The Contributions of Alan Mycroft to Abstract Interpretation

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Mycroft's PhD thesis

Abstract Interpretation and Optimising Transformations

for Applicative Programs

Alan Mycroft

Doctor of Philosophy

University of Edinburgh

1981

- Alan's second most cited paper!

Strictness analysis

Strictness analysis

Chapter 3 gives an example of the ease with which we can talk about function objects within abstract interpretive schemes. It uses this to show how a simple language using call-by-need semantics can be augmented with a system that annotates places in a program at which call-by-value can be used without violating the call-by-need semantics.

State of the art in the 80's

- Abstraction of the reachable states of a transition system
  - $\langle \Sigma, \tau \rangle$
  - Collecting semantics $\mathsf{post}(\tau^*)P = \mathsf{lfp}^{\subseteq} F_P, \quad P \in \wp(\Sigma), \quad F_P(X) = P \cup \mathsf{post}(\tau)X$
  - Galois connection $\langle \wp(\Sigma),\ \subseteq \rangle \xrightleftharpoons[\alpha]{\gamma} \langle A,\ \preceq \rangle$
  - Abstract reachability is " find Q such that $\alpha(\mathrm{lfp}^{\subseteq} F_P) \subseteq Q$ "

State of the art in the 80's

- Fixpoint abstraction theorem:

Proposition 3. *if* $\langle C,\ \preceq\rangle$ *and* $\langle A,\ \preccurlyeq\rangle$ *are CPO's (every increasing chain has a lub, including the empty chain, so has an infimum),* $f \in C \xrightarrow{c} C$ *and* $\bar{f} \in A \xrightarrow{c} A$ *are continuous,* $\langle C,\ \preceq\rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle A,\ \preccurlyeq\rangle$ *is a Galois connection, then the commutation condition* $\alpha \circ f = \bar{f} \circ \alpha$ *(respectively semi-commutation* $\alpha \circ f \mathrel{\dot{\preccurlyeq}} \bar{f} \circ \alpha$*, pointwise) implies that* $\alpha(\mathsf{lfp}^{\preceq} f) = \mathsf{lfp}^{\preceq} \bar{f}$ *(resp.* $\alpha(\mathsf{lfp}^{\preceq} f) \preccurlyeq \mathsf{lfp}^{\preccurlyeq} \bar{f}$*).*

Mycroft's strictness analysis problem

- Denotational semantics of a recursive function ($\sqsubseteq$ is Scott ordering, $\dot{\sqsubseteq}$ is pointwise, F continuous on a CPO $\langle \mathscr{D}_\perp, \sqsubseteq \rangle$)

$$\mathsf{lfp}^{\dot{\sqsubseteq}} F \in \mathscr{D}_\perp \longrightarrow \mathscr{D}_\perp$$

- Collecting semantics of $f \in \mathscr{D}_\perp \longrightarrow \mathscr{D}_\perp$ is $\mathsf{post}(f)P = \{f(x) \mid x \in P\}$

- In fixpoint form ($\dot{\hat{\sqsubseteq}}$ is Egli-Milner ordering)

$$\mathsf{post}(\mathsf{lfp}^{\dot{\sqsubseteq}} F) = \mathsf{lfp}^{\dot{\hat{\sqsubseteq}}} \hat{F}$$

with $\hat{F}(\phi)P \triangleq \mathsf{post}(F(\hat{\gamma}(\phi)))P$ and $\hat{\gamma}(\phi) \triangleq \boldsymbol{\lambda} x \bullet \mathsf{let}\ \{y\} = \phi(\{x\})\ \mathsf{in}\ y$

- Strictness analysis is " find Q such that $\alpha^\sharp(\mathsf{lfp}^{\dot{\hat{\sqsubseteq}}} \hat{F})) \subseteq Q$ "

Mycroft's strictness analysis solution

Proposition 7. *Let* $\langle C,\ \bot,\ \sqsubseteq,\ \sqcup\rangle$ *be a concrete CPO for the computational ordering* $\sqsubseteq$ *and* $f \in C \xrightarrow{c} C$ *be continuous. Let* $\langle C,\ \leq\rangle$ *be a poset for the approximation ordering* $\leq$.

Let $\langle A,\ \bot^\sharp,\ \sqsubseteq^\sharp,\ \sqcup^\sharp\rangle$ *be an abstract CPO and* $f^\sharp \in A \xrightarrow{c} A$ *be continuous.*

Let $\langle C,\ \leq\rangle \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \langle A,\ \sqsubseteq^\sharp\rangle$ *be an abstraction such that*

$$\bot \leq \gamma(\bot^\sharp) \tag{2}$$

$$\forall x \in C, y \in A\ .\ (x \leq \gamma(y)) \Rightarrow (f(x) \leq \gamma(f^\sharp(y)) \tag{3}$$

for all increasing chains $\langle x_i,\ i \in \mathbb{N}\rangle$ *for* $\sqsubseteq$ *and* $\langle y_i,\ i \in \mathbb{N}\rangle$ *for* $\sqsubseteq^\sharp$.

$$(\forall i \in \mathbb{N}\ .\ x_i \leq \gamma(y_i)) \Rightarrow \bigsqcup_{i\in\mathbb{N}} x_i \leq \gamma(\bigsqcup_{j\in\mathbb{N}}^{\sharp} y_j) \tag{4}$$

Then $\mathsf{lfp}^{\sqsubseteq} f \leq \gamma(\mathsf{lfp}^{\sqsubseteq^\sharp} f^\sharp)$.

Strictness analysis after Mycroft

- Alan’s strictness analysis originated an enormous amount of work on the subject in the 80’s and early 90’s

- Strictness analysis is found in modern compilers for lazy purely functional languages such as Haskell

Sharing analysis

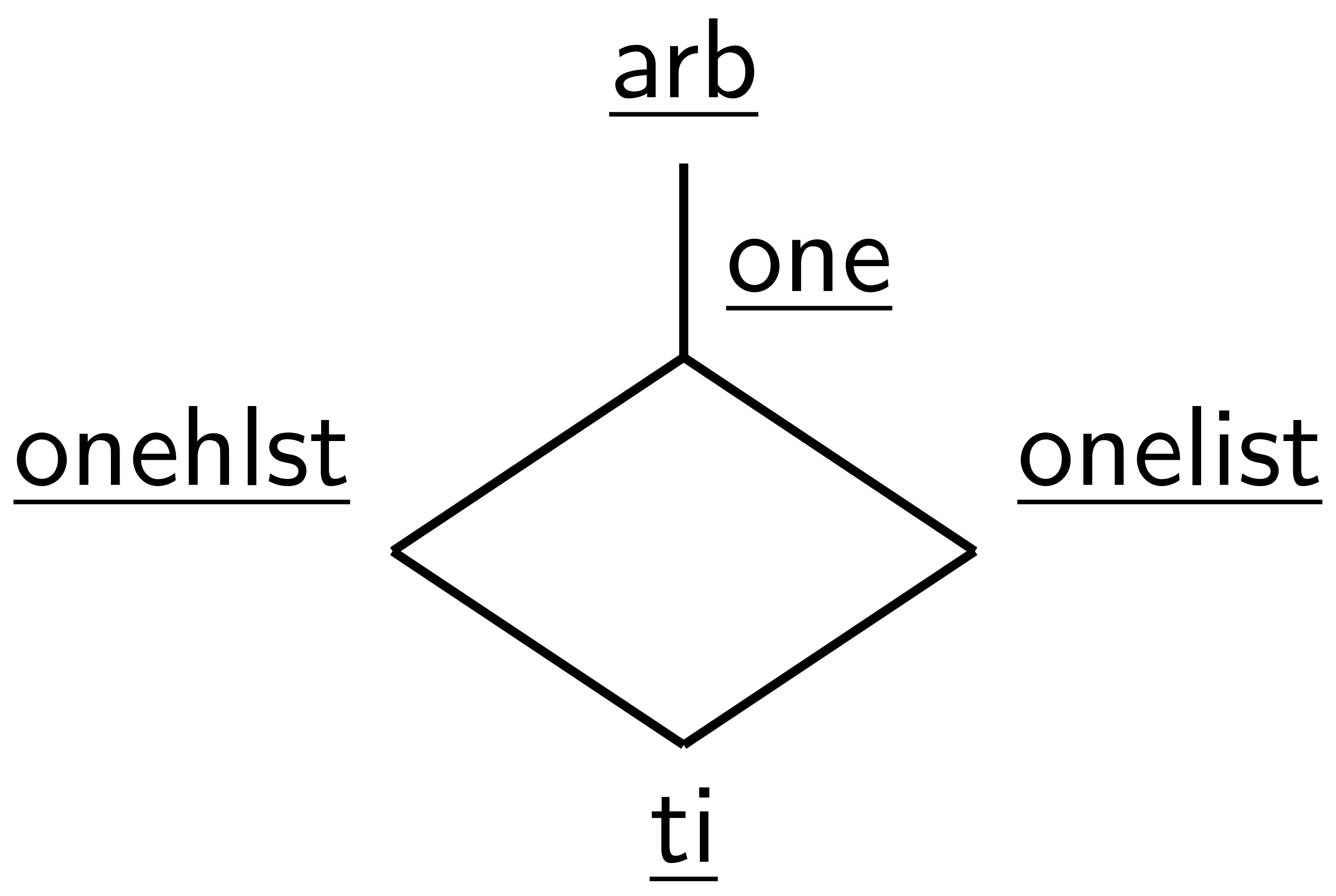
Sharing analysis

Chapter 5 is an attempt to apply the concepts of abstract interpretation to a completely different problem, that of incorporating destructive operators into an applicative program. We do this in order to increase the efficiency of implementation without violating the applicative semantics by introducing destructive operators into our language.

Collecting information on LISP data structures

- Fisrt-order functional language with atoms and operations `cons`, `car`, `cdr`, `free`, `atom`
- The heap runtime data structure is a DAG
- Denotational semantics of a function is parameter x heap $\longrightarrow$ result x heap
- The static analysis infers the set of heap locations descending from heap roots going exclusively through heads only (resp. through tails only, through heads or tails).

Mycroft's abstract domain



- arb: atom, nontermination, or any heap element
- one : atom, nontermination, or heap element accessible from the heap roots by one path only
- onehlst : idem, going through `car` only
- onelist : idem, through `cdr` only
- ti : atom, nontermination, or heap element accessible from the heap roots by one path only, as well as all of its descendants

Alan contributions on abstract interpretation

Mycroft thesis is the origin of static analysis of functional programs

- Alan originated fundamental ideas in abstract interpretation
  - The use of denotational semantics in static analysis
  - Strictness analysis
  - Shape analysis (still a very difficult and active research area)
  - Completeness in abstract interpretation (progressing, but not yet solved)
  - Types and effects (inexhaustible subject)

The End, Thank You

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Happy Retirement to Alan!

