

Static Analysis and Verification of Synchronous Embedded Code by Abstract Interpretation

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Motivation (1 mn)

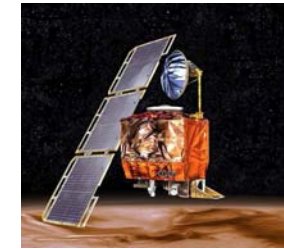
All Computer Scientists Have Experienced Bugs



Ariane 5.01 failure
(overflow)



Patriot failure
(float rounding)



Mars orbiter loss
(unit error)

It is preferable to verify that mission/safety-critical programs do not go wrong before running them.

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Static Analysis by Abstract Interpretation

Static analysis: analyze the program at compile-time to verify a program runtime property (e.g. the absence of some categories of bugs)

Undecidability →

Abstract interpretation: effectively compute an abstraction/
sound approximation of the program semantics,

- which is **precise** enough to imply the desired property, and
- coarse enough to be **efficiently computable**.

Abstract Interpretation, Reminder (10 mn)

Reference

[POPL '77] P. Cousot and R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In 4th ACM POPL.

[Thesis '78] P. Cousot. Méthodes itératives de construction et d'approximation de points fixes d'opérateurs monotones sur un treillis, analyse sémantique de programmes. Thèse ès sci. math. Grenoble, march 1978.

[POPL '79] P. Cousot & R. Cousot. Systematic design of program analysis frameworks. In 6th ACM POPL.

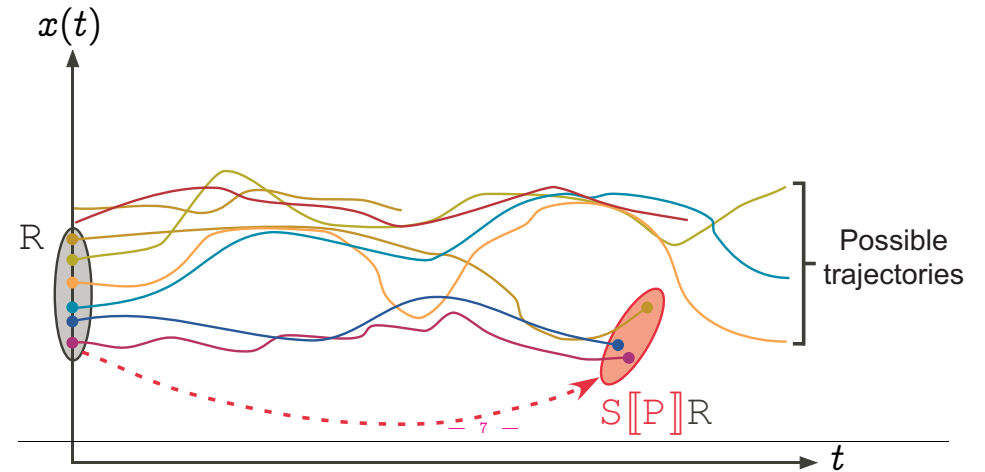
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Syntax of programs

X	variables $X \in \mathbb{X}$
T	types $T \in \mathbb{T}$
E	arithmetic expressions $E \in \mathbb{E}$
B	boolean expressions $B \in \mathbb{B}$
$D ::= T X ;$ $T X ; D'$	
$C ::= X = E ;$ while $B C'$ if $B C'$ else C'' $\{ C_1 \dots C_n \}, (n \geq 0)$	commands $C \in \mathbb{C}$
$P ::= D C$	program $P \in \mathbb{P}$

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Postcondition semantics



States

Values of given type:

$\mathcal{V}[T]$: values of type $T \in \mathbb{T}$

$\mathcal{V}[\text{int}] \stackrel{\text{def}}{=} \{z \in \mathbb{Z} \mid \text{min_int} \leq z \leq \text{max_int}\}$

Program states $\Sigma[P]$ ¹:

$\Sigma[D C] \stackrel{\text{def}}{=} \Sigma[D]$

$\Sigma[T X ;] \stackrel{\text{def}}{=} \{X\} \mapsto \mathcal{V}[T]$

$\Sigma[T X ; D] \stackrel{\text{def}}{=} (\{X\} \mapsto \mathcal{V}[T]) \cup \Sigma[D]$

¹ States $\rho \in \Sigma[P]$ of a program P map program variables X to their values $\rho(X)$

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Concrete Semantic Domain of Programs

Concrete semantic domain for reachability properties:

$$\mathcal{D}[P] \stackrel{\text{def}}{=} \wp(\Sigma[P]) \quad \text{sets of states}$$

i.e. program properties where \sqsubseteq is implication, \emptyset is false, \sqcup is disjunction.

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Concrete Reachability Semantics of Programs

$$S[X = E;]R \stackrel{\text{def}}{=} \{\rho[X \leftarrow \mathcal{E}[E]\rho] \mid \rho \in R \cap \text{dom}(E)\}$$

$$\rho[X \leftarrow v](X) \stackrel{\text{def}}{=} v, \quad \rho[X \leftarrow v](Y) \stackrel{\text{def}}{=} \rho(Y)$$

$$S[\text{if } B \text{ } C']R \stackrel{\text{def}}{=} S[C'](\mathcal{B}[B]R) \cup \mathcal{B}[\neg B]R$$

$$\mathcal{B}[B]R \stackrel{\text{def}}{=} \{\rho \in R \cap \text{dom}(B) \mid B \text{ holds in } \rho\}$$

$$S[\text{if } B \text{ } C' \text{ else } C'']R \stackrel{\text{def}}{=} S[C'](\mathcal{B}[B]R) \cup S[C''](\mathcal{B}[\neg B]R)$$

$$S[\text{while } B \text{ } C']R \stackrel{\text{def}}{=} \text{let } \mathcal{W} = \text{lfp}_0^{\subseteq} \lambda \mathcal{X}. R \cup S[C'](\mathcal{B}[B]\mathcal{X}) \\ \text{in } (\mathcal{B}[\neg B]\mathcal{W})$$

$$S[\{\}]R \stackrel{\text{def}}{=} R$$

$$S[\{C_1 \dots C_n\}]R \stackrel{\text{def}}{=} S[C_n] \circ \dots \circ S[C_1] \quad n > 0$$

$$S[D \text{ } C]R \stackrel{\text{def}}{=} S[C](\Sigma[D]) \quad (\text{uninitialized variables})$$

Not computable (undecidability).

Abstract Semantic Domain of Programs

$$\langle \mathcal{D}^\sharp[P], \sqsubseteq, \perp, \sqcup \rangle$$

such that:

$$\langle \mathcal{D}[P], \sqsubseteq \rangle \xleftrightarrow[\alpha]{\gamma} \langle \mathcal{D}^\sharp[P], \sqsubseteq \rangle$$

i.e.

$$\forall X \in \mathcal{D}[P], Y \in \mathcal{D}^\sharp[P] : \alpha(X) \sqsubseteq Y \iff X \sqsubseteq \gamma(Y)$$

hence $\langle \mathcal{D}^\sharp[P], \sqsubseteq, \perp, \sqcup \rangle$ is a complete lattice such that $\perp = \alpha(\emptyset)$ and $\sqcup X = \alpha(\cup \gamma(X))$

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Example 1 of Abstraction

Traces: set of finite or infinite maximal sequences of states for the operational transition semantics

$\xrightarrow{\alpha}$ **Strongest liberal postcondition:** final states s reachable from a given precondition P

$$\alpha(X) = \lambda P. \{s \mid \exists \sigma_0 \sigma_1 \dots \sigma_n \in X : \sigma_0 \in P \wedge s = \sigma_n\}$$

We have (Σ : set of states, $\dot{\subseteq}$ pointwise):

$$\langle \wp(\Sigma^\infty), \dot{\subseteq} \rangle \xleftrightarrow[\alpha]{\gamma} \langle \wp(\Sigma) \xrightarrow{\cup} \wp(\Sigma), \dot{\subseteq} \rangle$$

Example 2 of Abstraction

Traces: set of finite or infinite maximal sequences of states for the operational transition semantics

α_1 **Set of reachable states:** set of states appearing at least once along one of these traces (global invariant)

$$\alpha_1(X) = \{\sigma_i \mid \sigma \in X \wedge 0 \leq i < |\sigma|\}$$

α_2 **Partitionned set of reachable states:** project along each control point (local invariant)

$$\alpha_2(\{\langle c_i, \rho_i \rangle \mid i \in \Delta\}) = \lambda c. \{\rho_i \mid i \in \Delta \wedge c = c_i\}$$

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α_3 **Partitionned cartesian set of reachable states:** project along each program variable (relationships between variables are now lost)

$$\alpha_3(\lambda c. \{\rho_i \mid i \in \Delta_c\}) = \lambda c. \lambda x. \{\rho_i(x) \mid i \in \Delta_c\}$$

α_4 **Partitionned cartesian interval of reachable states:** take min and max of the values of the variables²

$$\alpha_4(\lambda c. \lambda x. \{v_i \mid i \in \Delta_{c,x}\}) = \lambda c. \lambda x. \langle \min\{v_i \mid i \in \Delta_{c,x}\}, \max\{v_i \mid i \in \Delta_{c,x}\} \rangle$$

$\alpha_1, \alpha_2, \alpha_3$ and α_4 , whence $\alpha_4 \circ \alpha_3 \circ \alpha_2 \circ \alpha_1$ are lower-adjoints of Galois connections

² assuming these values to be totally ordered.

Example 3: Reduced Product of Abstract Domains

To combine abstractions

$$\langle \mathcal{D}, \sqsubseteq \rangle \xleftrightarrow[\alpha_1]{\gamma_1} \langle \mathcal{D}_1^\#, \sqsubseteq_1 \rangle \text{ and } \langle \mathcal{D}, \sqsubseteq \rangle \xleftrightarrow[\alpha_2]{\gamma_2} \langle \mathcal{D}_2^\#, \sqsubseteq_2 \rangle$$

the **reduced product** is

$$\alpha(X) \stackrel{\text{def}}{=} \sqcap \{ \langle x, y \rangle \mid X \subseteq \gamma_1(x) \wedge X \subseteq \gamma_2(y) \}$$

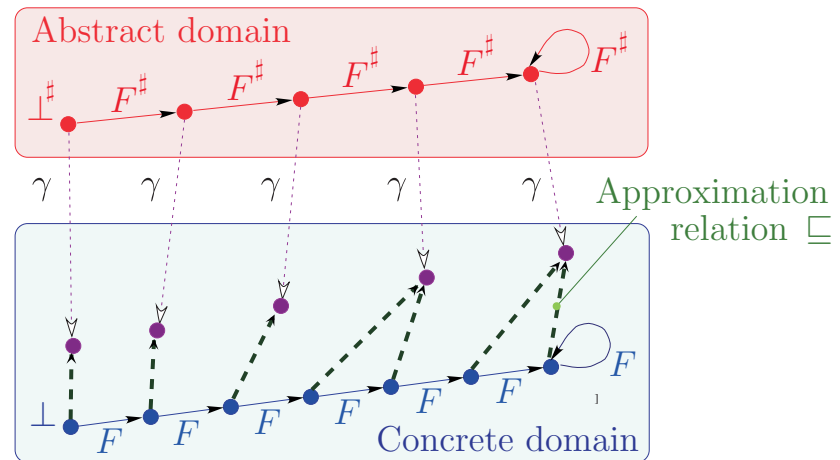
such that $\sqsubseteq \stackrel{\text{def}}{=} \sqsubseteq_1 \times \sqsubseteq_2$ and

$$\langle \mathcal{D}, \sqsubseteq \rangle \xleftrightarrow[\alpha]{\gamma_1 \times \gamma_2} \langle \alpha(\mathcal{D}), \sqsubseteq \rangle$$

Example: $x \in [1, 9] \wedge x \bmod 2 = 0$ reduces to $x \in [2, 8] \wedge x \bmod 2 = 0$

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Approximate Fixpoint Abstraction



$$F \circ \gamma \sqsubseteq \gamma \circ F^\# \Rightarrow \text{lfp } F \sqsubseteq \gamma(\text{lfp } F^\#)$$

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Abstract Reachability Semantics of Programs

$$\begin{aligned}
 S^\sharp[X = E;]R &\stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[E]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\}) \\
 S^\sharp[\text{if } B \ C']R &\stackrel{\text{def}}{=} S^\sharp[C'](B^\sharp[B]R) \sqcup B^\sharp[\neg B]R \\
 B^\sharp[B]R &\stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\}) \\
 S^\sharp[\text{if } B \ C' \ \text{else } C'']R &\stackrel{\text{def}}{=} S^\sharp[C'](B^\sharp[B]R) \sqcup S^\sharp[C''](B^\sharp[\neg B]R) \\
 S^\sharp[\text{while } B \ C']R &\stackrel{\text{def}}{=} \text{let } \mathcal{W} = \text{lfp}_{\perp}^{\sqsubseteq} \lambda \mathcal{X}. R \sqcup S^\sharp[C'](B^\sharp[B]\mathcal{X}) \\
 &\quad \text{in } (B^\sharp[\neg B]\mathcal{W}) \\
 S^\sharp[\{\}]R &\stackrel{\text{def}}{=} R \\
 S^\sharp[\{C_1 \dots C_n\}]R &\stackrel{\text{def}}{=} S^\sharp[C_n] \circ \dots \circ S^\sharp[C_1] \quad n > 0 \\
 S^\sharp[D \ C]R &\stackrel{\text{def}}{=} S^\sharp[C](\top) \quad (\text{uninitialized variables})
 \end{aligned}$$

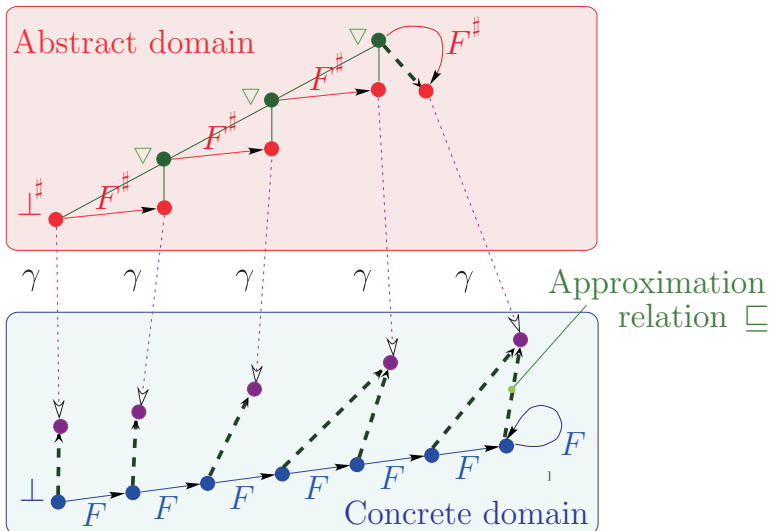
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Abstract Semantics with Convergence Acceleration³

$$\begin{aligned}
 S^\sharp[X = E;]R &\stackrel{\text{def}}{=} \alpha(\{\rho[X \leftarrow \mathcal{E}[E]\rho] \mid \rho \in \gamma(R) \cap \text{dom}(E)\}) \\
 S^\sharp[\text{if } B \ C']R &\stackrel{\text{def}}{=} S^\sharp[C'](B^\sharp[B]R) \sqcup B^\sharp[\neg B]R \\
 B^\sharp[B]R &\stackrel{\text{def}}{=} \alpha(\{\rho \in \gamma(R) \cap \text{dom}(B) \mid B \text{ holds in } \rho\}) \\
 S^\sharp[\text{if } B \ C' \ \text{else } C'']R &\stackrel{\text{def}}{=} S^\sharp[C'](B^\sharp[B]R) \sqcup S^\sharp[C''](B^\sharp[\neg B]R) \\
 S^\sharp[\text{while } B \ C']R &\stackrel{\text{def}}{=} \text{let } \mathcal{F}^\sharp = \lambda \mathcal{X}. \text{let } \mathcal{Y} = R \sqcup S^\sharp[C'](B^\sharp[B]\mathcal{X}) \\
 &\quad \text{in if } \mathcal{Y} \sqsubseteq \mathcal{X} \text{ then } \mathcal{X} \text{ else } \mathcal{X} \nabla \mathcal{Y} \\
 &\quad \text{and } \mathcal{W} = \text{lfp}_{\perp}^{\sqsubseteq} \mathcal{F}^\sharp \quad \text{in } (B^\sharp[\neg B]\mathcal{W}) \\
 S^\sharp[\{\}]R &\stackrel{\text{def}}{=} R \\
 S^\sharp[\{C_1 \dots C_n\}]R &\stackrel{\text{def}}{=} S^\sharp[C_n] \circ \dots \circ S^\sharp[C_1] \quad n > 0 \\
 S^\sharp[D \ C]R &\stackrel{\text{def}}{=} S^\sharp[C](\top) \quad (\text{uninitialized variables})
 \end{aligned}$$

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Convergence Acceleration with Widening



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Applications of Abstract Interpretation
(2 mn)

³ Note: \mathcal{F}^\sharp not monotonic!

Applications of Abstract Interpretation

- **Static Program Analysis** [POPL '77], [POPL '78], [POPL '79] including
 - **Dataflow Analysis** [POPL '79], [POPL '00],
 - **Set-based Analysis** [FPCA '95],
 - **Predicate Abstraction** [Manna's festschrift '03],
 - ...
- **Grammar Analysis and Parsing** [TCS 290(1) 2002], [Wilhelm's festschrift '07]

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Applications of Abstract Interpretation (Cont'd)

- **Hierarchies of Semantics (including Proofs)** [POPL '92], [TCS 277(1-2) 2002]
- **Typing & Type Inference** [POPL '97]
- **(Abstract) Model Checking** [POPL '00]
- **Bisimulations** [RT-ESOP '04]
- **Non-interference** [POPL '04]
- **Models of Security Protocols** [IPL'05]

Applications of Abstract Interpretation (Cont'd)

- **Program Transformation** [POPL '02] including
 - **Software Watermarking** [POPL '04]
 - **(Semantic/Abstract) Slicing** [14]
 - **Code Obfuscation** [DPG-ICALP '05]
- **Malware Detection** [POPL '07]
- ...

All these techniques involve **sound approximations** that can be formalized by **abstract interpretation**

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**Application to the
Astrée Static Analyzer (15 mn)**

Reference

- [1] <http://www.astree.ens.fr/>

Project Members



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Programs analysed by Astrée

- **Application Domain**: large safety critical embedded real-time synchronous software for non-linear control of very complex control/command systems.
- **C programs**:
 - with
 - basic numeric datatypes, structures and arrays
 - pointers (including on functions),
 - floating point computations
 - tests, loops and function calls
 - limited branching (forward goto, break, continue)

⁴ Nov. 2001 — Nov. 2003.

- with (cont'd)
 - union **NEW** [LCTES '06]
 - pointer arithmetics & casts **NEW** [LCTES '06]
- without
 - dynamic memory allocation
 - recursive function calls
 - unstructured/backward branching
 - conflicting side effects
 - C libraries, system calls (parallelism)

Such limitations are quite common for embedded safety-critical software.

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Concrete Operational Semantics

- International **norm of C** (ISO/IEC 9899:1999)
- *restricted by implementation-specific behaviors* depending upon the machine and compiler (e.g. representation and size of integers, IEEE 754-1985 norm for floats and doubles)
- *restricted by user-defined programming guidelines* (such as no modular arithmetic for signed integers, even though this might be the hardware choice)
- *restricted by program specific user requirements* (e.g. assert, execution stops on first runtime error⁵)

⁵ semantics of C unclear after an error, equivalent if no alarm

Abstract Semantics

- **Reachable states** for the concrete trace operational semantics
- **Volatile environment** is specified by a *trusted* configuration file.

Requirements:

- **Soundness**: absolutely essential
- **Precision**: few or no false alarm⁶ (full certification)
- **Efficiency**: rapid analyses and fixes during development

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Implicit Specification: Absence of Runtime Errors

- No violation of the **norm of C** (e.g. array index out of bounds, division by zero)
- **No implementation-specific undefined behaviors** (e.g. maximum short integer is 32767, NaN)
- No violation of the **programming guidelines** (e.g. static variables cannot be assumed to be initialized to 0)
- No violation of the **programmer assertions** (must all be statically verified).

⁶ Potential runtime error signaled by the analyzer due to overapproximation but impossible in any actual program run.

Example application

- **Primary flight control software** of the Airbus A340 family/A380 fly-by-wire system



- C program, automatically generated from a proprietary high-level specification (à la Simulink/SCADE)
- A340 family: 132,000 lines, **75,000 LOCs** after preprocessing, **10,000 global variables**, over **21,000** after expansion of small arrays, now $\times 2$
- A380: $\times 3/7$

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The Class of Considered Periodic Synchronous Programs

```
declare volatile input, state and output variables;
initialize state and output variables;
loop forever
  - read volatile input variables,
  - compute output and state variables,
  - write to output variables;
  __ASTREE_wait_for_clock ();
end loop
```

Task scheduling is static:

- **Requirements**: the only interrupts are clock ticks;
- **Execution time of loop body less than a clock tick, as verified by the aiT WCET Analyzers [EMSOFT '01].**

Challenging aspects

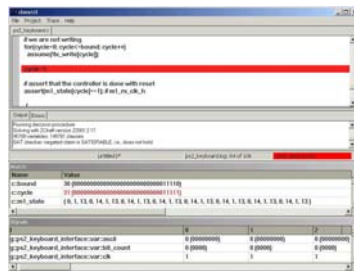
- **Size:** 100/1000 kLOC, 10/150 000 global variables
- **Floating point computations**
including interconnected networks of filters, non linear control with feedback, interpolations...
- **Interdependencies among variables:**
 - Stability of computations should be established
 - Complex relations should be inferred among numerical and boolean data
 - Very long data paths from input to outputs

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Example 1: CBMC

- **CBMC** is a **Bounded Model Checker** for ANSI-C programs (started at CMU in 1999).
- Allows verifying array bounds (buffer overflows), pointer safety, exceptions and user-specified assertions.
- Aimed for embedded software, also supports dynamic memory allocation using malloc.
- Done by unwinding the loops in the program and passing the resulting equation to a SAT solver.

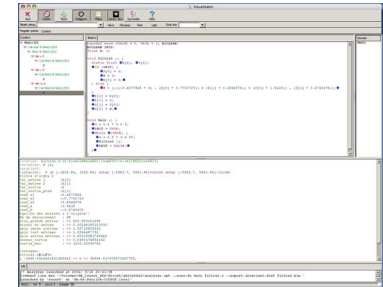
– **Problem (a.o.): does not scale up!**



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Example 2: Astrée

- **ASTRÉE** is an abstract interpretation-based **static analyzer** for ANSI-C programs (started at ENS in 2001).
- Allows verifying array bounds (buffer overflows), pointer safety, exceptions and user-specified assertions.
- Aimed for embedded software, does not support dynamic memory allocation.
- Done by abstracting the reachability fixpoint equations for the program operational semantics.
- **Advantage (a.o.): does scale up!**



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Characteristics of the Astrée Analyzer

- **Sound:** –ASTRÉE is a **bug eradicator**: finds all bugs in a well-defined class (runtime errors)
- ASTRÉE is not a **bug hunter**: finding some bugs in a well-defined class (e.g. by *bug pattern detection* like FindBugs™, PRefast or PMD)
- ASTRÉE is **exhaustive**: covers the whole state space (\neq MAGIC, CBMC)
- ASTRÉE is **comprehensive**: never omits potential errors (\neq UNO, CMC from coverity.com) or sort most probable ones to avoid overwhelming messages (\neq Splint)

21st March 2007

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Characteristics of the Astrée Analyzer (Cont'd)

Static: compile time analysis (\neq run time analysis **Rational Purify**, **Parasoft Insure++**)

Program Analyzer: analyzes programs not micromodels of programs (\neq **PROMELA** in **SPIN** or **Alloy** in the **Alloy Analyzer**)

Automatic: no end-user intervention needed (\neq **ESC Java**, **ESC Java 2**), or **PREfast** (annotate functions with intended use)

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Characteristics of the Astrée Analyzer (Cont'd)

Multiabstraction: uses many numerical/symbolic abstract domains (\neq symbolic constraints in **Bane** or the canonical abstraction of **TVLA**)

Infinitary: all abstractions use infinite abstract domains with widening/narrowing (\neq model checking based analyzers such as **Bandera**, **Bogor**, **Java PathFinder**, **Spin**, **VeriSoft**)

Efficient: always terminate (\neq counterexample-driven automatic abstraction refinement **BLAST**, **SLAM**)

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Characteristics of the Astrée Analyzer (Cont'd)

Extensible/Specializable: can easily incorporate new abstractions (and reduction with already existing abstract domains) (\neq general-purpose analyzers **PolySpace Verifier**)

Domain-Aware: knows about control/command (e.g. digital filters) (as opposed to specialization to a mere programming style in **C Global Surveyor**)

Parametric: the precision/cost can be tailored to user needs by options and directives in the code

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Characteristics of the Astrée Analyzer (Cont'd)

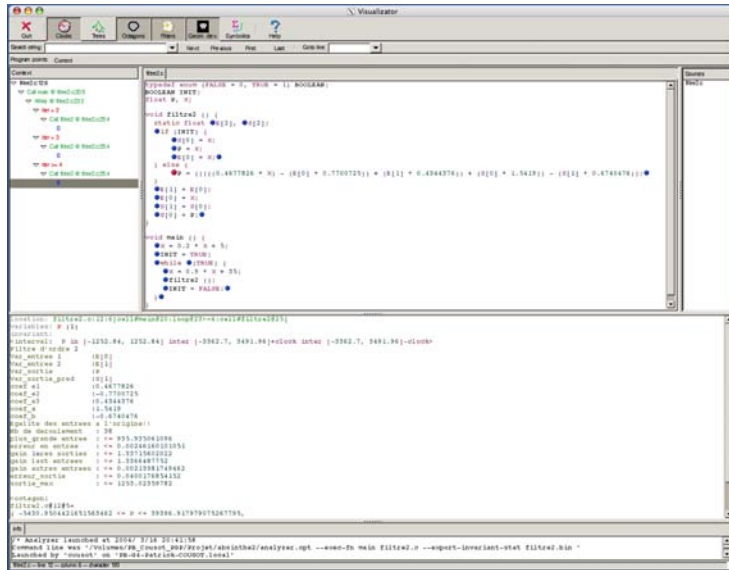
Automatic Parametrization: the generation of parametric directives in the code can be programmed (to be specialized for a specific application domain)

Modular: an analyzer instance is built by selection of **O-CAML** modules from a collection each implementing an abstract domain

Precise: very few or no false alarm when adapted to an application domain \rightarrow it is a **VERIFIER!**

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Example of Analysis Session



Examples of Abstractions in Astrée (15 mn)

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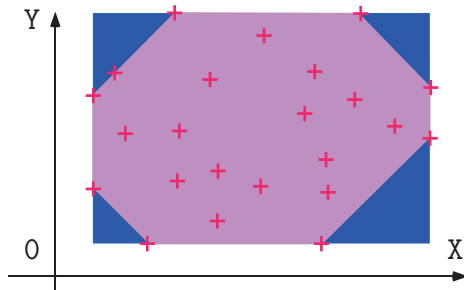
Benchmarks (Airbus A340 Primary Flight Control Software)⁷

- 132,000 lines, 75,000 LOCs after preprocessing
- Comparative results (commercial software):
 - 4,200 (false?) alarms, 3.5 days;
- Our results:
 - 0 alarms,
 - 40mn on 2.8 GHz PC, 300 Megabytes
 - A world première in Nov. 2003!

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⁷ “Flight Control and Guidance Unit” (FCGU) running on the “Flight Control Primary Computers” (FCPC). The A340 electrical flight control system is placed between the pilot’s controls (sidesticks, rudder pedals) and the control surfaces of the aircraft, whose movement they control and monitor.

General-Purpose Abstract Domains: Intervals and Octagons



Intervals:

$$\begin{cases} 1 \leq x \leq 9 \\ 1 \leq y \leq 20 \end{cases}$$

Octagons [11]:

$$\begin{cases} 1 \leq x \leq 9 \\ x + y \leq 77 \\ 1 \leq y \leq 20 \\ x - y \leq 04 \end{cases}$$

Difficulties: many global variables, arrays (smashed or not), IEEE 754 floating-point arithmetic (in program and analyzer) [POPL'77, 11, 12]

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Floating-Point Computations

```
/* float-error.c */
int main () {
    float x, y, z, r;
    x = 1.000000019e+38;
    y = x + 1.0e21;
    z = x - 1.0e21;
    r = y - z;
    printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
    double x; float y, z, r;
    /* x = ldexp(1.,50)+ldexp(1.,26); */
    x = 1125899973951487.0;
    y = x + 1;
    z = x - 1;
    r = y - z;
    printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
0.000000
```

$$(x + a) - (x - a) \neq 2a$$

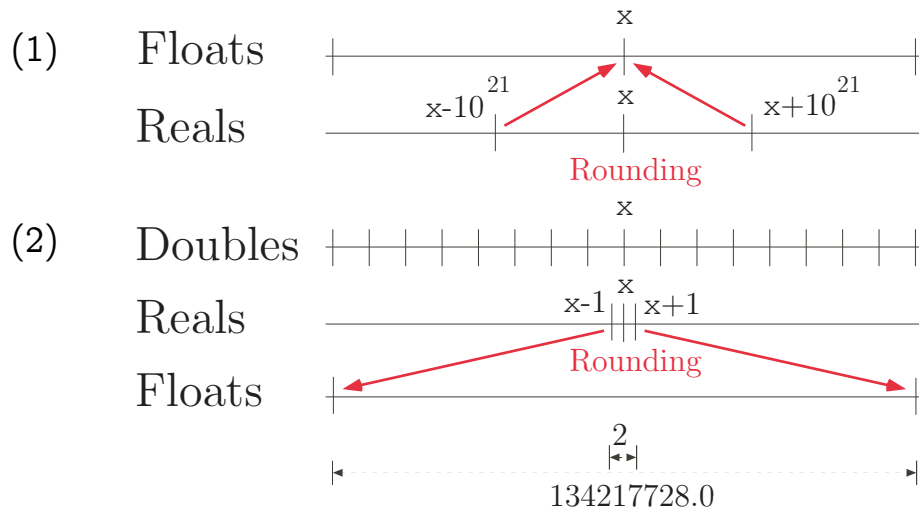
Floating-Point Computations

```
/* float-error.c */
int main () {
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    x = 1.000000019e+38;
    y = x + 1.0e21;
    z = x - 1.0e21;
    r = y - z;
    printf("%f\n", r);
}
% gcc float-error.c
% ./a.out
0.000000
```

```
/* double-error.c */
int main () {
    double x; float y, z, r;
    /* x = ldexp(1.,50)+ldexp(1.,26); */
    x = 1125899973951488.0;
    y = x + 1;
    z = x - 1;
    r = y - z;
    printf("%f\n", r);
}
% gcc double-error.c
% ./a.out
134217728.000000
```

$$(x + a) - (x - a) \neq 2a$$

Explanation of the huge rounding error



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Floating-point linearization [12, 13]

- Approximate arbitrary expressions in the form $[a_0, b_0] + \sum_k ([a_k, b_k] \times V_k)$
- Example:
 $Z = X - (0.25 * X)$ is linearized as
 $z = ([0.749 \dots, 0.750 \dots] \times x) + (2.35 \dots 10^{-38} \times [-1, 1])$
- Allows **simplification** even in the interval domain
 if $x \in [-1, 1]$, we get $|Z| \leq 0.750 \dots$ instead of $|Z| \leq 1.25 \dots$
- Allows using a **relational abstract domain** (octagons)
- Example of good compromise between cost and precision

Symbolic abstract domain [12, 13]

- **Interval analysis**: if $x \in [a, b]$ and $y \in [c, d]$ then $x - y \in [a - d, b - c]$ so if $x \in [0, 100]$ then $x - x \in [-100, 100]$!!!
- The **symbolic abstract domain** propagates the symbolic values of variables and performs simplifications;
- Must maintain the **maximal possible rounding error** for float computations (overestimated with intervals);

```
% cat -n x-x.c
1 void main () { int X, Y;
2   __ASTREE_known_fact(((0 <= X) && (X <= 100)));
3   Y = (X - X);
4   __ASTREE_log_vars((Y));
5 }

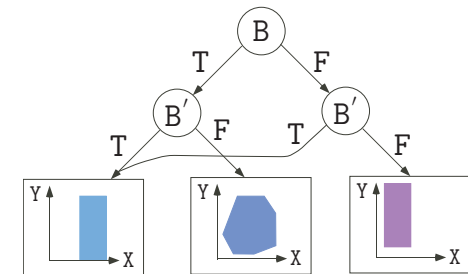
astree -exec-fn main -no-relational x-x.c      astree -exec-fn main x-x.c
Call main@x-x.c:1:5-x-x.c:1:9:                Call main@x-x.c:1:5-x-x.c:1:9:
<interval: Y in [-100, 100]>                  <interval: Y in {0}> <symbolic: Y = (X -i X)>
```

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Boolean Relations for Boolean Control

- **Code Sample:**

```
/* boolean.c */
typedef enum {F=0,T=1} BOOL;
BOOL B;
void main () {
  unsigned int X, Y;
  while (1) {
    ...
    B = (X == 0);
    ...
    if (!B) {
      Y = 1 / X;
    }
    ...
  }
}
```



The boolean relation abstract domain is parameterized by the height of the decision tree (an analyzer option) and the abstract domain at the leaves

Control Partitioning for Case Analysis

Code Sample:

```

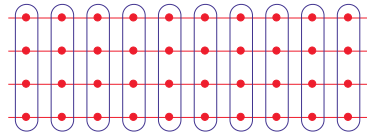
/* trace_partitioning.c */
void main() {
  float t[5] = {-10.0, -10.0, 0.0, 10.0, 10.0};
  float c[4] = {0.0, 2.0, 2.0, 0.0};
  float d[4] = {-20.0, -20.0, 0.0, 20.0};
  float x, r;
  int i = 0;

  ... found invariant -100 ≤ x ≤ 100 ...

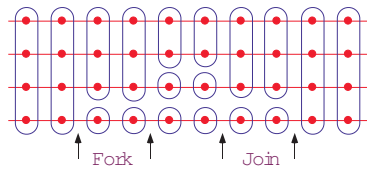
  while ((i < 3) && (x >= t[i+1])) {
    i = i + 1;
  }
  r = (x - t[i]) * c[i] + d[i];
}

```

Control point partitioning:



Trace partitioning:



Delaying abstract unions in tests and loops is more precise for non-distributive abstract domains (and much less expensive than disjunctive completion).

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Filter Example [8]

```

typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
BOOLEAN INIT; float P, X;

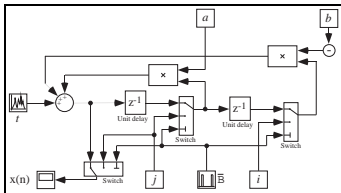
void filter () {
  static float E[2], S[2];
  if (INIT) { S[0] = X; P = X; E[0] = X; }
  else { P = (((((0.5 * X) - (E[0] * 0.7)) + (E[1] * 0.4))
              + (S[0] * 1.5)) - (S[1] * 0.7)); }
  E[1] = E[0]; E[0] = X; S[1] = S[0]; S[0] = P;
  /* S[0], S[1] in [-1327.02698354, 1327.02698354] */
}

void main () { X = 0.2 * X + 5; INIT = TRUE;
  while (1) {
    X = 0.9 * X + 35; /* simulated filter input */
    filter (); INIT = FALSE; }
}

```

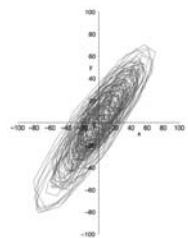
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2nd Order Digital Filter:

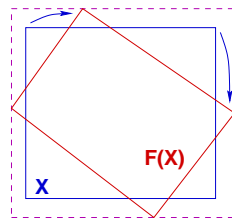


Ellipsoid Abstract Domain for Filters

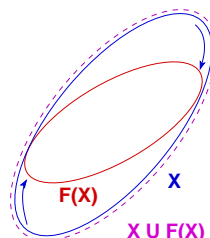
- Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$
- The concrete computation is bounded, which must be proved in the abstract.
- There is no stable interval or octagon.
- The simplest stable surface is an ellipsoid.



execution trace



$X \cup F(X)$
unstable interval



$X \cup F(X)$
stable ellipsoid

Arithmetic-geometric progressions⁸ [9]

– Abstract domain: $(\mathbb{R}^+)^5$

– Concretization:

$$\gamma \in (\mathbb{R}^+)^5 \mapsto \wp(\mathbb{N} \mapsto \mathbb{R})$$

$$\gamma(M, a, b, a', b') =$$

$$\{f \mid \forall k \in \mathbb{N} : |f(k)| \leq (\lambda x \cdot ax + b \circ (\lambda x \cdot a'x + b')^k) (M)\}$$

i.e. any function bounded by the arithmetic-geometric progression.

⁸ here in \mathbb{R}

Arithmetic-Geometric Progressions (Example 1)

```
% cat count.c
typedef enum {FALSE = 0, TRUE = 1} BOOLEAN;
volatile BOOLEAN I; int R; BOOLEAN T;
void main() {
    R = 0;
    while (TRUE) {
        __ASTREE_log_vars((R));
        if (I) { R = R + 1; } ← potential overflow!
        else { R = 0; }
        T = (R >= 100);
        __ASTREE_wait_for_clock();
    }
}

% cat count.config
__ASTREE_volatile_input((I [0,1]));
__ASTREE_max_clock((3600000));
% astree -exec-fn main -config-sem count.config count.c|grep '|R|'
|R| <= 0. + clock *1. <= 3600001.
```

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Arithmetic-geometric progressions (Example 2)

```
% cat retro.c
typedef enum {FALSE=0, TRUE=1} BOOL;
BOOL FIRST;
volatile BOOL SWITCH;
volatile float E;
float P, X, A, B;

void dev( )
{ X=E;
  if (FIRST) { P = X; }
  else
    { P = (P - (((2.0 * P) - A) - B)
      * 4.491048e-03)); };
  B = A;
  if (SWITCH) {A = P;}
  else {A = X;}
}

void main()
{ FIRST = TRUE;
  while (TRUE) {
    dev();
    FIRST = FALSE;
    __ASTREE_wait_for_clock();
  }
}

% cat retro.config
__ASTREE_volatile_input((E [-15.0, 15.0]));
__ASTREE_volatile_input((SWITCH [0,1]));
__ASTREE_max_clock((3600000));
|P| <= (15. + 5.87747175411e-39
/ 1.19209290217e-07) * (1
+ 1.19209290217e-07)^clock
- 5.87747175411e-39 /
1.19209290217e-07 <=
23.0393526881
```

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(Automatic) Parameterization

- All abstract domains of ASTRÉE are **parameterized**, e.g.
 - variable packing for octagones and decision trees,
 - partition/merge program points,
 - loop unrollings,
 - thresholds in widenings, ...;
- End-users can either **parameterize by hand** (analyzer options, directives in the code), or
- choose the **automatic parameterization** (default options, directives for pattern-matched predefined program schemata).

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The main loop invariant for the A340

A textual file over 4.5 Mb with

- **6,900** boolean interval assertions ($x \in [0; 1]$)
 - **9,600** interval assertions ($x \in [a; b]$)
 - **25,400** clock assertions ($x + \text{clk} \in [a; b] \wedge x - \text{clk} \in [a; b]$)
 - **19,100** additive octagonal assertions ($a \leq x + y \leq b$)
 - **19,200** subtractive octagonal assertions ($a \leq x - y \leq b$)
 - **100** decision trees
 - **60** ellipse invariants, etc ...
- involving over **16,000** floating point constants (only **550** appearing in the program text) \times **75,000** LOCs.

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Possible origins of imprecision and how to fix it

In case of false alarm, the imprecision can come from:

- **Abstract transformers** (not best possible) → improve algorithm;
- **Automatized parametrization** (e.g. variable packing) → improve pattern-matched program schemata;
- **Iteration strategy** for fixpoints → fix widening ⁹;
- **Inexpressivity** i.e. indispensable local inductive invariant are inexpressible in the abstract → add a **new abstract domain** to the reduced product (e.g. filters).

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Demo of Astrée (10mn)

Conclusion (2mn)

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Conclusion

- Most applications of abstract interpretation **tolerate a small rate** (typically 5 to 15%) **of false alarms**:
 - Program transformation → do not optimize,
 - Typing → reject some correct programs, etc,
 - WCET analysis → overestimate;
- Some applications **require no false alarm** at all:
 - **Program verification**.
- **Theoretically possible** [SARA '00], **practically feasible** [PLDI '03]

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⁹ This can be very hard since at the limit only a precise infinite iteration might be able to compute the proper abstract invariant. In that case, it might be better to design a more refined abstract domain.

The Future & Grand Challenges

Forthcoming (1/2 years):

– Industrialization

Future (5 years):

– Asynchronous concurrency (for less critical software)

– Functional properties (reactivity)

Grand challenge:

– Verification from specifications to machine code (verifying compiler)

– Verification of systems (quasi-synchrony, distribution)

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THE END, THANK YOU

More references at URL www.di.ens.fr/~cousot
www.astree.ens.fr.

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