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A Static Analyzer for Large Safety-Critical Software

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Automatic Program Verification by Abstract Interpretation

Result:

- Can produce **zero or very few false alarms** while checking **non-trivial properties** (absence of Run-Time Error);
- **Does scale up**.

How ?

- We **specialize** the abstract interpreter for a **family of programs** (which correctness proofs would be similar).
- The abstract domains are **generic** invariants **automatically** instantiated by the analyzer (to make these proofs).

Considered Programs and Semantics

Which Programs are Considered ?

- Embedded avionic programs;
- Automatically generated from a proprietary graphical system control language (à la Simulink);
- Synchronous real-time critical programs:

```
declare volatile input, state, and output variables;
initialize state variables;
loop forever
  read volatile input variables,
  compute output and state variables,
  write to volatile output variables;
  wait for next clock tick
end loop
```

Main Characteristics of the Programs

Difficulties:

- **Many global variables and arrays** ($>$ **10 000**);
- A **huge loop** ($>$ **75 000 lines** after simplification);
- Each iteration depends on the state of the previous iterations (state variables);
- **Floating-point** computations
  (**80% of the code** implements non-linear control with feed-back);
- Everything is **interdependent** (live variables analysis, slicing ineffective);
- Abstraction by elimination of any variable is too imprecise.

Simplicities:

- All data is **statically allocated**;
- Pointers are restricted to call-by-reference, **no pointer arithmetics**;
- Structured, **recursion-free** control flow.

Semantics

- The standard **ISO C99 semantics**:
  - arrays should not be accessed out of their bounds, ...

  restricted by:
- The **machine semantics:**
  - integer arithmetics is 2's complement,
  - floating point arithmetics is IEEE 754-1985,
  - `int` and `float` are 32-bit, `short` is 16-bit, ...

  restricted by:
- The **user's semantics:**
  - integer arithmetics should not wrap-around,
  - some IEEE exceptions (invalid operation, overflow, division by zero) should not occur, ...

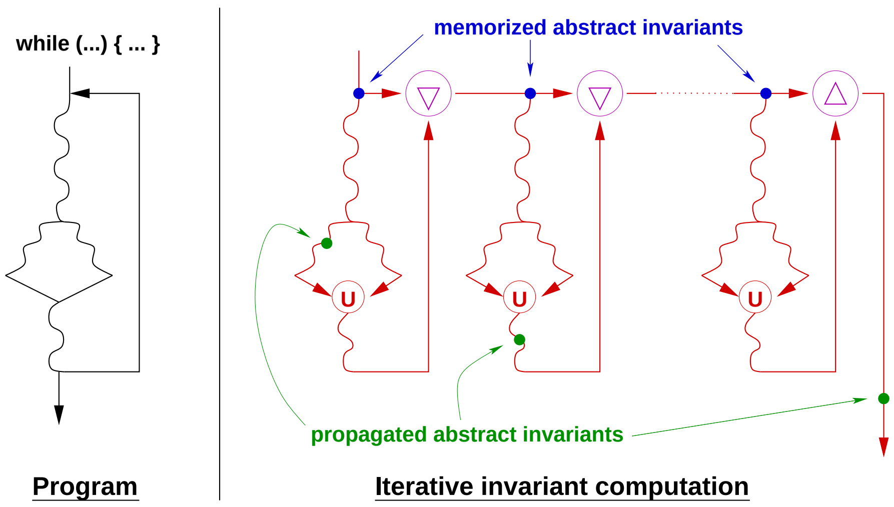
Goal of the Program Static Analyzer

- **Correctness verification**.
- Nothing can go wrong at execution:
  - no integer overflow or division by zero,
  - no exception, NaN, or $\pm\infty$ generated by IEEE floating-point arithmetics,
  - no out of bounds array access,
  - no erroneous type conversion.
- The execution semantics on the machine **never reaches an indetermination or an error case** in the standard / machine / user semantics.

Information about the Program Execution Automatically Inferred by the Analyzer

- The analyzer effectively computes a **finitely represented**, **compact** over-approximation of the **immense** reachable state space.
- The information is **valid for any execution** interacting with **any possible environment** (through undetermined volatiles).
- It is inferred **automatically** by abstract interpretation of the collecting semantics and convergence acceleration (∇, Δ).

Iterations to Over-Approximate the Reachable States



Abstract Domains

Choice of the Abstract Domains

Abstract Domain:

- Computer representation of a **class** of **program properties**;
- **Transformers** for propagation through expressions and commands;
- Primitives for **convergence acceleration**: ∇, Δ.

Composition of Abstract Domains:

- Essentially approximate **reduced product** (conjunction with simplification).

Design of Abstract Domains:

- Know-how;
- **Experimentation**.

Interval Abstract Domain

- Classical domain [Cousot Cousot 76];
- Minimum information needed to check the correctness conditions;
- **Not precise enough** to express a useful inductive invariant (thousands of false alarms);
- $\Longrightarrow$ must be refined by:
  - combining with existing domains through reduced product,
  - designing **new domains**, until all false alarms are eliminated.

Clock Abstract Domain

Code Sample:

```
R = 0;
while (1) {
  if (I)
    { R = R+1; }
  else
    { R = 0; }
  T = (R>=n);
  wait_for_clock ();
}
```

- Output `T` is true iff the volatile input `I` has been true for the last `n` clock ticks.
- The clock ticks every `s` seconds for at most `h` hours, thus `R` **is bounded**.
- To prove that `R` **cannot overflow**, we must prove that `R` **cannot exceed the elapsed clock ticks** (impossible using only intervals).

Solution:

- We add a phantom variable `clock` in the concrete user semantics to track elapsed clock ticks.
- For each variable `X`, we abstract **three intervals**: `X`, `X+clock`, and `X-clock`.
- If `X+clock` or `X-clock` is bounded, so is `X`.

Octagon Abstract Domain

Code Sample:

```
while (1) {
  R = A-Z;
  L = A;
  if (R>V)
    { ★ L = Z+V; }
  ★
}
```

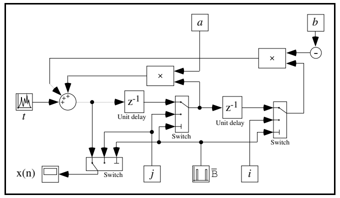
- At ★, the interval domain gives `L` ≤ max(max `A`, (max `Z`)+(max `V`)).
- In fact, we have `L` ≤ `A`.
- To discover this, we must know at ★ that `R` = `A-Z` and `R` > `V`.

Solution: we need a numerical **relational** abstract domain.

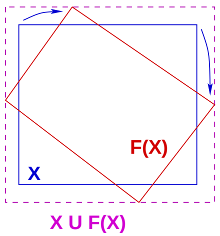
- The **octagon** abstract domain [Miné 03] is a good cost / precision trade-off.
- Invariants of the form $\pm x \pm y \le c$, with $\mathcal{O}(N^2)$ memory and $\mathcal{O}(N^3)$ time cost.
- Here, `R` = `A-Z` cannot be discovered, but we get `L-Z` ≤ max `R` which is sufficient.
- We use many octagons on **small packs** of variables instead of a large one using all variables to cut costs.

Ellipsoid Abstract Domain

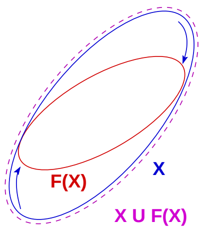
2^d Order Filter Sample:



- Computes $X_n = \begin{cases} \alpha X_{n-1} + \beta X_{n-2} + Y_n \\ I_n \end{cases}$
- The concrete computation is **bounded**, which must be proved in the abstract.
- There is no stable interval or octagon.
- The simplest stable surface is an **ellipsoid**.



unstable interval



stable ellipsoid

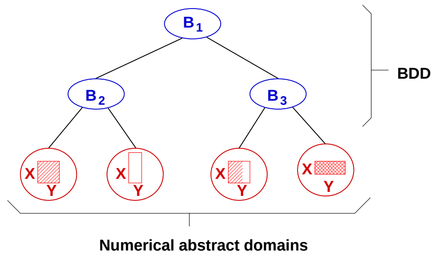
Decision Tree Abstract Domain

Synchronous reactive programs **encode control flow in boolean variables**.

Code Sample:

```
bool B1,B2,B3;
float N,X,Y;
N = f(B1);
if (B1)
  { X = g(N); }
else
  { Y = h(N); }
```

Decision Tree:



There are too many booleans (**4 000**) to build one big tree so we:

- limit the **BDD height** to 3 (analysis parameter);
- use a **syntactic criterion** to select variables in the BDD and the numerical parts.

Relational Domains on Floating-Point

Problems:

- Relational numerical abstract domains rely on a **perfect mathematical concrete semantics** (in $\mathbb{R}$ or $\mathbb{Q}$).
- Perfect arithmetics in $\mathbb{R}$ or $\mathbb{Q}$ is costly.
- IEEE 754-1985 floating-point concrete semantics **incurs rounding**.

Solution:

- Build an **abstract mathematical semantics in $\mathbb{R}$** that over-approximates the concrete floating-point semantics, including rounding.
- Implement the abstract domains on $\mathbb{R}$ **using floating-point numbers** rounded in a sound way.

Iteration Strategies for Fixpoint Approximation

Iteration Refinement: Loop Unrolling

Principle:

- Semantically equivalent to:

```
while (B) { C }   ⟹   if (B) { C }; while (B) { C }
```

- More precise in the abstract:
  - **less** concrete execution paths are **merged** in the abstract.

Application:

- Isolate the **initialization phase** in a loop (e.g. first iteration).

Iteration Refinement: Trace Partitioning

Principle:

- Semantically equivalent to:

```
if (B) { C1 } else { C2 }; C3
              ⇓
if (B) { C1; C3 } else { C2; C3 };
```

- More precise in the abstract:
  - concrete execution paths are **merged later**.

Application:

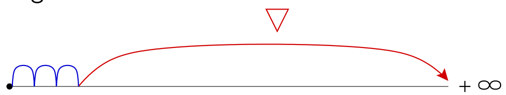
```
if (B)
  { X=0; Y=1; }
else
  { X=1; Y=0; }
R = 1 / (X-Y);
```

/ cannot result in a division by zero

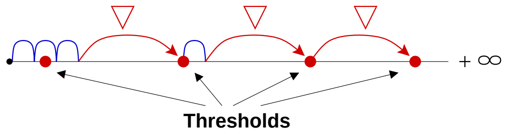
Convergence Accelerator: Widening

Principle:

- Brute-force widening:



- Widening **with thresholds**:



Examples:

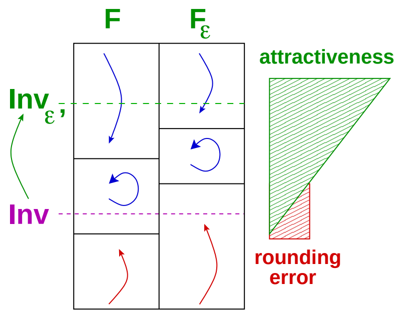
- 1., 10., 100., 1000., etc. for floating-point variables;
- maximal values of data types;
- syntactic program constants, etc.

Fixpoint Stabilization for Floating-point

Problem:

- Mathematically, we look for an abstract invariant **inv** such that F(**inv**) $\subseteq$ **inv**.
- Unfortunately, abstract computation uses floating-point and incurs rounding: maybe F_ε(**inv**) $\not\subseteq$ **inv**!

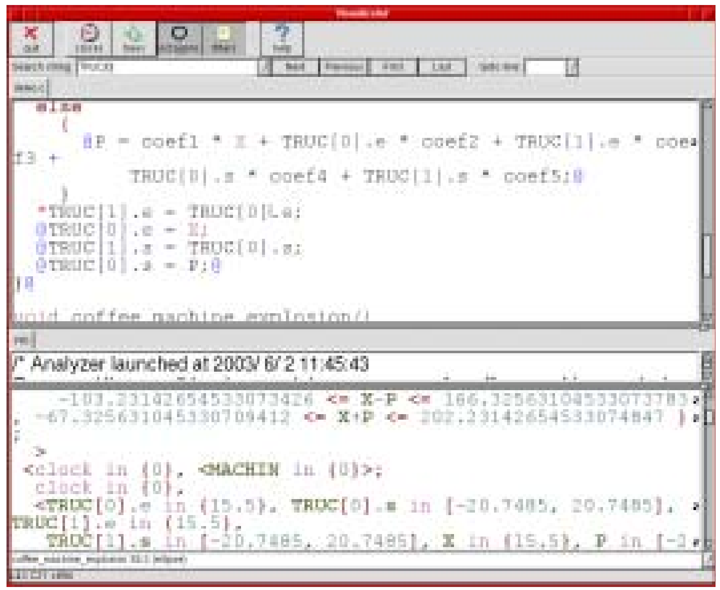
Solution:



- Widen **inv** to **inv**$_{\varepsilon'}$ with the hope to jump into a **stable zone of** F_ε.
- Works if F has some **attractiveness** property that fights against rounding errors (otherwise iteration goes on).
- ε' is an analysis parameter.

Results

Example of Analysis Session



Results

- **Efficient:**
  - tested on two **75 000** lines programs,
  - **120 min** and **37 min** computation time on a 2.8GHz PC,
  - **200 Mb** memory usage.
- **Precise:**
  - **11** and **3** lines containing a warning.
- **Exhaustive:**
  - full control and data **coverage** (unlike checking, testing, simulation).

Conclusion

- **Success story**:
  - we succeed where **a commercial abstract interpretation-based static analysis tool failed**
    (because of prohibitive time and memory consumption and very large number of false alarms);
- **Usable** in practice for verification:
  - **directly applicable** to other similar programs
    by changing some analyzer parameters,
  - approach **generalizable** to other program families
    by including new abstract domains and specializing the iteration strategy.
    (Work in progress: power-on self-test for a family of embedded systems.)