  - At ②, $\mathtt{x} \in [-10, 10];\ \mathtt{y} \in [0, 5]$
  - Alarm (assert not proved)
- A relation between $\mathtt{x}$ and $\mathtt{y}$ is required:
  $\Rightarrow$ We need a relational abstraction
- Octagons:
  - Express constraints of the form $\pm x \pm y \leq c$.
    Above example:
    - At ①, $0 \leq \mathtt{y} - \mathtt{x} \leq 20;\ 0 \leq \mathtt{y} + \mathtt{x} \leq 20$
    - At ②, $\mathtt{y} \in [0, 5];\ 0 \leq \mathtt{y} - \mathtt{x} \leq 20;\ 0 \leq \mathtt{y} + \mathtt{x} \leq 20$,
      so we derive $\mathtt{x} \in [-5, 5]$
  - Reasonable cost: $\mathcal{O}(n^2)$ memory and $\mathcal{O}(n^3)$ time complexity

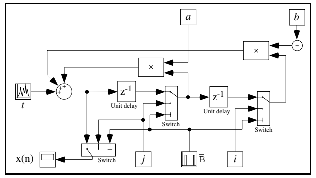
Using Octagons

Several issues should be addressed:

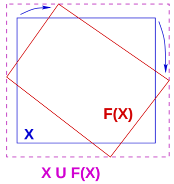
- Scalability: $\mathcal{O}(n^3)$ time, $n \equiv 10\,000$ will not scale:
  $\Rightarrow$ Use many small octagons instead of a big one
  - Packs: small group of variables relations are required for
  - Strategy: determines packs, required relations represented
  - Complexity: linear in the number of packs
    Size of packs: bounded by a constant
    Number of packs: linear in the size of the code
    $\Rightarrow$ Linear complexity
- Floating point rounding errors in the concrete computations
  solved by linearization of expressions:
  - Expressions approximated with real interval linear forms
  - Relational domain: semantics in terms of real numbers
  - Rounding errors in concrete computations accounted for at linearization time

Analyzing Filters

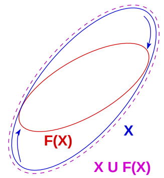
Simplified 2nd Order Filter:



- Computes $\boxed{X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}}$
- The concrete computation is bounded, which must be proved in the abstract
- No stable interval or octagon
- Simplest stable surface is an ellipsoid



Unstable interval



Stable ellipsoid

Bounding Slow Divergences

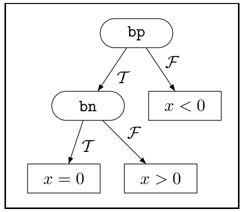
```
x = 1.0;
while(TRUE){①
  x = x/3.0;
  x = x*3.0;
}
```

- With real numbers: $x = 1.0$ at ①
- Floating point computations
  $\Rightarrow$ rounding errors in the concrete
- Rounding errors accumulate;
  possible cause of divergence
- Solution: an arithmetico-geometric progression domain, aimed at bounding the rounding errors using the number of iterations:
  - Relation $|\, x \,| \leq A \cdot B^n + C$,
    where A, B, C are constants, n is the iteration number
  - Number of iterations: bounded by N: $\Rightarrow |\, x \,| \leq A \cdot B^N + C$
- Ellipsoids, progressions:
  - Domains based on external mathematical theorems
  - Beyond what automatic refinement could do

Decision Trees

```
bp = x ≤ 0.;
bn = x ≥ 0.;
①if(bp && bn)
  ②y = 0.0;
else y = 1.0/x;
```

- Non relational analysis: alarm at ② (div. 0)
- Relations needed at ①:
  - bp = FALSE $\Rightarrow$ x $\neq 0$
  - bn = FALSE $\Rightarrow$ x $\neq 0$



- Domain similar to BDDs:
  - Nodes labeled with booleans variables
  - Leaves: values in an underlying domain
    e.g. interval in the example
- Scalability problems:
  Packing, similar to octagons

Trace Partitioning

```
assume(x ∈ [0, 15]);
float t = {0, 10, 20, 30};
int i = 0;
while(x < t[i])
   {i + +; }
①y = 1.0/(t[i] − t[i − 1]);
```

- No partitioning, interval analysis, at ①:
  - $\texttt{i} \in [0, 2]$
  - $(\texttt{t[i]} - \texttt{t[i} - 1\texttt{]}) \in [-10, 20]$
  - Alarm at ① (divide by 0)
- We need to do case-by-case analysis:
  - Relate $\texttt{x}$ and $\texttt{i}$
  - Relate $\texttt{x}$ and the number of iterations in the loop
- General, control-based partitioning domain
- Partitioning strategy (choice of partitions)
- Subject of the previous talk...

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Use of ASTRÉE

- The analyzer:
  - Full automatic mode:
    Should do well for the families of programs ASTRÉE is designed for
  - $\simeq 150$ options, so as to set
    - The input (one or many files...)
    - The iteration strategy, the packing strategy
    - The domains to enable or disable, domain parameters
    - The export of invariants to disk
  - Standard output: alarms, invariants
  - Can be run in parallel mode
- A graphical interface:
  Navigation through invariants (saved invariants)

Main Practical Results

- Used on 2 families of synchronous embedded programs
- Results: 3 development versions in the second family
- 2.2 GHz bi-opteron, 1 processor used, 64-bit architecture

Nb of lines	70 000	226 000	400 000
Number of iterations	32	51	88
Memory (Gb)	0.6	1.3	2.2
Time	46mn	3h57mn	11h48mn
False alarms	0	0	0

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Main Project Results

- The proof of strong safety properties is amenable to static analysis methods:
  - Very few or no false alarms
  - Reasonable resource usage
  - Thanks to a specialized abstract interpreter
- Many practical and theoretical advances:
  - Relational numerical domains and floating point
  - Packing, linearization and relational domains
  - Development of new, specialized domains
  - Implementation of symbolic domains, e.g. partitioning, symbolic...

Perspectives

- Allow for the parallelization of the analysis:
  It works, allows cutting down the analysis time
- Extension of the memory model (work in progress):
  Unions, pointer arithmetic
- Analyze asynchronous programs
- Certify the assembly code (work in progress):
  Validation of the translation (successful prototype)
- Prove formally some ASTRÉE components
- Tracking semi-automatically the source of alarms:
  - Either prove an alarm false
  - Or restrict the alarm context
    (help to find a scenario or the imprecision)

  Encouraging early results: alarms successfully diagnosed