Proof of mutual-exclusion and non-starvation of a program with weak memory model: PostgreSQL

Patrick Cousot (NYU, Emer. ENS, PSL)
(joint work with Jade Alglave)

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PostgreSQL

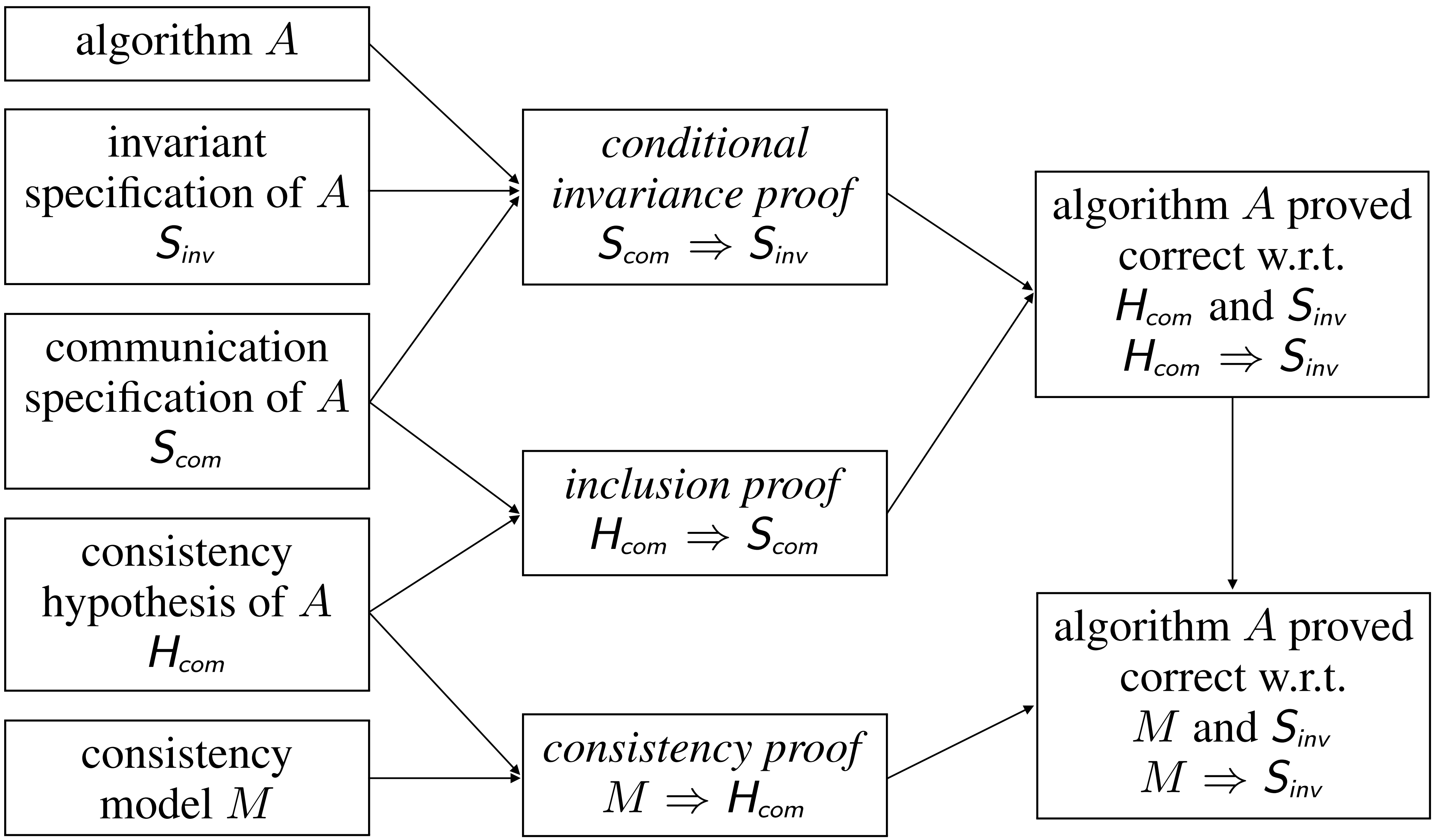
```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

```
1: do
2:   do
3:     r[] Rl0 latch0
4:   while (Rl0=0)
5:   w[] latch0 0
6:   r[] Rf0 flag0
7:   if (Rf0≠0) then
8:     (* critical section *)
       w[] flag0 0
9:     w[] flag1 1
10:    w[] latch1 1
11:  fi
12:while true
13:
```

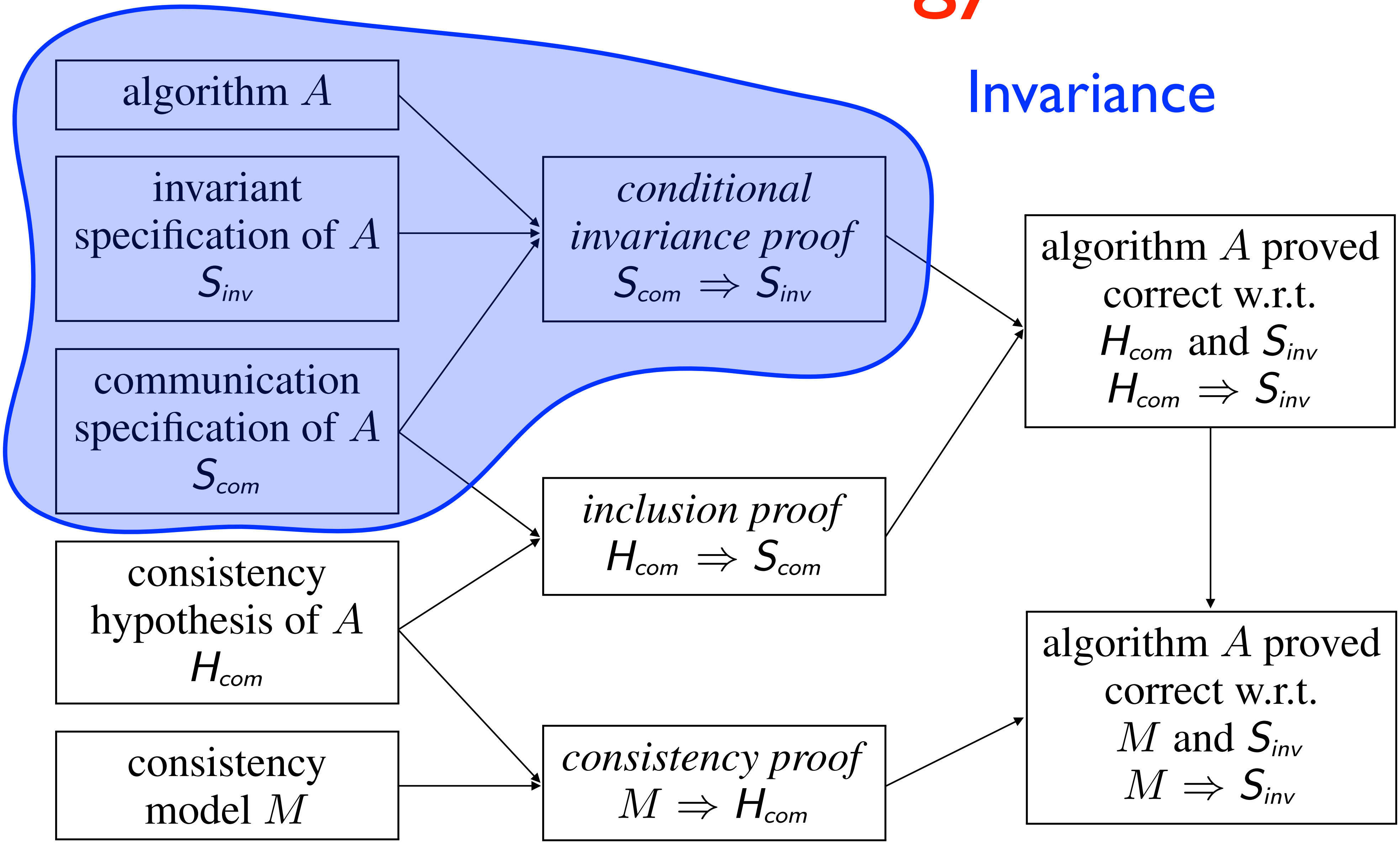
||

```
21:do
22:  do
23:    r[] Rl1 latch1
24:  while (Rl1=0)
25:  w[] latch1 0
26:  r[] Rf1 flag1
27:  if (Rf1≠0) then
28:    (* critical section *)
       w[] flag1 0
29:    w[] flag0 1
30:    w[] latch0 1
31:  fi
32:while true
33:
```

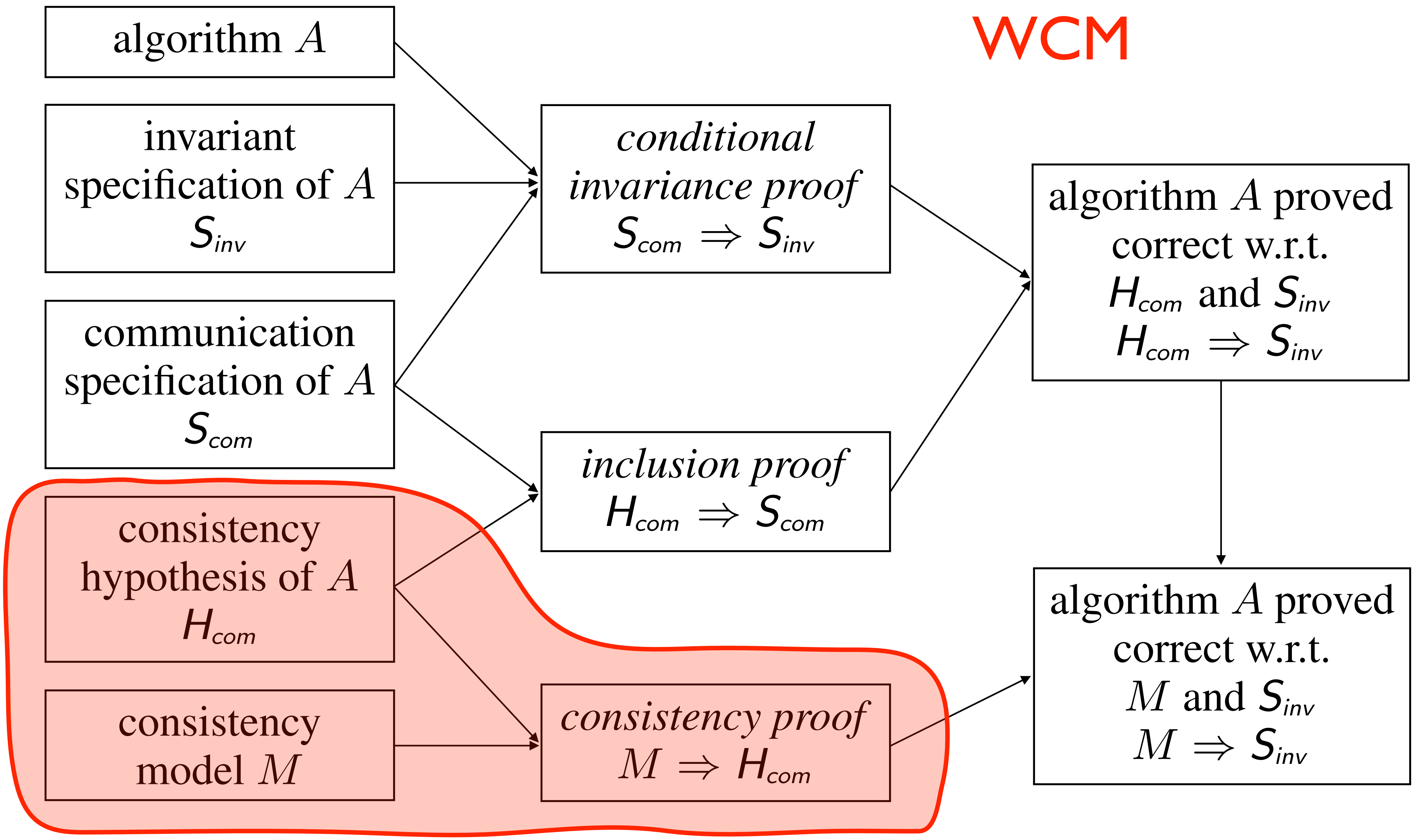
Methodology



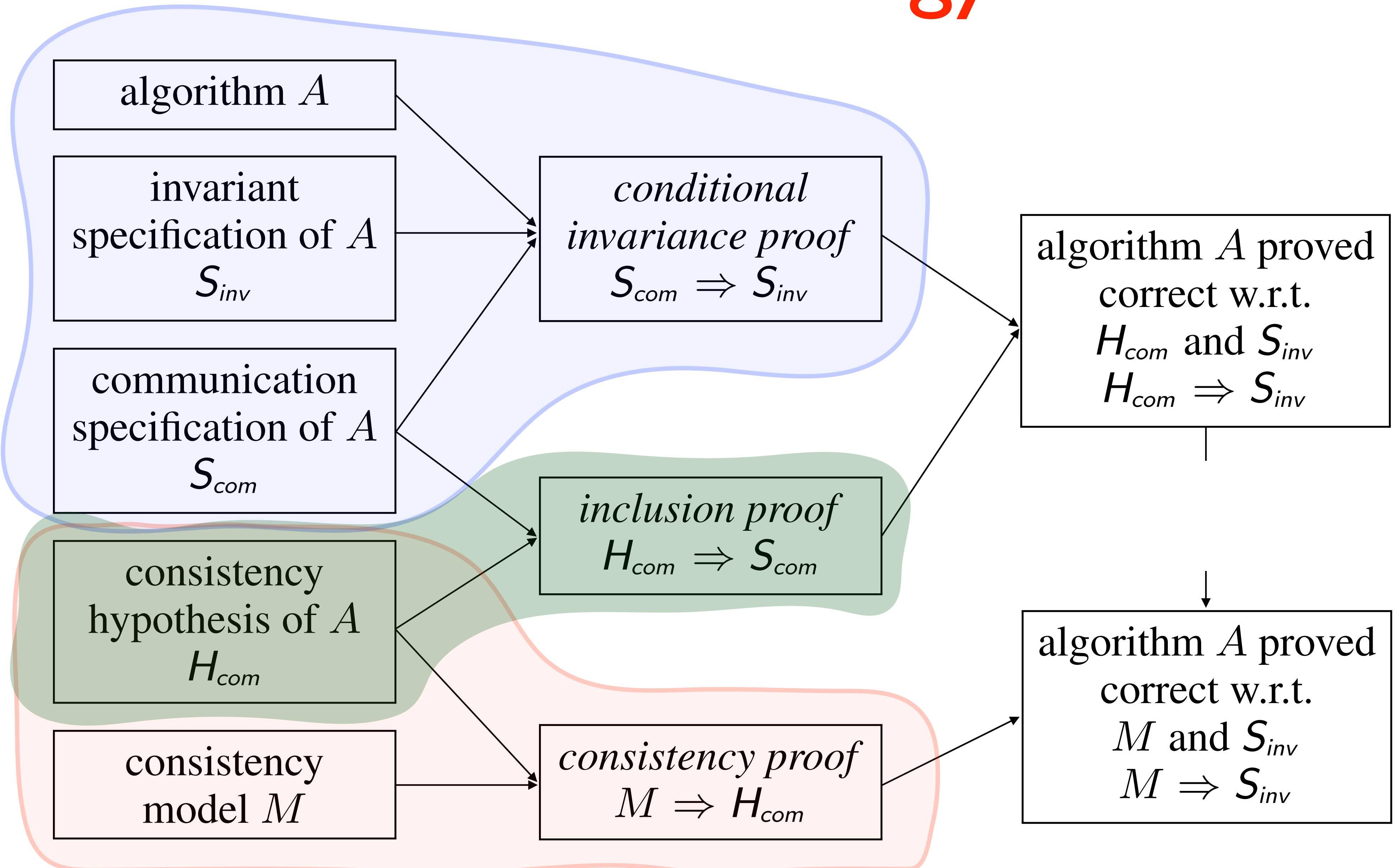
Methodology



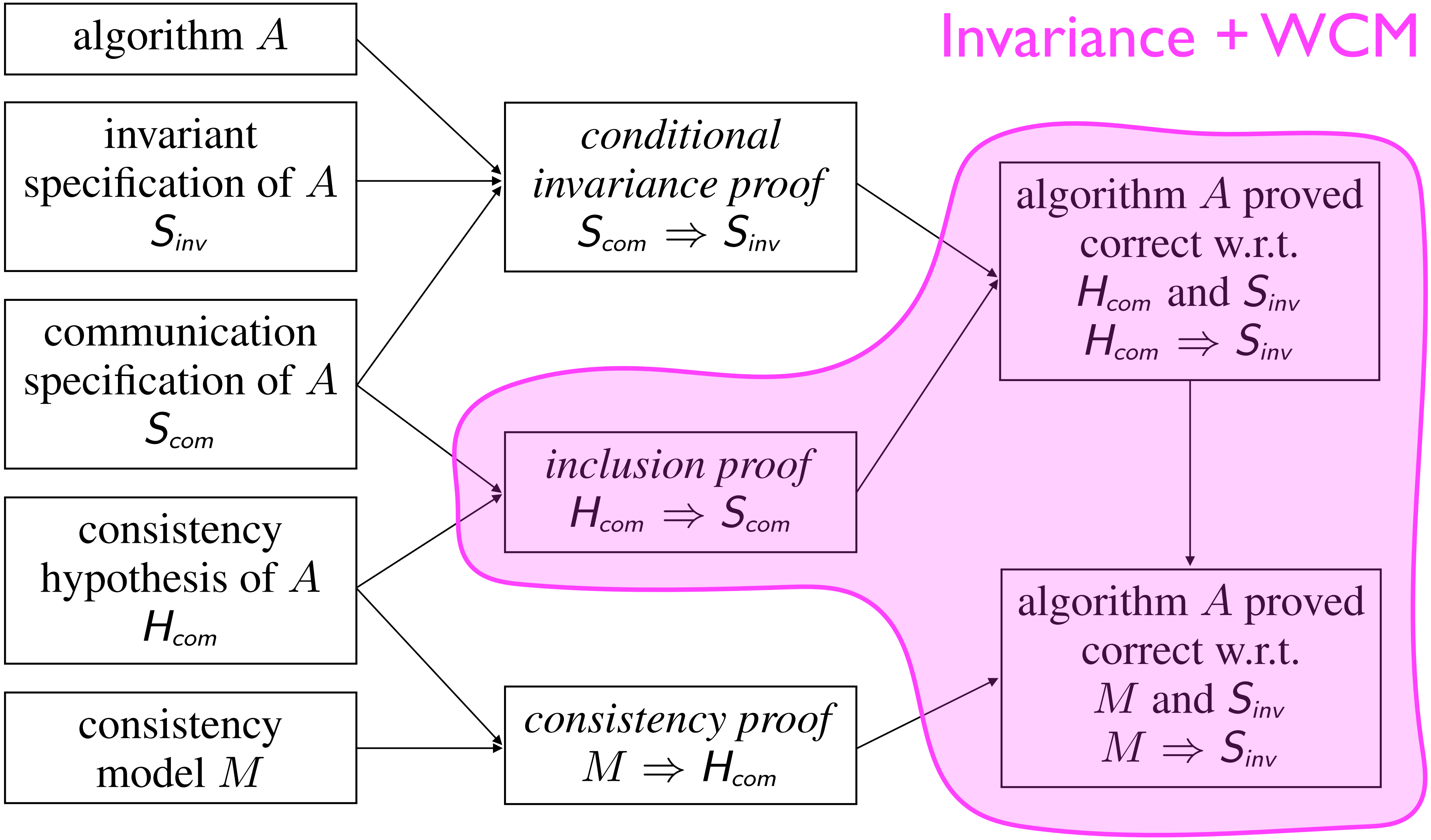
Methodology



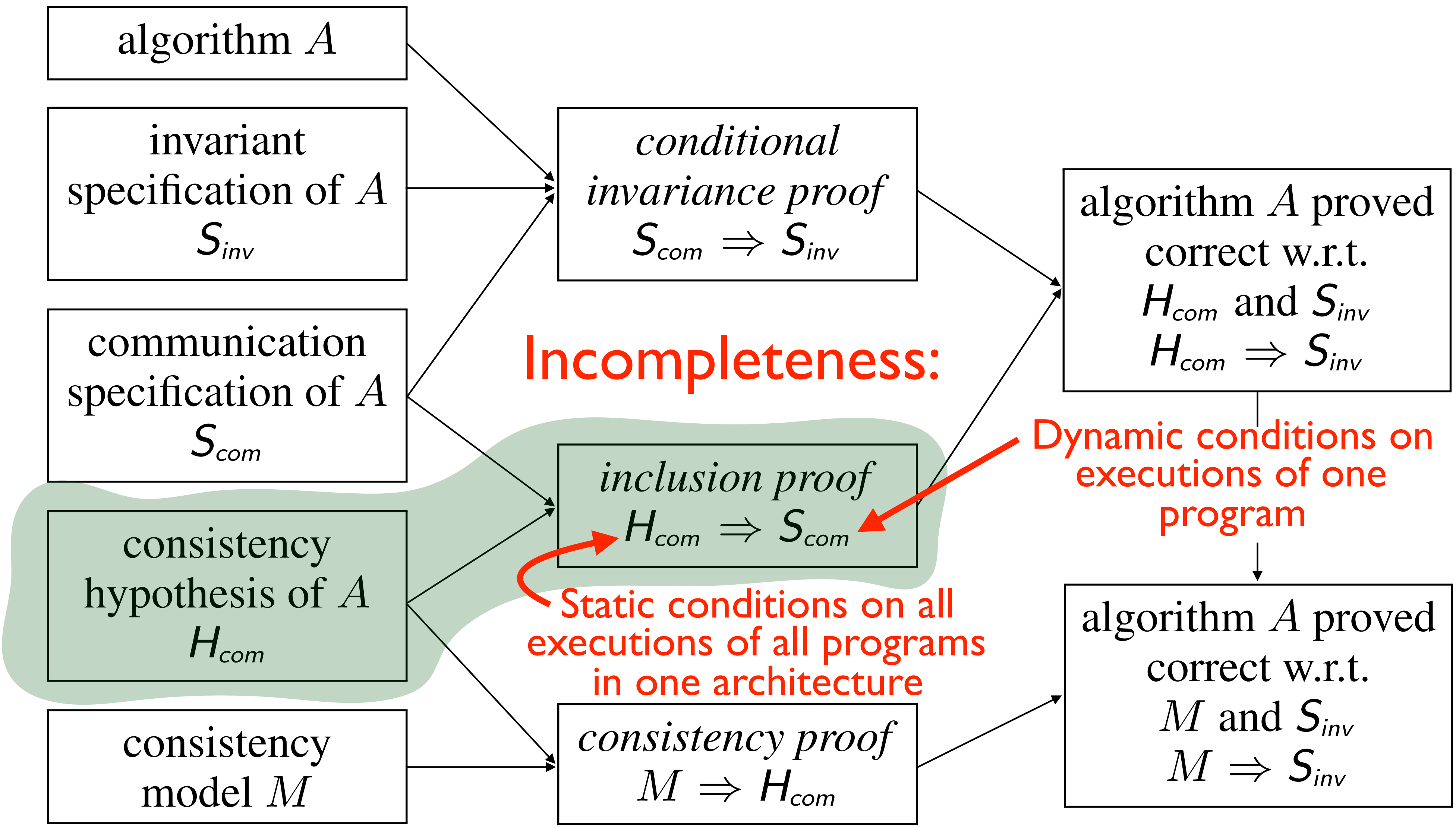
Methodology



Methodology

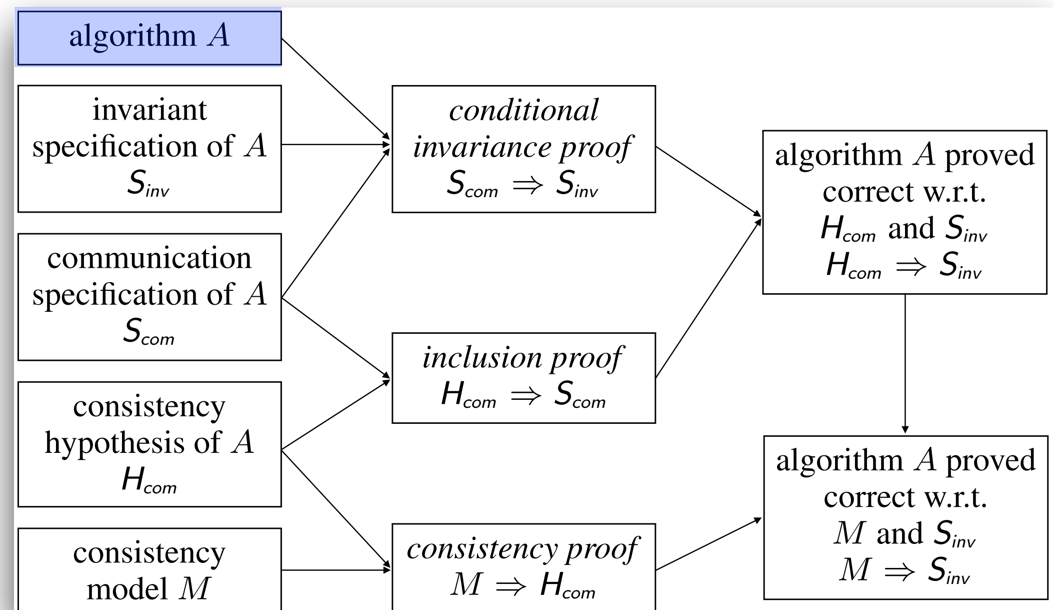


Methodology



Conditional invariance proof: Mutual exclusion

Algorithm


algorithm A
invariant specification of A S_{inv}
communication specification of A S_{com}
consistency hypothesis of A H_{com}
consistency model M
conditional invariance proof $S_{com} \Rightarrow S_{inv}$
inclusion proof $H_{com} \Rightarrow S_{com}$
consistency proof $M \Rightarrow H_{com}$
algorithm A proved correct w.r.t. H_{com} and S_{inv} $H_{com} \Rightarrow S_{inv}$
algorithm A proved correct w.r.t. M and S_{inv} $M \Rightarrow S_{inv}$

PostgreSQL

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: do $\{i\}$	‖	21:do $\{\ell\}$
2: do $\{j_i\}$		22: do $\{m_\ell\}$
3: r[] Rl0 latch0 $\{\rightsquigarrow L0^i_{j_i}\}$		23: r[] Rl1 latch1 $\{\rightsquigarrow L1^\ell_{m_\ell}\}$
4: while (Rl0=0) $\{k_i\}$		24: while (Rl1=0) $\{n_\ell\}$
5: w[] latch0 0		25: w[] latch1 0
6: r[] Rf0 flag0 $\{\rightsquigarrow F0^i\}$		26: r[] Rf1 flag1 $\{\rightsquigarrow F1^\ell\}$
7: if (Rf0$\neq$0) then		27: if (Rf1$\neq$0) then
8: (* critical section *)		28: (* critical section *)
w[] flag0 0		w[] flag1 0
9: w[] flag1 1		29: w[] flag0 1
10: w[] latch1 1		30: w[] latch0 1
11: fi		31: fi
12:while true		32:while true
13:		33:

Stamps

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: do $\{i\}$
2: do $\{j_i\}$
3: r[] Rl0 latch0 $\{\rightsquigarrow L0^{i}_{j_i}\}$
4: while (Rl0=0) $\{k_i\}$
5: w[] latch0 0
6: r[] Rf0 flag0 $\{\rightsquigarrow F0^{i}\}$
7: if (Rf0$\neq$0) then
8: (* critical section *)
w[] flag0 0
9: w[] flag1 1
10: w[] latch1 1
11: fi
12:while true
13:

||

21:do $\{\ell\}$
22: do $\{m_\ell\}$
23: r[] Rl1 latch1 $\{\rightsquigarrow L1^{\ell}_{m_\ell}\}$
24: while (Rl1=0) $\{n_\ell\}$
25: w[] latch1 0
26: r[] Rf1 flag1 $\{\rightsquigarrow F1^{\ell}\}$
27: if (Rf1$\neq$0) then
28: (* critical section *)
w[] flag1 0
29: w[] flag0 1
30: w[] latch0 1
31: fi
32:while true
33:

Ensure that events are unique (your choice)

Variables in Hoare logic & L/O-G

- program variables: `int x;`
- in predicates you need to name the value of variable `x` to express properties of this value of x:
  - $\mathrm{valueof}(\mathrm{x})$
  - x
- WCM: no notion of "the" value of a shared variable `x`
- The only way to know something about "the" value of a shared variable `x` is to read it
- Pythia variable: name given to the read value
- Not necessary in the semantics, only in assertions (but we put them in the semantics)

Pythia variables

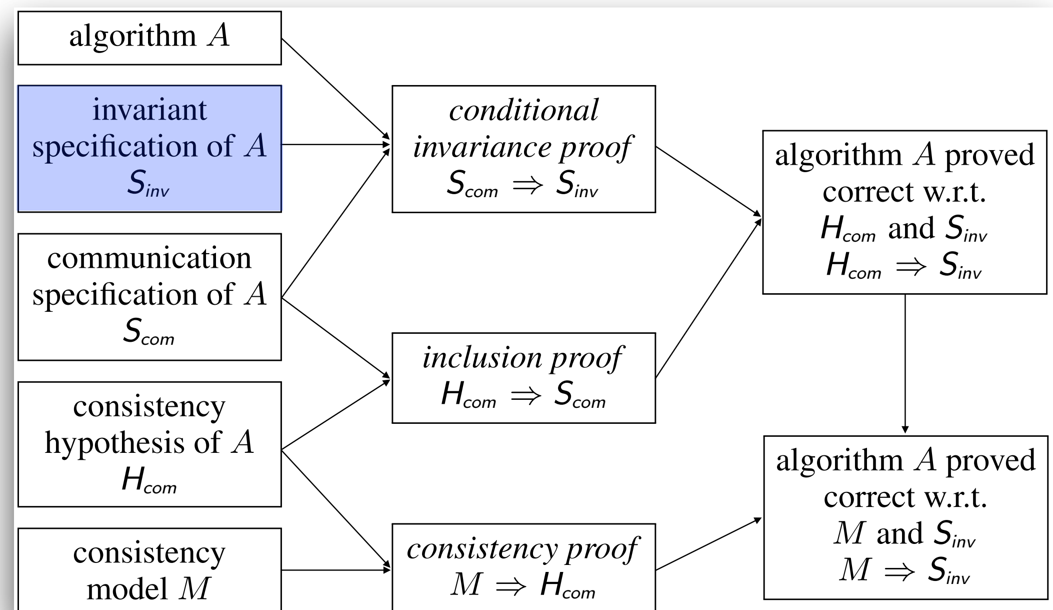
`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

`1: do` $\{i\}$
`2:   do` $\{j_i\}$
`3:     r[] Rl0 latch0` $\{\rightsquigarrow\ L0^{i}_{j_i}\}$
`4:   while (Rl0=0)` $\{k_i\}$
`5:   w[] latch0 0`
`6:   r[] Rf0 flag0` $\{\rightsquigarrow\ F0^{i}\}$
`7:   if (Rf0`$\neq$`0) then`
`8:     (* critical section *)`
`      w[] flag0 0`
`9:     w[] flag1 1`
`10:    w[] latch1 1`
`11:  fi`
`12:while true`
`13:`

$\|$

`21:do` $\{\ell\}$
`22:  do` $\{m_\ell\}$
`23:    r[] Rl1 latch1` $\{\rightsquigarrow\ L1^{\ell}_{m_\ell}\}$
`24:  while (Rl1=0)` $\{n_\ell\}$
`25:  w[] latch1 0`
`26:  r[] Rf1 flag1` $\{\rightsquigarrow\ F1^{\ell}\}$
`27:  if (Rf1`$\neq$`0) then`
`28:    (* critical section *)`
`     w[] flag1 0`
`29:    w[] flag0 1`
`30:    w[] latch0 1`
`31:  fi`
`32:while true`
`33:`

Invariant specification S_{inv}



Mutual exclusion

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: do $\{i\}$
2:   do $\{j_i\}$
3:     r[] Rl0 latch0 $\{\rightsquigarrow L0^{i}_{j_i}\}$
4:   while (Rl0=0) $\{k_i\}$
5:   w[] latch0 0
6:   r[] Rf0 flag0 $\{\rightsquigarrow F0^{i}\}$
7:   if (Rf0≠0) then
8:     ¬at{28}
      (* critical section *)
      w[] flag0 0
9:     w[] flag1 1
10:    w[] latch1 1
11: fi
12:while true
13:

||

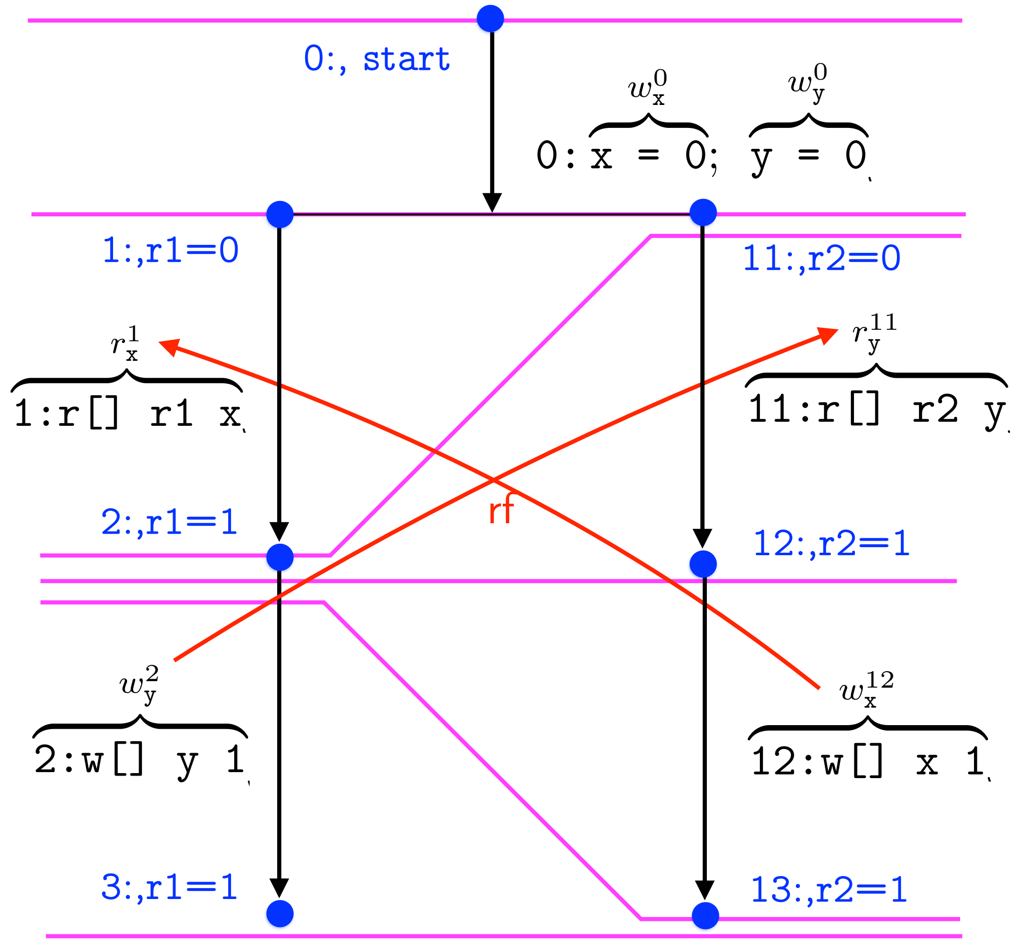
21:do $\{\ell\}$
22:   do $\{m_\ell\}$
23:     r[] Rl1 latch1 $\{\rightsquigarrow L1^{\ell}_{m_\ell}\}$
24:   while (Rl1=0) $\{n_\ell\}$
25:   w[] latch1 0
26:   r[] Rf1 flag1 $\{\rightsquigarrow F1^{\ell}\}$
27:   if (Rf1≠0) then
28:     ¬at{8}
      (* critical section *)
      w[] flag1 0
29:     w[] flag0 1
30:     w[] latch0 1
31:   fi
32:while true
33:

(invariant $S\mathrm{i}_{\mathrm{nv}}$ is elsewhere `true`)

Analytic semantics =
Anarchic semantics +
communication constraints

Analytics semantics with cuts

```
0:{ x = 0; y = 0; }
P0             || P1          ;
1:r[] r1 x     || 11:r[] r2 y;
2:w[] y 1      || 12:w[] x 1 ;
3:             || 13:         ;
```



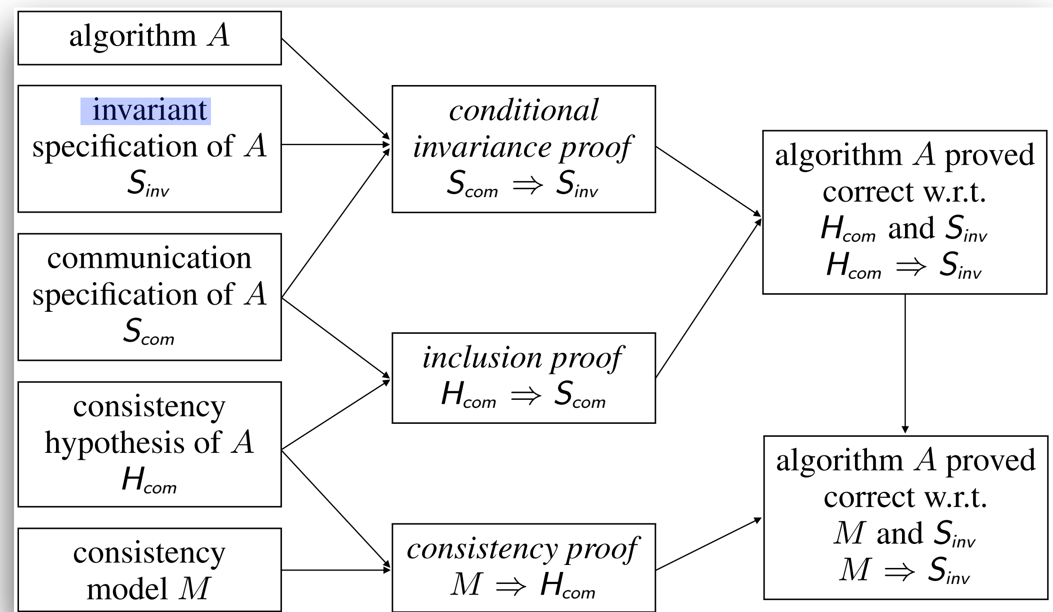
- Anarchic semantics: set of executions:

  $\pi = \varsigma \times \pi \times \mathsf{rf}$

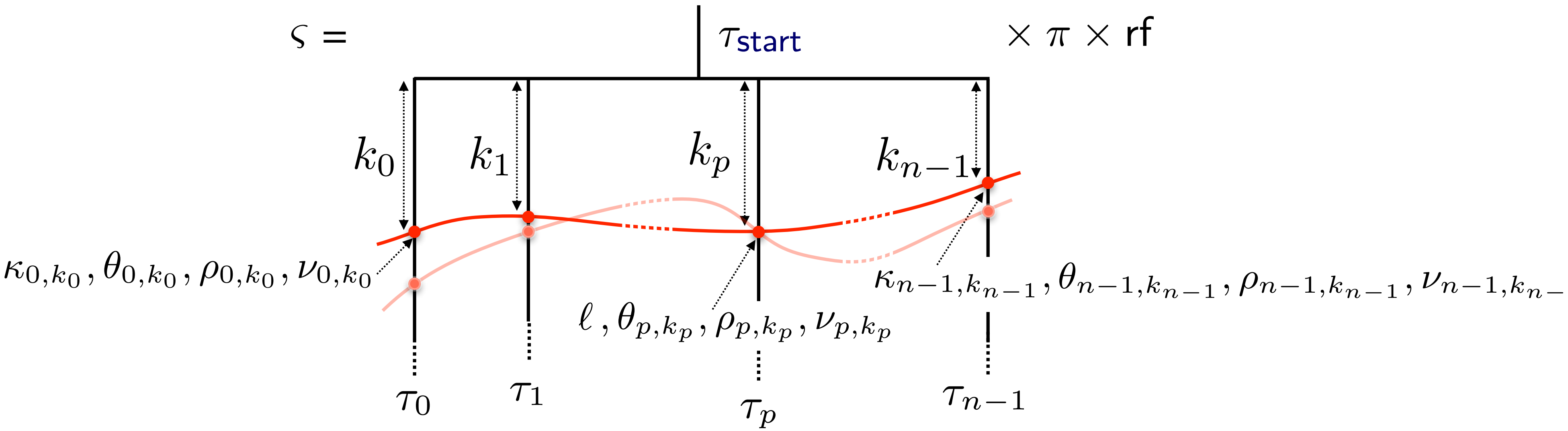
  - ς is the *computation*
  - π is the *cut sequence*
  - rf is the *communication*

- Communication semantics: restrictions on rf in `cat`

Local invariants



Local invariant



- Attached to each program point ℓ of each process p
- Depends on
  - Program points of all other processes κ
  - Stamps θ of all processes
  - Local registers of all processes ρ
  - Pythia variables ν
  - Communications (rf)

Communication relation rf

- rf: relation between write and read events

- Each rf is encoded by $\mathit{\Gamma}$, a set of pairs

$$\mathfrak{rf}\langle x_\theta,\ \langle \ell :,\ \theta',\ v\rangle\rangle$$

Pythia variable of the read event

Program label of the write action

Stamp of the write event

Value write

- $\mathit{\Gamma} \in \Gamma$ (the set of all possible communications rf)

Anarchic communications

Anarchic communications

- Any read can read from any write on the same shared variable (location)

$$\mathrm{RL0}^i_{j_i} \triangleq \{\mathfrak{rf}\langle L0^i_{j_i},\ \langle 0:,\ \_,\ 0\rangle\rangle, \mathfrak{rf}\langle L0^i_{j_i},\ \langle 5:,\ i_5,\ 0\rangle\rangle, \mathfrak{rf}\langle L0^i_{j_i},\ \langle 30:,\ \ell_{30},\ 1\rangle\rangle \mid i_5 \in \mathbb{N} \wedge \ell_{30} \in \mathbb{N}\}$$

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do {i}
2:   do {j_i}
3:     r[] Rl0 latch0 {⇝ L0^i_{j_i}}
4:   while (Rl0=0) {k_i}
5:   w[] latch0 0
6:   r[] Rf0 flag0 {⇝ F0^i}
7:   if (Rf0≠0) then
8:     (* critical section *)
       w[] flag0 0
9:     w[] flag1 1
10:    w[] latch1 1
11:  fi
12:while true
13:
```

||

```
21:do {ℓ}
22:  do {m_ℓ}
23:    r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:  while (Rl1=0) {n_ℓ}
25:  w[] latch1 0
26:  r[] Rf1 flag1 {⇝ F1^ℓ}
27:  if (Rf1≠0) then
28:    (* critical section *)
       w[] flag1 0
29:    w[] flag0 1
30:    w[] latch0 1
31:  fi
32:while true
33:
```

Anarchic communications

- Possible communications for each read at each stamp (point in the execution):

$$\begin{aligned}
\mathrm{RL0}^i_{j_i} &\triangleq \{\mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{0:},\ _,\ 0\rangle\rangle, \mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{5:},\ i_5,\ 0\rangle\rangle, \mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{30:},\ \ell_{30},\ 1\rangle\rangle \mid i_5 \in \mathbb{N} \wedge \ell_{30} \in \mathbb{N}\} \\
\mathrm{RF0}^i &\triangleq \{\mathfrak{rf}\langle F0^i,\ \langle \mathtt{0:},\ _,\ 0\rangle\rangle, \mathfrak{rf}\langle F0^i,\ \langle \mathtt{8:},\ i_8,\ 0\rangle\rangle, \mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \mid i_8 \in \mathbb{N} \wedge \ell_{29} \in \mathbb{N}\} \\
\mathrm{RL1}^\ell_{m_\ell} &\triangleq \{\mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{0:},\ _,\ 1\rangle\rangle, \mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{25:},\ \ell_{25},\ 0\rangle\rangle, \mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{10:},\ i_{10},\ 1\rangle\rangle \mid \ell_{25} \in \mathbb{N} \wedge i_{10} \in \mathbb{N}\} \\
\mathrm{RF1}^\ell &\triangleq \{\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{0:},\ _,\ 1\rangle\rangle, \mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{28:},\ \ell_{28},\ 0\rangle\rangle, \mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{9:},\ i_9,\ 1\rangle\rangle \mid \ell_{28} \in \mathbb{N} \wedge i_9 \in \mathbb{N}\}
\end{aligned}$$

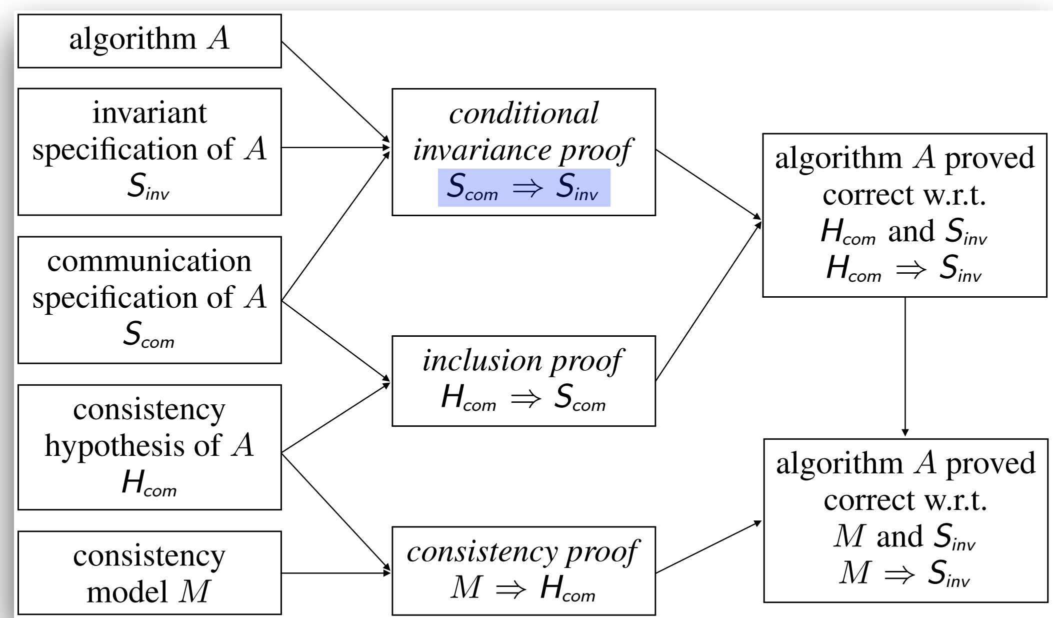
- Anarchic communications:

$$\begin{aligned}
\overline{\Gamma} = \{\{&\mathrm{rl0}^i_{j_i}, \mathrm{rf0}^i, \mathrm{rl1}^\ell_{m_\ell}, \mathrm{rf1}^\ell \mid i \in \mathbb{N} \wedge j_i \in [0, k_i] \wedge \ell \in \mathbb{N} \wedge j \in [0, n_\ell]\} \mid \forall i \in \mathbb{N} \mathrel{.} \forall j_i \in [1, k_i] \mathrel{.} \\
&\mathrm{rl0}^i_{j_i} \in \mathrm{RL0}^i_{j_i} \wedge \mathrm{rf0}^i \in \mathrm{RF0}^i \wedge \forall \ell \in \mathbb{N} \mathrel{.} \forall m_\ell \in [1, m_\ell] \mathrel{.} \mathrm{rl1}^\ell_{m_\ell} \in \mathrm{RL1}^\ell_{m_\ell} \wedge \mathrm{rf1}^\ell \in \mathrm{RF1}^\ell\}
\end{aligned}$$

- Anarchic semantics: $\mathit{\Gamma} \in \overline{\Gamma}$

- WCM semantics: $\mathit{\Gamma} \in \Gamma, \Gamma \subseteq \overline{\Gamma}$

Inductive invariant S_{ind}



Inductive invariant

- S_{ind} is inductive under hypothesis S_{com} iff, assuming S_{com}, we have:
  - S_{ind} is true at the beginning of an execution
  - If S_{ind} is true during execution is remains true after one more computation or communication step
- $$\frac{S_{inv} \text{ holds under hypothesis } S_{com} \qquad S_{ind} \Rightarrow S_{inv}}{S_{com} \Rightarrow S_{inv}}$$

Inductive invariant

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: $\{\varGamma \in \Gamma\}$
  do $\{i\}$
2:   $\{\varGamma \in \Gamma\}$
    do $\{j_i\}$
3:     $\{\varGamma \in \Gamma\}$
      r[] R10 latch0 $\{\leadsto L0^i_{j_i}\}$
4:     $\{\varGamma \in \Gamma \wedge \texttt{R10} = L0^i_{j_i} \wedge (\mathrm{r0R10}^i_{j_i}[\varGamma] \vee \mathrm{r1R10}^i_{j_i}[\varGamma])\}$
    while (R10=0) $\{k_i\}$
5:   $\{\varGamma \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\varGamma]\}$
    w[] latch0 0
6:   $\{\varGamma \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\varGamma]\}$
    r[] Rf0 flag0 $\{\leadsto F0^i\}$
7:   $\{\varGamma \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\varGamma] \wedge \texttt{Rf0} = F0^i$
      $\wedge (\mathrm{r0Rf0}^i[\varGamma] \vee \mathrm{r1Rf0}^i[\varGamma])\}$
    if (Rf0≠0) then
8:     $\{\varGamma \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\varGamma] \wedge \mathrm{r1Rf0}^i[\varGamma]\}$
      (* critical section *)
      w[] flag0 0
9:     $\{\varGamma \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\varGamma] \wedge \mathrm{r1Rf0}^i[\varGamma]\}$
      w[] flag1 1
10:     $\{\varGamma \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\varGamma] \wedge \mathrm{r1Rf0}^i[\varGamma]\}$
      w[] latch1 1
11:     $\{\varGamma \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\varGamma] \wedge \mathrm{r1Rf0}^i[\varGamma]\}$
    fi
12:   $\{\varGamma \in \Gamma\}$
  while true
13: $\{\mathsf{false}\}$

||

21: $\{\varGamma \in \Gamma\}$
  do $\{\ell\}$
22:   $\{\varGamma \in \Gamma\}$
    do $\{m_\ell\}$
23:     $\{\varGamma \in \Gamma\}$
      r[] R11 latch1 $\{\leadsto L1^\ell_{m_\ell}\}$
24:     $\{\varGamma \in \Gamma \wedge \texttt{R11} = L1^\ell_{m_\ell} \wedge (\mathrm{r0R11}^\ell_{m_\ell}[\varGamma] \vee \mathrm{r1R11}^\ell_{m_\ell}[\varGamma])\}$
    while (R11=0) $\{n_\ell\}$
25:   $\{\varGamma \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\varGamma]\}$
    w[] latch1 0
26:   $\{\varGamma \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\varGamma]\}$
    r[] Rf1 flag1 $\{\leadsto F1^\ell\}$
27:   $\{\varGamma \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\varGamma] \wedge \texttt{Rf1} = F1^\ell$
      $\wedge (\mathrm{r0Rf1}^\ell[\varGamma] \vee \mathrm{r1Rf1}^\ell[\varGamma])\}$
    if (Rf1≠0) then
28:     $\{\varGamma \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\varGamma] \wedge \mathrm{r1Rf1}^\ell[\varGamma]\}$
      (* critical section *)
      w[] flag1 0
29:     $\{\varGamma \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\varGamma] \wedge \mathrm{r1Rf1}^\ell[\varGamma]\}$
      w[] flag0 1
30:     $\{\varGamma \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\varGamma] \wedge \mathrm{r1Rf1}^\ell[\varGamma]\}$
      w[] latch0 1
31:     $\{\varGamma \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\varGamma] \wedge \mathrm{r1Rf1}^\ell[\varGamma]\}$
    fi
32:   $\{\varGamma \in \Gamma\}$
  while true
33: $\{\mathsf{false}\}$

Inductive invariant

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: $\{\mathit{\Gamma} \in \Gamma\}$
   do $\{i\}$
2:    $\{\mathit{\Gamma} \in \Gamma\}$
      do $\{j_i\}$
3:       $\{\mathit{\Gamma} \in \Gamma\}$
         r[] Rl0 latch0 $\{\rightsquigarrow L0^i_{j_i}\}$
4:       $\{\mathit{\Gamma} \in \Gamma \wedge \texttt{Rl0} = L0^i_{j_i} \wedge (\text{r0Rl0}^i_{j_i}[\mathit{\Gamma}] \vee \text{r1}$
      while (Rl0=0) $\{k_i\}$
5:    $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl0}^i_{k_i}[\mathit{\Gamma}]\}$
      w[] latch0 0
6:    $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl0}^i_{k_i}[\mathit{\Gamma}]\}$
      r[] Rf0 flag0 $\{\rightsquigarrow F0^i\}$
7:    $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \texttt{Rf0} = F0^i \wedge (\text{r0Rf0}^i[\mathit{\Gamma}] \vee \text{r1Rf0}^i[\mathit{\Gamma}])\}$
      if (Rf0≠0) then
8:       $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \text{r1Rf0}^i[\mathit{\Gamma}]\}$
         (* critical section *)
         w[] flag0 0
9:       $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \text{r1Rf0}^i[\mathit{\Gamma}]\}$
         w[] flag1 1
10:      $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \text{r1Rf0}^i[\mathit{\Gamma}]\}$
         w[] latch1 1
11:      $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \text{r1Rf0}^i[\mathit{\Gamma}]\}$
      fi
12:   $\{\mathit{\Gamma} \in \Gamma\}$
   while true
13: {false}

||

21: $\{\mathit{\Gamma} \in \Gamma\}$
   do $\{\ell\}$
22:   $\{\mathit{\Gamma} \in \Gamma\}$

$\text{Rl1}^\ell_{m_\ell}[\mathit{\Gamma}])\}$

      w[] latch1 0
26:   $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}]\}$
      r[] Rf1 flag1 $\{\rightsquigarrow F1^\ell\}$
27:   $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \texttt{Rf1} = F1^\ell \wedge (\text{r0Rf1}^\ell[\mathit{\Gamma}] \vee \text{r1Rf1}^\ell[\mathit{\Gamma}])\}$
      if (Rf1≠0) then
28:      $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \text{r1Rf1}^\ell[\mathit{\Gamma}]\}$
         (* critical section *)
         w[] flag1 0
29:      $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \text{r1Rf1}^\ell[\mathit{\Gamma}]\}$
         w[] flag0 1
30:      $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \text{r1Rf1}^\ell[\mathit{\Gamma}]\}$
         w[] latch0 1
31:      $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \text{r1Rf1}^\ell[\mathit{\Gamma}]\}$
      fi
32:   $\{\mathit{\Gamma} \in \Gamma\}$
   while true
33: {false}

Possible communications

Inductive invariant

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
r[] R10 latch0 $\{\leadsto L0^i_{j_i}\}$
4: $\{\mathit{\Gamma} \in \Gamma \wedge \texttt{R10} = L0^i_{j_i} \wedge (\text{r0R10}^i_{j_i}[\mathit{\Gamma}] \vee \text{r1R10}^i_{j_i}[\mathit{\Gamma}])\}$
while (R10=0) $\{k_i\}$
5: $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1R10}^i_{k_i}[\mathit{\Gamma}]\}$
w[] latch0 0
6: $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1R10}^i_{k_i}[\mathit{\Gamma}]\}$
r[] Rf0 flag0 $\{\leadsto F0^i\}$
7: $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \texttt{Rf0} = F0^i$
$\wedge (\text{r0Rf0}^i[\mathit{\Gamma}] \vee \text{r1Rf0}^i[\mathit{\Gamma}])\}$
if (Rf0≠0) then
8: $\{\mathit{\Gamma} \in$
(* c
w[]
9: $\{\mathit{\Gamma} \in$
w[]
10: $\{\mathit{\Gamma} \in$
w[]
11: $\{\mathit{\Gamma} \in$
fi
12: $\{\mathit{\Gamma} \in \Gamma\}$
while true
13: {false}

||

21: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{\ell\}$
22: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{m_\ell\}$
23: $\{\mathit{\Gamma} \in \Gamma\}$
r[] R11 latch1 $\{\leadsto L1^\ell_{m_\ell}\}$
24: $\{\mathit{\Gamma} \in \Gamma \wedge \texttt{R11} = L1^\ell_{m_\ell} \wedge (\text{r0R11}^\ell_{m_\ell}[\mathit{\Gamma}] \vee \text{r1R11}^\ell_{m_\ell}[\mathit{\Gamma}])\}$
while (R11=0) $\{n_\ell\}$
25: $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}]\}$
w[] latch1 0
26: $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}]\}$
r[] Rf1 flag1 $\{\leadsto F1^\ell\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \text{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \texttt{Rf1} = F1^\ell$
$\wedge (\text{r0Rf1}^\ell[\mathit{\Gamma}] \vee \text{r1Rf1}^\ell[\mathit{\Gamma}])\}$
if (Rf1≠0) then
$\Gamma \wedge \text{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \text{r1Rf1}^\ell[\mathit{\Gamma}]\}$
itical section *)
lag1 0
$\Gamma \wedge \text{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \text{r1Rf1}^\ell[\mathit{\Gamma}]\}$
lag0 1
$\Gamma \wedge \text{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \text{r1Rf1}^\ell[\mathit{\Gamma}]\}$
atch0 1
$\Gamma \wedge \text{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \text{r1Rf1}^\ell[\mathit{\Gamma}]\}$
fi
32: $\{\mathit{\Gamma} \in \Gamma\}$
while true
33: {false}

Register assignment of the Pythia variable after read event

Inductive invariant

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
r[] R10 latch0 $\{\rightsquigarrow L0^i_{j_i}\}$
4: $\{\mathit{\Gamma} \in \Gamma \wedge \mathtt{R10} = L0^i_{j_i} \wedge (\mathrm{r0R10}^i_{j_i}[\mathit{\Gamma}] \vee \mathrm{r1R10}^i_{j_i}[\mathit{\Gamma}])\}$
while (R10=0) $\{k_i\}$
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}]\}$
w[] latch0 0

w[] flag0 0
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
w[] flag1 1
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
w[] latch1 1
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
fi
12: $\{\mathit{\Gamma} \in \Gamma\}$
while true
13: {false}

21: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{\ell\}$
22: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{m_\ell\}$
23: $\{\mathit{\Gamma} \in \Gamma\}$
r[] R11 latch1 $\{\rightsquigarrow L1^\ell_{m_\ell}\}$
24: $\{\mathit{\Gamma} \in \Gamma \wedge \mathtt{R11} = L1^\ell_{m_\ell} \wedge (\mathrm{r0R11}^\ell_{m_\ell}[\mathit{\Gamma}] \vee \mathrm{r1R11}^\ell_{m_\ell}[\mathit{\Gamma}])\}$
while (R11=0) $\{n_\ell\}$
25: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}]\}$
w[] latch1 0

w[] flag1 0
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
w[] flag0 1
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
w[] latch0 1
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
fi
32: $\{\mathit{\Gamma} \in \Gamma\}$
while true
33: {false}

Possible values of Pythia variables depending on communications

$$\mathrm{r0R10}^i_{j_i}[\mathit{\Gamma}] \triangleq (\mathfrak{rf}\langle L0^i_{j_i},\ \langle 0\!:,\ \_,\ 0\rangle\rangle \in \mathit{\Gamma} \wedge L0^i_{j_i} = 0) \vee (\exists i_5 \in \mathbb{N}\ .\ \mathfrak{rf}\langle L0^i_{j_i},\ \langle 5\!:,\ i_5,\ 0\rangle\rangle \in \mathit{\Gamma} \wedge L0^i_{j_i} = 0)$$

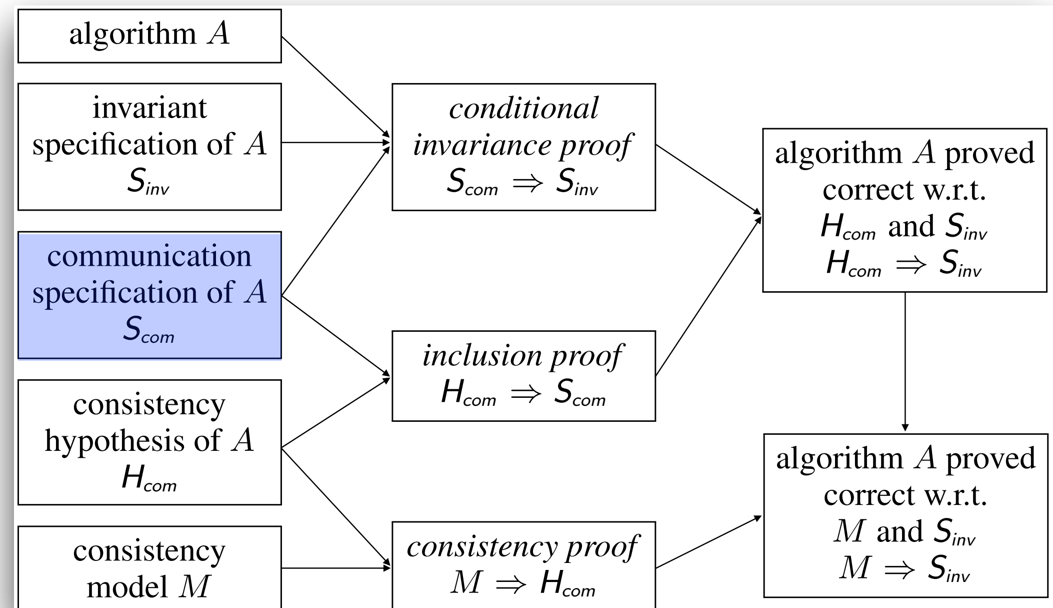
$$\mathrm{r1R10}^i_{j_i}[\mathit{\Gamma}] \triangleq (\exists \ell_{30} \in \mathbb{N}\ .\ \mathfrak{rf}\langle L0^i_{j_i},\ \langle 30\!:,\ \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge L0^i_{j_i} = 1)$$

Communicated values

- Notation: $\underbrace{\mathrm{r}}_{\text{read}}\underbrace{(0|1)}_{\text{value 0 or 1}}\underbrace{\mathrm{R}(\overbrace{\mathrm{l}}^{\text{latch}},\overbrace{\mathrm{f}}^{\text{fetch}})\overbrace{(0|1)}^{\text{process}}}_{\text{register}}$

$$
\begin{aligned}
\mathrm{r0Rl0}^i_{j_i}[\Gamma] &\triangleq (\mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{0}{:},\ \_,\ 0\rangle\rangle \in \Gamma \wedge L0^i_{j_i} = 0) \vee (\exists i_5 \in \mathbb{N}\ .\ \mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{5}{:},\ i_5,\ 0\rangle\rangle \in \Gamma \wedge L0^i_{j_i} = 0)\\
\mathrm{r1Rl0}^i_{j_i}[\Gamma] &\triangleq (\exists \ell_{30} \in \mathbb{N}\ .\ \mathfrak{rf}\langle L0^i_{j_i},\ \langle \mathtt{30}{:},\ \ell_{30},\ 1\rangle\rangle \in \Gamma \wedge L0^i_{j_i} = 1)\\
\mathrm{r0Rf0}^i[\Gamma] &\triangleq (\mathfrak{rf}\langle F0^i,\ \langle \mathtt{0}{:},\ \_,\ 0\rangle\rangle \in \Gamma \wedge F0^i = 0) \vee (\exists i_8 \in \mathbb{N}\ .\ \mathfrak{rf}\langle F0^i,\ \langle \mathtt{8}{:},\ i_8,\ 0\rangle\rangle \in \Gamma \wedge F0^i = 0)\\
\mathrm{r1Rf0}^i[\Gamma] &\triangleq (\exists \ell_{29} \in \mathbb{N}\ .\ \mathfrak{rf}\langle F0^i,\ \langle \mathtt{29}{:},\ \ell_{29},\ 1\rangle\rangle \in \Gamma \wedge F0^i = 1)\\
\mathrm{r0Rl1}^\ell_{m_\ell}[\Gamma] &\triangleq (\exists \ell_{25} \in \mathbb{N}\ .\ \mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{25}{:},\ \ell_{25},\ 0\rangle\rangle \in \Gamma \wedge L1^\ell_{m_\ell} = 0)\\
\mathrm{r1Rl1}^\ell_{m_\ell}[\Gamma] &\triangleq (\mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{0}{:},\ \_,\ 1\rangle\rangle \in \Gamma \wedge L1^\ell_{m_\ell} = 1) \vee (\exists i_{10} \in \mathbb{N}\ .\ \mathfrak{rf}\langle L1^\ell_{m_\ell},\ \langle \mathtt{10}{:},\ i_{10},\ 1\rangle\rangle \in \Gamma \wedge L1^\ell_{m_\ell} = 1)\\
\mathrm{r0Rf1}^\ell[\Gamma] &\triangleq (\exists m_{28} \in \mathbb{N}\ .\ \mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{28}{:},\ m_{28},\ 0\rangle\rangle \in \Gamma \wedge F1^\ell = 0)\\
\mathrm{r1Rf1}^\ell[\Gamma] &\triangleq (\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{0}{:},\ \_,\ 1\rangle\rangle \in \Gamma \wedge F1^\ell = 1) \vee (\exists i_9 \in \mathbb{N}\ .\ \mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{9}{:},\ i_9,\ 1\rangle\rangle \in \Gamma \wedge F1^\ell = 1)
\end{aligned}
$$

Communication specification



Calculational design of the communication specification

$$
\begin{array}{ll}
& (\neg \mathsf{S}_{inv}(\mathit{\Gamma}, \Gamma)) \wedge \mathsf{S}_{ind}(\mathit{\Gamma}, \Gamma) \\
\triangleq & \mathsf{at}\{8\} \wedge \mathsf{at}\{28\} \wedge \mathsf{S}_{ind}(\mathit{\Gamma}, \Gamma) \qquad \text{(def. invariance specification } \mathsf{S}_{inv}) \\
\Rightarrow & \mathsf{at}\{8\} \wedge \mathsf{at}\{28\} \wedge (\exists i, k_i, \ell, n_\ell \in \mathbb{N} \,.\, \mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \\
& \mathrm{r1Rf0}^i[\mathit{\Gamma}] \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]) \qquad \text{(by invariant } \mathsf{S}_{ind}(\mathit{\Gamma}, \Gamma)) \\
\Rightarrow & \mathsf{at}\{8\} \wedge \mathsf{at}\{28\} \wedge \Big(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29} \in \mathbb{N} \,.\, \mathit{\Gamma} \in \Gamma \wedge (\mathfrak{rf}\langle L0^i_{k_i}, \\
& \langle \mathtt{30:},\ \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle L1^\ell_{n_\ell}, \\
& \langle \mathtt{0:},\ \_,\ 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{0:},\ \_,\ 1\rangle\rangle \in \mathit{\Gamma})\Big) \vee \\
& \Big(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_9 \in \mathbb{N} \,.\, \mathit{\Gamma} \in \Gamma \wedge (\mathfrak{rf}\langle L0^i_{k_i},\ \langle \mathtt{30:},\ \ell_{30}, \\
& 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle \mathtt{0:},\ \_, \\
& 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{9:},\ i_9,\ 1\rangle\rangle \in \mathit{\Gamma})\Big) \vee \\
& \Big(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10} \in \mathbb{N} \,.\, \mathit{\Gamma} \in \Gamma \wedge (\mathfrak{rf}\langle L0^i_{k_i},\ \langle \mathtt{30:},\ \ell_{30}, \\
& 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle \mathtt{10:},\ i_{10}, \\
& 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{0:},\ \_,\ 1\rangle\rangle \in \mathit{\Gamma})\Big) \vee \\
& \Big(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10}, i_9 \in \mathbb{N} \,.\, \mathit{\Gamma} \in \Gamma \wedge (\mathfrak{rf}\langle L0^i_{k_i},\ \langle \mathtt{30:},\ \ell_{30}, \\
& 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle F0^i,\ \langle \mathtt{29:},\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle \mathtt{10:},\ i_{10}, \\
& 1\rangle\rangle \in \mathit{\Gamma}) \wedge (\mathfrak{rf}\langle F1^\ell,\ \langle \mathtt{9:},\ i_9,\ 1\rangle\rangle \in \mathit{\Gamma})\Big) \\
& \quad \text{(def. } \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}], \mathrm{r1Rf0}^i[\mathit{\Gamma}], \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}], \text{ and } \mathrm{r1Rf1}^\ell[\mathit{\Gamma}], \mathfrak{rf}\langle x_\theta, \\
& \quad \langle \ell\mathtt{:},\ \theta',\ v\rangle\rangle \text{ implies that } x_\theta = v,\ A \wedge (B \vee C) = (A \wedge B) \vee \\
& \quad (A \wedge C),\ \exists \text{ distributes over } \vee, \text{ and } (\exists x \,.\, A(x)) \wedge B = \exists x \,. \\
& \quad (A(x) \wedge B) \text{ when } x \text{ is not free in } B) \\
\Rightarrow & \mathsf{at}\{8\} \wedge \mathsf{at}\{28\} \wedge (\neg \mathsf{S}_{com_1}(\mathit{\Gamma}, \Gamma) \vee \neg \mathsf{S}_{com_2}(\mathit{\Gamma}, \Gamma) \vee \neg \mathsf{S}_{com_3}(\mathit{\Gamma}, \Gamma) \vee \\
& \neg \mathsf{S}_{com_4}(\mathit{\Gamma}, \Gamma)) \\
\Rightarrow & \neg \mathsf{S}_{com}(\mathit{\Gamma}, \Gamma)
\end{array}
$$

Calculational design of the communication specification

- where

$$S_{com}(\Gamma,\overline{\Gamma}) \triangleq (\mathsf{at}\{8\} \wedge \mathsf{at}\{28\}) \Longrightarrow (S_{com_1}(\Gamma,\overline{\Gamma}) \wedge S_{com_2}(\Gamma,\overline{\Gamma}) \wedge S_{com_3}(\Gamma,\overline{\Gamma}) \wedge S_{com_4}(\Gamma,\overline{\Gamma}))$$

$$\begin{aligned}
S_{com_1} \triangleq\ & \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29} \in \mathbb{N} \ .\ \mathit{\Gamma} \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\ \langle \mathsf{30}:,\\
& \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F0^i,\ \langle \mathsf{29}:,\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell},\\
& \langle \mathsf{0}:,\ _,\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F1^\ell,\ \langle \mathsf{0}:,\ _,\ 1\rangle\rangle \in \mathit{\Gamma}\\
S_{com_2} \triangleq\ & \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_9 \in \mathbb{N} \ .\ \mathit{\Gamma} \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\ \langle \mathsf{30}:,\\
& \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F0^i,\ \langle \mathsf{29}:,\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell},\\
& \langle \mathsf{0}:,\ _,\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F1^\ell,\ \langle \mathsf{9}:,\ i_9,\ 1\rangle\rangle \in \mathit{\Gamma}\\
S_{com_3} \triangleq\ & \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10} \in \mathbb{N} \ .\ \mathit{\Gamma} \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\ \langle \mathsf{30}:,\\
& \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F0^i,\ \langle \mathsf{29}:,\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell},\\
& \langle \mathsf{10}:,\ i_{10},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F1^\ell,\ \langle \mathsf{0}:,\ _,\ 1\rangle\rangle \in \mathit{\Gamma}\\
S_{com_4} \triangleq\ & \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10}, i_9 \in \mathbb{N} \ .\ \mathit{\Gamma} \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\\
& \langle \mathsf{30}:,\ \ell_{30},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F0^i,\ \langle \mathsf{29}:,\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge\\
& \mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle \mathsf{10}:,\ i_{10},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge \mathfrak{rf}\langle F1^\ell,\ \langle \mathsf{9}:,\ i_9,\ 1\rangle\rangle \in \mathit{\Gamma}
\end{aligned}$$

- This proves S_{com} sufficient for correctness
- Counter-examples prove S_{com} necessary $\Rightarrow$ S_{com} is the weakest WCM requirement for correctness

Example of counter-example to S_{com_1}

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1:
   do {i}
2:
      do {j_i}
3:
         r[] R10 latch0 {⇝ L0^i_{j_i}}
4:

      while (R10=0) {k_i}
5:
      w[] latch0 0
6:
      r[] Rf0 flag0 {⇝ F0^i}
7:

      if (Rf0≠0) then
8:
         (* critical section *)
         w[] flag0 0
9:
         w[] flag1 1
10:
         w[] latch1 1
11:
      fi
12:
   while true
13:
```

```
21:
   do {ℓ}
22:
      do {m_ℓ}
23:
         r[] R11 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:

      while (R11=0) {n_ℓ}
25:
      w[] latch1 0
26:
      r[] Rf1 flag1 {⇝ F1^ℓ}
27:

      if (Rf1≠0) then
28:
         (* critical section *)
         w[] flag1 0
29:
         w[] flag0 1
30:
         w[] latch0 1
31:
      fi
32:
   while true
33:
```

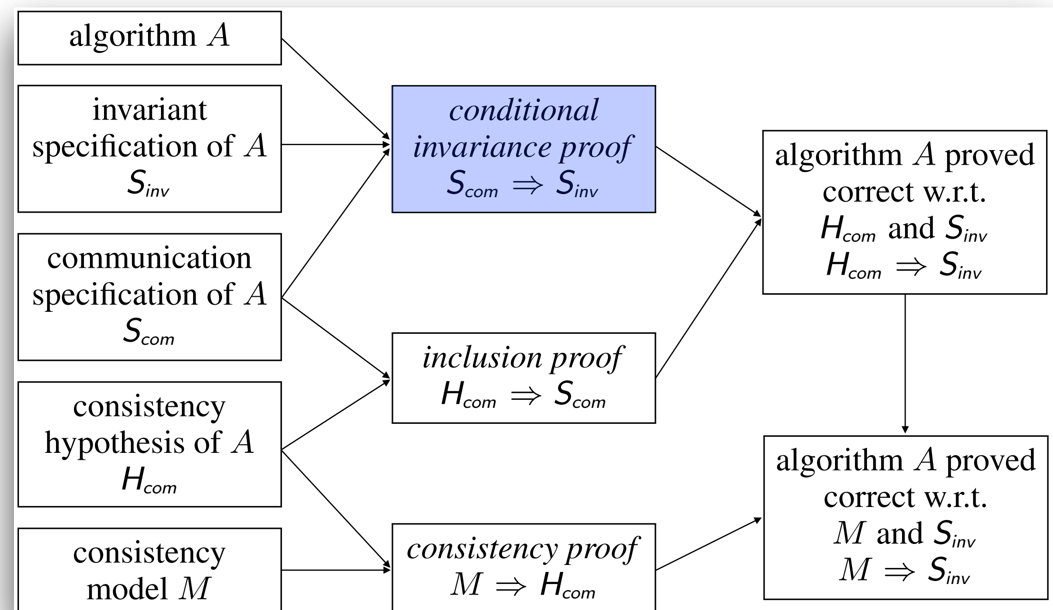
cut

Proof of mutual exclusion

- S_{com} implies mutual exclusion (for any Γ)

$$
\begin{array}{ll}
& (\neg S_{inv}(\Gamma,\Gamma) \wedge S_{ind}(\Gamma,\Gamma)) \Longrightarrow \neg(S_{com}(\Gamma,\Gamma)) \\
\Longrightarrow & S_{com}(\Gamma,\Gamma) \Longrightarrow (S_{inv}(\Gamma,\Gamma) \vee \neg S_{ind}(\Gamma,\Gamma)) \quad \wr\text{contraposition}\wr \\
\Longrightarrow & S_{com}(\Gamma,\Gamma) \Longrightarrow (S_{ind}(\Gamma,\Gamma) \Longrightarrow S_{inv}(\Gamma,\Gamma)) \quad \wr\text{implication}\wr \\
\Longrightarrow & (S_{com}(\Gamma,\Gamma) \wedge S_{ind}(\Gamma,\Gamma)) \Longrightarrow S_{inv}(\Gamma,\Gamma) \quad \wr\text{implication}\wr \\
\Longrightarrow & S_{com}(\Gamma,\overline{\Gamma}) \Rightarrow S_{inv}(\Gamma,\overline{\Gamma}) \quad \wr\text{since } S_{com}(\Gamma,\overline{\Gamma}) \Rightarrow S_{ind}(\Gamma,\overline{\Gamma})\wr
\end{array}
$$

Conditional invariance proof



Sequential proof $\ell = \kappa$ and $p = q$

{0: `latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1;` }

For a ***read instruction*** κ : `r[`*ts*`] R x` κ': (read)

$$\mathsf{PRE}^{\ell,\kappa}_{p,r}[\theta_r, \rho_r, \nu_r, \mathsf{rf}] \wedge \mathit{rf}[\mathfrak{w}(\langle q,\ \ell',\ \mathtt{w}[ts]\ \mathtt{x}\ \textit{r-value},\ \theta'\rangle, v),\ \mathfrak{r}(\langle r,\ \ell,\ \mathtt{r}[ts]\ \mathtt{R}\ \mathtt{x},\ \theta_r\rangle, \mathtt{x}_{\theta_r})] \in \mathsf{rf}$$

$$\Rightarrow \mathsf{POST}^{\ell,\kappa'}_{p,r}[\rho_r \leftarrow \rho_r[\mathtt{R} := \mathtt{x}_{\theta_r}], \nu_r \leftarrow \nu_r[\mathtt{x}_{\theta_r} := v]]$$

1: $\{\mathit{\Gamma} \in \Gamma\}$
   do $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
   do $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
   `r[] Rl0 lat`
4: $\{\mathit{\Gamma} \in \Gamma \wedge \mathtt{Rl0}$
   `while (Rl0=0)`
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^{i}_{k_i}[\mathit{\Gamma}]\}$
   `w[] latch0 0`
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^{i}_{k_i}[\mathit{\Gamma}]\}$
   `r[] Rf0 flag0` $\{\leadsto F0^i\}$
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathtt{Rf0} = F0^i \wedge (\mathrm{r0Rf0}^i[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^i[\mathit{\Gamma}])\}$
   `if (Rf0`$\neq$`0) then`
8: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
   `(* critical section *)`
   `w[] flag0 0`
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
   `w[] flag1 1`
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
   `w[] latch1 1`
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
   `fi`
12: $\{\mathit{\Gamma} \in \Gamma\}$
   `while true`
13: {false}

|| 21: $\{\mathit{\Gamma} \in \Gamma\}$
25: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}]\}$
   `w[] latch1 0`
26: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}]\}$
   `r[] Rf1 flag1` $\{\leadsto F1^\ell\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathtt{Rf1} = F1^\ell \wedge (\mathrm{r0Rf1}^\ell[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^\ell[\mathit{\Gamma}])\}$
   `if (Rf1`$\neq$`0) then`
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
   `(* critical section *)`
   `w[] flag1 0`
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
   `w[] flag0 1`
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
   `w[] latch0 1`
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
   `fi`
32: $\{\mathit{\Gamma} \in \Gamma\}$
   `while true`
33: {false}

Sequential proof $\ell = \kappa$ and $p = q$

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathit{\Gamma} \in \Gamma\}$
 do $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
 do $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
 r[] R10 lat
4: $\{\mathit{\Gamma} \in \Gamma \wedge$ R1C
 while (R10=0)
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}$
 w[] latch0 0
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}]\}$
 r[] Rf0 flag0 $\{\leadsto F0^{i}\}$
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathtt{Rf0} = F0^{i} \wedge (\mathrm{r0Rf0}^{i}[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^{i}[\mathit{\Gamma}])\}$
 if (Rf0$\neq$0) then
8: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^{i}[\mathit{\Gamma}]\}$
 (* critical section *)
 w[] flag0 0
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^{i}[\mathit{\Gamma}]\}$
 w[] flag1 1
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^{i}[\mathit{\Gamma}]\}$
 w[] latch1 1
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^{i}_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^{i}[\mathit{\Gamma}]\}$
 fi
12: $\{\mathit{\Gamma} \in \Gamma\}$
 while true
13: $\{\mathsf{false}\}$

21: $\{\mathit{\Gamma} \in \Gamma\}$
 do $\{\ell\}$
22: $\{\mathit{\Gamma} \in \Gamma\}$
 do $\{m_\ell\}$
26: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}]\}$
 r[] Rf1 flag1 $\{\leadsto F1^{\ell}\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathtt{Rf1} = F1^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}])\}$
 if (Rf1$\neq$0) then
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
 (* critical section *)
 w[] flag1 0
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
 w[] flag0 1
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
 w[] latch0 1
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
 fi
32: $\{\mathit{\Gamma} \in \Gamma\}$
 while true
33: $\{\mathsf{false}\}$

For a ***test instruction*** κ : b[*ts*] *operation* l_t κ': (test)

$$\mathsf{PRE}^{\ell,\kappa}_{p,r}[\rho_r, \nu_r] \wedge \mathsf{sat}(E[\![\mathit{operation}]\!](\rho_r, \nu_r) \neq 0) \Rightarrow \mathsf{POST}^{\ell,l_t}_{p,r}$$

$$\mathsf{PRE}^{\ell,\kappa}_{p,r}[\rho_r, \nu_r] \wedge \mathsf{sat}(E[\![\mathit{operation}]\!](\rho_r, \nu_r) = 0) \Rightarrow \mathsf{POST}^{\ell,\kappa'}_{p,r}$$

Sequential proof $\ell = \kappa$ and $p = q$

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
r[] R10 latch0 $\{\leadsto L0^i_{j_i}\}$
4: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{R10} = L0^i_{j_i} \wedge (\mathrm{r0R10}^i_{j_i}[\mathit{\Gamma}] \vee \mathrm{r1R10}^i_{j_i}[\mathit{\Gamma}])\}$
while (R10=0)
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}$
w[] latch0 0
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}$
r[] Rf0 flag0
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{Rf0} = F0$
$\wedge (\mathrm{r0Rf0}^i[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^i[\mathit{\Gamma}])\}$
if (Rf0≠0) then
8: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
(* critical section *)
w[] flag0 0
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
w[] flag1 1
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
w[] latch1 1
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R10}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
fi
12: $\{\mathit{\Gamma} \in \Gamma\}$
while true
13: {false}

||

21: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{\ell\}$
22: $\{\mathit{\Gamma} \in \Gamma\}$
do $\{m_\ell\}$
23: $\{\mathit{\Gamma} \in \Gamma\}$
r[] R11 latch1 $\{\leadsto L1^\ell_{m_\ell}\}$
24: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{R11} = L1^\ell_{m_\ell} \wedge (\mathrm{r0R11}^\ell_{m_\ell}[\mathit{\Gamma}] \vee \mathrm{r1R11}^\ell_{m_\ell}[\mathit{\Gamma}])\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{Rf1} = F1$
$\wedge (\mathrm{r0Rf1}^\ell[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^\ell[\mathit{\Gamma}])\}$
if (Rf1≠0) then
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
(* critical section *)
w[] flag1 0
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
w[] flag0 1
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
w[] latch0 1
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1R11}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
fi
32: $\{\mathit{\Gamma} \in \Gamma\}$
while true
33: {false}

For local side-effect free ***marker instructions*** κ : *instr* κ' where *instr* = f [*ts*] $\left[\{l^0_1 \ldots l^m_1\}\ \{l^0_2 \ldots l^q_2\}\right]$, w [*ts*] x *r-value*, beginrmw [*ts*] x, endrmw [*ts*] x: (marker)

$$\mathsf{PRE}^{\ell,\kappa}_{p,r} \Rightarrow \mathsf{POST}^{\ell,\kappa'}_{p,r}$$

Non-interference proof

{0: `latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1;` }

1: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{i\}$
2: $\{\mathit{\Gamma} \in \Gamma\}$
`do` $\{j_i\}$
3: $\{\mathit{\Gamma} \in \Gamma\}$
`r[] Rl0 latch0` $\{\leadsto\ L0$
4: $\{\mathit{\Gamma} \in \Gamma \wedge \mathtt{Rl0} = L0^i_{j_i} \wedge (\mathrm{r}$
`while (Rl0=0)` $\{k_i\}$
5: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}]\}$
`w[] latch0 0`
6: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}]\}$
`r[] Rf0 flag0` $\{\leadsto\ F0^i\}$
7: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathtt{Rf0} = F0^i \wedge (\mathrm{r0Rf0}^i[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^i[\mathit{\Gamma}])\}$
`if (Rf0`$\neq$`0) then`
8: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`(* critical section *)`
`w[] flag0 0`
9: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`w[] flag1 1`
10: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`w[] latch1 1`
11: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
`fi`
12: $\{\mathit{\Gamma} \in \Gamma\}$
`while true`
13: {false}

||

26: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}]\}$
`r[] Rf1 flag1` $\{\leadsto\ F1^\ell\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathtt{Rf1} = F1^\ell \wedge (\mathrm{r0Rf1}^\ell[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^\ell[\mathit{\Gamma}])\}$
`if (Rf1`$\neq$`0) then`
28: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`(* critical section *)`
`w[] flag1 0`
29: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`w[] flag0 1`
30: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`w[] latch0 1`
31: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^\ell_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^\ell[\mathit{\Gamma}]\}$
`fi`
32: $\{\mathit{\Gamma} \in \Gamma\}$
`while true`
33: {false}

The local invariants of process p depend only on Γ and local registers or Pythia variables unchanged by a step in the other process

Communication proof

$\{$0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; $\}$

1: $\{\Gamma \in \Gamma\}$
  do $\{i\}$
2:   $\{\Gamma \in \Gamma\}$
   do $\{j_i\}$ • • •
3:    $\{\Gamma \in \Gamma\}$
    r[] R10 latch0 $\{\rightsquigarrow L0$
4:    $\{\Gamma \in \Gamma \wedge \mathtt{R10} = L0^i_{j_i} \wedge (\mathrm{r}$
   while (R10=0) $\{k_i\}$
5:   $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma]\}$
   w[] latch0 0
6:   $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma]\}$
   r[] Rf0 flag0 $\{\rightsquigarrow F0^i\}$
7:   $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma] \wedge \mathtt{Rf0} = F0^i \wedge (\mathrm{r0Rf0}^i[\Gamma] \vee \mathrm{r1Rf0}^i[\Gamma])\}$
   if (Rf0$\neq$0) then
8:    $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma] \wedge \mathrm{r1Rf0}^i[\Gamma]\}$
    (* critical section *)
    w[] flag0 0
9:    $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma] \wedge \mathrm{r1Rf0}^i[\Gamma]\}$
    w[] flag1 1
10:    $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma] \wedge \mathrm{r1Rf0}^i[\Gamma]\}$
    w[] latch1 1
11:    $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma] \wedge \mathrm{r1Rf0}^i[\Gamma]\}$
   fi
12:   $\{\Gamma \in \Gamma\}$
  while true
13: $\{$false$\}$

• *Communication condition*

$$\mathsf{COM}^{\ell}_{p}[\mathsf{rf}] \quad \triangleq \quad S_{ind\,p}(\ell)[\mathsf{rf}] \wedge S_{comp}(\ell)[\mathsf{rf}]$$

• A read event can read from only one write event.

$$\mathsf{COM}^{\ell}_{p}[\mathsf{rf}] \wedge \mathit{rf}[r, w_1] \in \mathsf{rf} \wedge \mathit{rf}[r, w_2] \in \mathsf{rf} \Rightarrow w_1 = w_2 \,. \qquad \text{(singleness)}$$

||

26:   $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma]\}$
   r[] Rf1 flag1 $\{\rightsquigarrow F1^{\ell}\}$
27:   $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma] \wedge \mathtt{Rf1} = F1^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\Gamma] \vee \mathrm{r1Rf1}^{\ell}[\Gamma])\}$
   if (Rf1$\neq$0) then
28:    $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma]\}$
    (* critical section *)
    w[] flag1 0
29:    $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma]\}$
    w[] flag0 1
30:    $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma]\}$
    w[] latch0 1
31:    $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma]\}$
   fi
32:   $\{\Gamma \in \Gamma\}$
  while true
33: $\{$false$\}$

Communication proof

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: $\{\mathit{\Gamma} \in \Gamma\}$
  do $\{i\}$
2:   $\{\mathit{\Gamma} \in \Gamma\}$
    do $\{j_i\}$
3:     $\{\mathit{\Gamma} \in \Gamma\}$
      r[] R10 latch0 $\{\leadsto L0$
4:     $\{\mathit{\Gamma} \in \Gamma \wedge \texttt{R10} = L0^i_{j_i} \wedge ($r
    while (R10=0) $\{k_i\}$
5:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}]\}$
    w[] latch0 0
6:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}]\}$
    r[] Rf0 flag0 $\{\leadsto F0^i\}$
7:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \texttt{Rf0} = F0^i \wedge (\mathrm{r0Rf0}^i[\mathit{\Gamma}] \vee \mathrm{r1Rf0}^i[\mathit{\Gamma}])\}$
    if (Rf0$\neq$0) then
8:     $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
      (* critical section *)
      w[] flag0 0
9:     $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$
      w[] flag1 1
10:     $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\mathit{\Gamma}] \wedge \mathrm{r1Rf0}^i[\mathit{\Gamma}]\}$

- All process read instructions ℓ : r[ts] R x ℓ' must read either from an initial or a reachable program write, allowed by the communication hypothesis ($\exists\,\mathsf{P}[X_1, \ldots, X_m]$ means that all free variables in

$$\mathsf{COM}^{\ell}_{p}[\theta_p, \mathsf{rf}] \wedge \mathsf{rf} \neq \emptyset \Rightarrow \exists\, \mathit{rf}[\mathfrak{w}(\langle q,\ \ell_q,\ \texttt{w}[ts]\ \texttt{x}\ \textit{r-value},\ \theta'\rangle, v),\ \mathfrak{r}(\langle p,\ \ell,\ \texttt{r}[ts]\ \texttt{R}\ \texttt{x},\ \theta_p\rangle, \texttt{x}_{\theta_p})] \in \mathsf{rf}\ . \qquad \text{(satisfaction)}$$
$$((q \in \mathbb{P}\mathrm{i} \wedge \exists\,\mathsf{PRE}^{\ell_q}_{q}[\theta_q \leftarrow \theta', \mathsf{rf}]) \vee (q = \mathsf{start} \wedge v = 0))\ .$$

26: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}]\}$
    r[] Rf1 flag1 $\{\leadsto F1^{\ell}\}$
27: $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \texttt{Rf1} = F1^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\mathit{\Gamma}] \vee \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}])\}$
    if (Rf1$\neq$0) then
28:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
      (* critical section *)
      w[] flag1 0
29:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$
      w[] flag0 1
30:   $\{\mathit{\Gamma} \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\mathit{\Gamma}] \wedge \mathrm{r1Rf1}^{\ell}[\mathit{\Gamma}]\}$

$\mathrm{r0Rf0}^i[\mathit{\Gamma}] \triangleq (\mathfrak{rf}\langle F0^i,\ \langle 0{:},\ \_,\ 0\rangle\rangle \in \mathit{\Gamma} \wedge F0^i = 0) \vee (\exists i_8 \in \mathbb{N}\ .\ \mathfrak{rf}\langle F0^i,\ \langle 8{:},\ i_8,\ 0\rangle\rangle \in \mathit{\Gamma} \wedge F0^i = 0)$

$\mathrm{r1Rf0}^i[\mathit{\Gamma}] \triangleq (\exists \ell_{29} \in \mathbb{N}\ .\ \mathfrak{rf}\langle F0^i,\ \langle 29{:},\ \ell_{29},\ 1\rangle\rangle \in \mathit{\Gamma} \wedge F0^i = 1)$

12: $\{\mathit{\Gamma} \in \Gamma\}$
  while true
13: $\{\mathsf{false}\}$

32: $\{\mathit{\Gamma} \in \Gamma\}$
  while true
33: $\{\mathsf{false}\}$

Communication proof

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

1: $\{\Gamma \in \Gamma\}$
do $\{i\}$
2: $\{\Gamma \in \Gamma\}$
do $\{j_i\}$
3: $\{\Gamma \in \Gamma\}$
`r[] Rl0 latch0` $\{\rightsquigarrow L0$
4: $\{\Gamma \in \Gamma \wedge \texttt{Rl0} = L0^i_{j_i} \wedge ($
`while (Rl0=0)` $\{k_i\}$
5: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma]\}$
`w[] latch0 0`
6: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma]\}$
`r[] Rf0 flag0` $\{\rightsquigarrow F0^i\}$
7: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma] \wedge \texttt{Rf0} = F0^i$
$\wedge (\mathrm{r0Rf0}^i[\Gamma] \vee \mathrm{r1Rf0}^i[\Gamma])\}$
`if (Rf0≠0) then`
8: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma] \wedge \mathrm{r1Rf0}^i[\Gamma]\}$
`(* critical section *)`
`w[] flag0 0`
9: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma] \wedge \mathrm{r1Rf0}^i[\Gamma]\}$
`w[] flag1 1`
10: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl0}^i_{k_i}[\Gamma] \wedge \mathrm{r1Rf0}^i[\Gamma]\}$

- The values v allowed to be read by the communication hypothesis must originate from reachable program write instructions $\ell :$ `w[`ts`] x` $r\text{-}value\ \ell'$:

$$\forall \mathsf{rf}\ .\ \forall\, rf[\mathfrak{w}(\langle q,\ \ell_q,\ \texttt{w[}ts\texttt{] x}\ r\text{-}value,\ \theta_p\rangle, v), r] \in \mathsf{rf}\ (\text{match})$$
$$\mathsf{COM}^{\ell}_{p}[\theta_q, \rho_q, \nu_q, \mathsf{rf}] \Rightarrow v = E[\![r\text{-}value]\!](\rho_q, \nu_q)$$

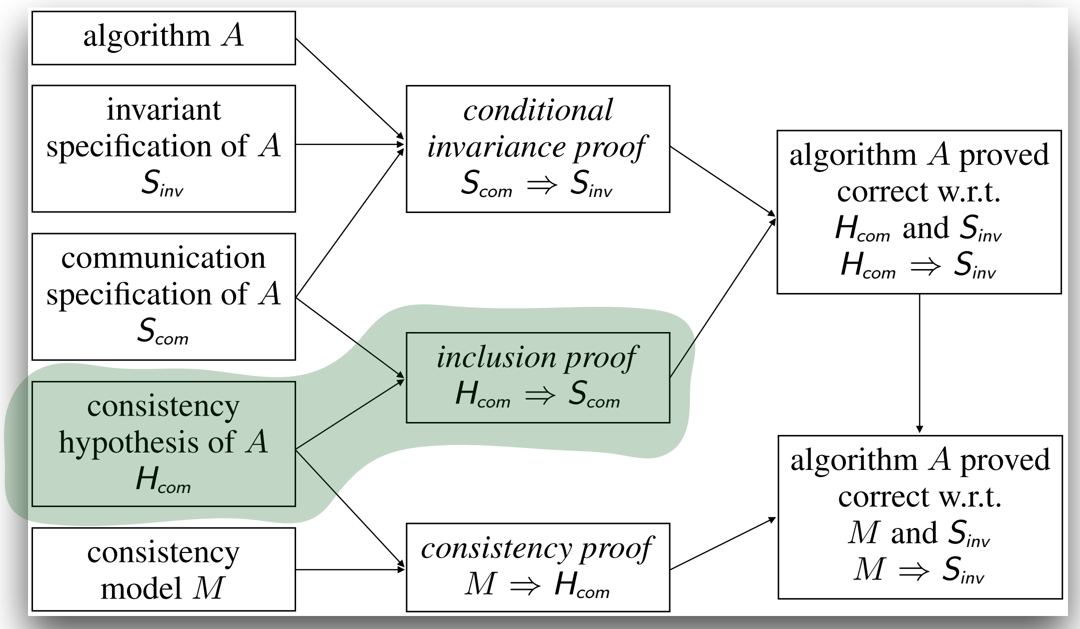
26: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma]\}$
`r[] Rf1 flag1` $\{\rightsquigarrow F1^{\ell}\}$
27: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma] \wedge \texttt{Rf1} = F1^{\ell}$
$\wedge (\mathrm{r0Rf1}^{\ell}[\Gamma] \vee \mathrm{r1Rf1}^{\ell}[\Gamma])\}$
`if (Rf1≠0) then`
28: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma]\}$
`(* critical section *)`
`w[] flag1 0`
29: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma]\}$
`w[] flag0 1`
30: $\{\Gamma \in \Gamma \wedge \mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma]\}$

$$\mathrm{r0Rf0}^i[\Gamma] \triangleq (\mathfrak{rf}\langle F0^i,\ \langle 0:,\ \_,\ 0\rangle\rangle \in \Gamma \wedge F0^i = 0) \vee (\exists i_8 \in \mathbb{N}\ .\ \mathfrak{rf}\langle F0^i,\ \langle 8:,\ i_8,\ 0\rangle\rangle \in \Gamma \wedge F0^i = 0)$$
$$\mathrm{r1Rf0}^i[\Gamma] \triangleq (\exists \ell_{29} \in \mathbb{N}\ .\ \mathfrak{rf}\langle F0^i,\ \langle 29:,\ \ell_{29},\ 1\rangle\rangle \in \Gamma \wedge F0^i = 1)$$

12: $\{\Gamma \in \Gamma\}$
`while true`
13: $\{\mathsf{false}\}$

32: $\{\Gamma \in \Gamma\}$
`while true`
33: $\{\mathsf{false}\}$

Inclusion proof



Method

- The communication specification is

$$S_{com}(\Gamma,\overline{\Gamma}) \triangleq (\mathsf{at}\{8\} \wedge \mathsf{at}\{28\}) \Longrightarrow (S_{com_1}(\Gamma,\overline{\Gamma}) \wedge S_{com_2}(\Gamma,\overline{\Gamma}) \wedge S_{com_3}(\Gamma,\overline{\Gamma}) \wedge S_{com_4}(\Gamma,\overline{\Gamma}))$$

- The consistency specification must satisfy

$$H_{com}(\Gamma,\overline{\Gamma}) \Rightarrow S_{com}(\Gamma,\overline{\Gamma}) \qquad \text{i.e.} \qquad \neg S_{com}(\Gamma,\overline{\Gamma}) \Rightarrow \neg H_{com}(\Gamma,\overline{\Gamma})$$

- So the design of $H_{com}(\Gamma,\overline{\Gamma})$ must forbid the erroneous communications specified by the communication specification

$$\Big(\mathsf{at}\{8\} \wedge \mathsf{at}\{28\} \wedge \bigvee_{i=1}^{4} \neg S_{com_i}(\Gamma,\overline{\Gamma})\Big) \Longrightarrow \bigvee_{i=1}^{4} \neg H_{com\,i}(\Gamma,\overline{\Gamma})$$

$$S_{com_1} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29} \in \mathbb{N} \,.\, \Gamma \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i}, \langle \mathsf{30:}, \ell_{30}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F0^i, \langle \mathsf{29:}, \ell_{29}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell}, \langle \mathsf{0:}, \_, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F1^\ell, \langle \mathsf{0:}, \_, 1\rangle\rangle \in \Gamma$$

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

```
1: do {i}
2:    do {j_i}
3:       r[] R10 latch0 {⇝ L0^i_{j_i}}
4:    while (R10=0) {k_i}
5:    w[] latch0 0
6:    r[] Rf0 flag0 {⇝ F0^i}
7:    if (Rf0≠0) then
8:       (* critical section *)
         w[] flag0 0
9:       w[] flag1 1
10:      w[] latch1 1
11:   fi
12:while true
13:
```

```
21:do {ℓ}
22:   do {m_ℓ}
23:      r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:   while (Rl1=0) {n_ℓ}
25:   w[] latch1 0
26:   r[] Rf1 flag1 {⇝ F1^ℓ}
27:   if (Rf1≠0) then
28:      (* critical section *)
         w[] flag1 0
29:      w[] flag0 1
30:      w[] latch0 1
31:   fi
32:while true
33:
```

cut

no prophecy beyond cut during execution

$$S_{com_2} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_9 \in \mathbb{N} \,.\, \Gamma \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i},\ \langle 30{:},\ \ell_{30},\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F0^i,\ \langle 29{:},\ \ell_{29},\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell},\ \langle 0{:},\ \_,\ 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F1^\ell,\ \langle 9{:},\ i_9,\ 1\rangle\rangle \in \Gamma$$

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
```

```
1: do {i}
2:    do {j_i}
3:       r[] R10 latch0 {⇝ L0^i_{j_i}}
4:    while (R10=0) {k_i}
5:    w[] latch0 0
6:    r[] Rf0 flag0 {⇝ F0^i}
7:    if (Rf0≠0) then
8:       (* critical section *)
         w[] flag0 0
9:       w[] flag1 1
10:      w[] latch1 1
11:   fi
12:while true
13:
```

```
21:do {ℓ}
22:   do {m_ℓ}
23:      r[] R11 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:   while (R11=0) {n_ℓ}
25:   w[] latch1 0
26:   r[] Rf1 flag1 {⇝ F1^ℓ}
27:   if (Rf1≠0) then
28:      (* critical section *)
         w[] flag1 0
29:      w[] flag0 1
30:      w[] latch0 1
31:   fi
32:while true
33:
```

cut

no prophecy beyond cut during execution

$$S_{com_3} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10} \in \mathbb{N} \,.\, \Gamma \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i}, \langle 30{:}, \ell_{30}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F0^i, \langle 29{:}, \ell_{29}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell}, \langle 10{:}, i_{10}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F1^\ell, \langle 0{:}, \_, 1\rangle\rangle \in \Gamma$$

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do {i}
2:   do {j_i}
3:     r[] R10 latch0 {⇝ L0^i_{j_i}}
4:   while (R10=0) {k_i}
5:   w[] latch0 0
6:   r[] Rf0 flag0 {⇝ F0^i}
7:   if (Rf0≠0) then
8:     (* critical section *)
       w[] flag0 0
9:     w[] flag1 1
10:    w[] latch1 1
11:  fi
12:while true
13:
```

```
21:do {ℓ}
22:  do {m_ℓ}
23:    r[] R11 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:  while (R11=0) {n_ℓ}
25:  w[] latch1 0
26:  r[] Rf1 flag1 {⇝ F1^ℓ}
27:  if (Rf1≠0) then
28:    (* critical section *)
       w[] flag1 0
29:    w[] flag0 1
30:    w[] latch0 1
31:  fi
32:while true
33:
```

cut

no prophecy beyond cut during execution

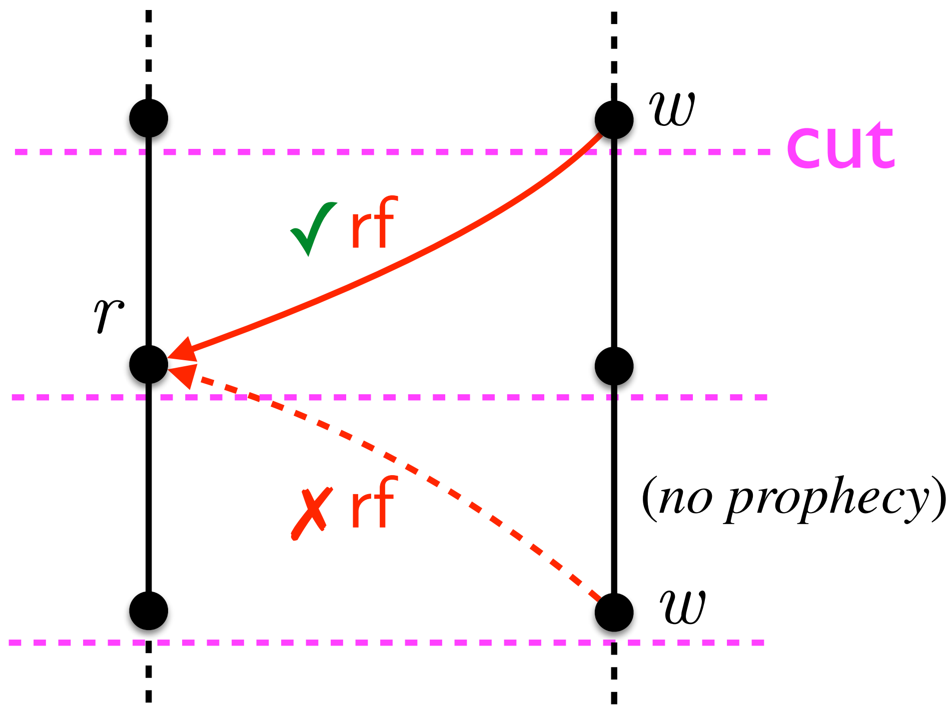
$$S_{com_4} \triangleq \neg(\exists i, k_i, \ell, n_\ell, \ell_{30}, \ell_{29}, i_{10}, i_9 \in \mathbb{N} \;.\; \Gamma \in \Gamma \wedge \mathfrak{rf}\langle L0^i_{k_i}, \langle 30:, \ell_{30}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F0^i, \langle 29:, \ell_{29}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle L1^\ell_{n_\ell}, \langle 10:, i_{10}, 1\rangle\rangle \in \Gamma \wedge \mathfrak{rf}\langle F1^\ell, \langle 9:, i_9, 1\rangle\rangle \in \Gamma$$

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do {i}                                   || 21:do {ℓ}
2:    do {j_i}                              || 22:   do {m_ℓ}
3:       r[] R10 latch0 {⇝ L0^i_{j_i}}      || 23:      r[] R11 latch1 {⇝ L1^ℓ_{m_ℓ}}
4:    while (R10=0) {k_i}                   || 24:   while (R11=0) {n_ℓ}
5:    w[] latch0 0                          || 25:   w[] latch1 0
6:    r[] Rf0 flag0 {⇝ F0^i}                || 26:   r[] Rf1 flag1 {⇝ F1^ℓ}
7:    if (Rf0≠0) then                       || 27:   if (Rf1≠0) then
8:       (* critical section *)             || 28:      (* critical section *)      cut
         w[] flag0 0                        ||          w[] flag1 0
9:       w[] flag1 1                        || 29:      w[] flag0 1
10:      w[] latch1 1                       || 30:      w[] latch0 1
11:   fi                                    || 31:   fi
12:while true                               || 32:while true
13:                                         || 33:
```

no prophecy beyond cut during execution

Conclusion on mutual exclusion

- PostgreSQL is correct on architectures satisfying the ``no prophecy beyond cut during execution'' property



- Intuition on necessity: when waiting for a spinlock, you should look at its current value, not at later ones!

in cat

- A static condition to impose a dynamic condition:

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1: do {i}
2:    do {j_i}
3:       r[] Rl0 latch0 {⇝ L0^i_{j_i}}
4:    while (Rl0=0) {k_i}
5:    w[] latch0 0
6:    r[] Rf0 flag0 {⇝ F0^i}
7:    if (Rf0≠0) then
8:       f[cut]
         (* critical section *)
         w[] flag0 0
9:       w[] flag1 1
10:      w[] latch1 1
11:   fi
12:while true
13:
```

```
21:do {ℓ}
22:   do {m_ℓ}
23:      r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:   while (Rl1=0) {n_ℓ}
25:   w[] latch1 0
26:   r[] Rf1 flag1 {⇝ F1^ℓ}
27:   if (Rf1≠0) then
28:      f[cut]
         (* critical section *)
         w[] flag1 0
29:      w[] flag0 1
30:      w[] latch0 1
31:   fi
32:while true
33:
```

po cut rf po

```
enum fences = 'cut
instructions F[{'cut}]
let cut = (tag2events('cut) * tag2events('cut)) & ext
irreflexive rf; po; cut; po
```

Prevents valid executions

- $\{0\text{: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }\}$

2 po 4

1: do $\{i\}$
2: do $\{j_i\}$
3: r[] Rl0 latch0 $\{\leadsto L0^{i}_{j_i}\}$
4: while (Rl0=0) $\{k_i\}$
5: w[] latch0 0
6: r[] Rf0 flag0 $\{\leadsto F0^{i}\}$
7: if (Rf0$\neq$0) then
8: f[cut]
(* critical section *)
w[] flag0 0
9: w[] flag1 1
10: w[] latch1 1
11: fi
12: while true
13:

||

21: do $\{\ell\}$
22: do $\{m_\ell\}$
23: r[] Rl1 latch1 $\{\leadsto L1^{\ell}_{m_\ell}\}$
24: while (Rl1=0) $\{n_\ell\}$
25: w[] latch1 0
26: r[] Rf1 flag1 $\{\leadsto F1^{\ell}\}$
27: if (Rf1$\neq$0) then
28: f[cut]
(* critical section *)
w[] flag1 0
29: w[] flag0 1
30: w[] latch0 1
31: fi
32: while true
33:

cut rf

1 po 3

Invalid

- $\{0\text{: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }\}$

3 po 4

1: do $\{i\}$
2: do $\{j_i\}$
3: r[] Rl0 latch0 $\{\leadsto L0^{i}_{j_i}\}$
4: while (Rl0=0) $\{k_i\}$
5: w[] latch0 0
6: r[] Rf0 flag0 $\{\leadsto F0^{i}\}$
7: if (Rf0$\neq$0) then
8: f[cut]
(* critical section *)
w[] flag0 0
9: w[] flag1 1
10: w[] latch1 1
11: fi
12: while true
13:

||

21: do $\{\ell\}$
22: do $\{m_\ell\}$
23: r[] Rl1 latch1 $\{\leadsto L1^{\ell}_{m_\ell}\}$
24: while (Rl1=0) $\{n_\ell\}$
25: w[] latch1 0
26: r[] Rf1 flag1 $\{\leadsto F1^{\ell}\}$
27: if (Rf1$\neq$0) then
28: f[cut]
(* critical section *)
w[] flag1 0
29: w[] flag0 1
30: w[] latch0 1
31: fi
32: while true
33:

cut rf

1 po 2

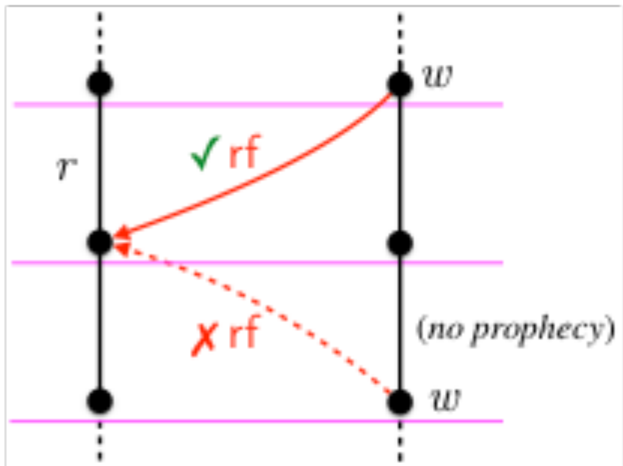
Valid

`irreflexive rf; po; cut; po`

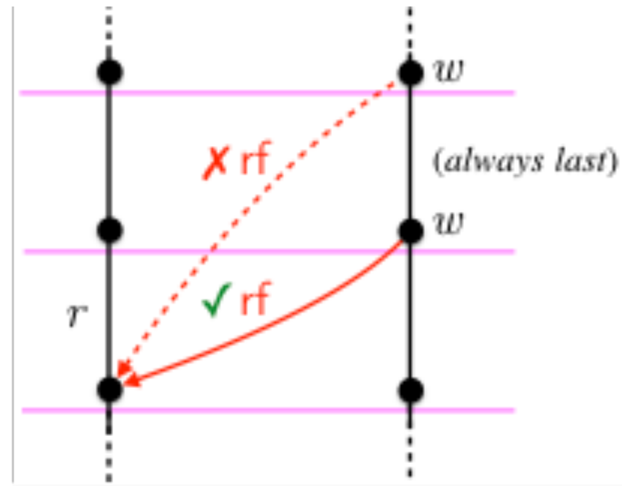
Non-starvation

Difference with Lamport/Owicki-Gries

- The communications in L/O-G are fixed in the semantics (SC) for <u>all</u> executions:



(a) No prophecy beyond cuts



(b) Read from last write

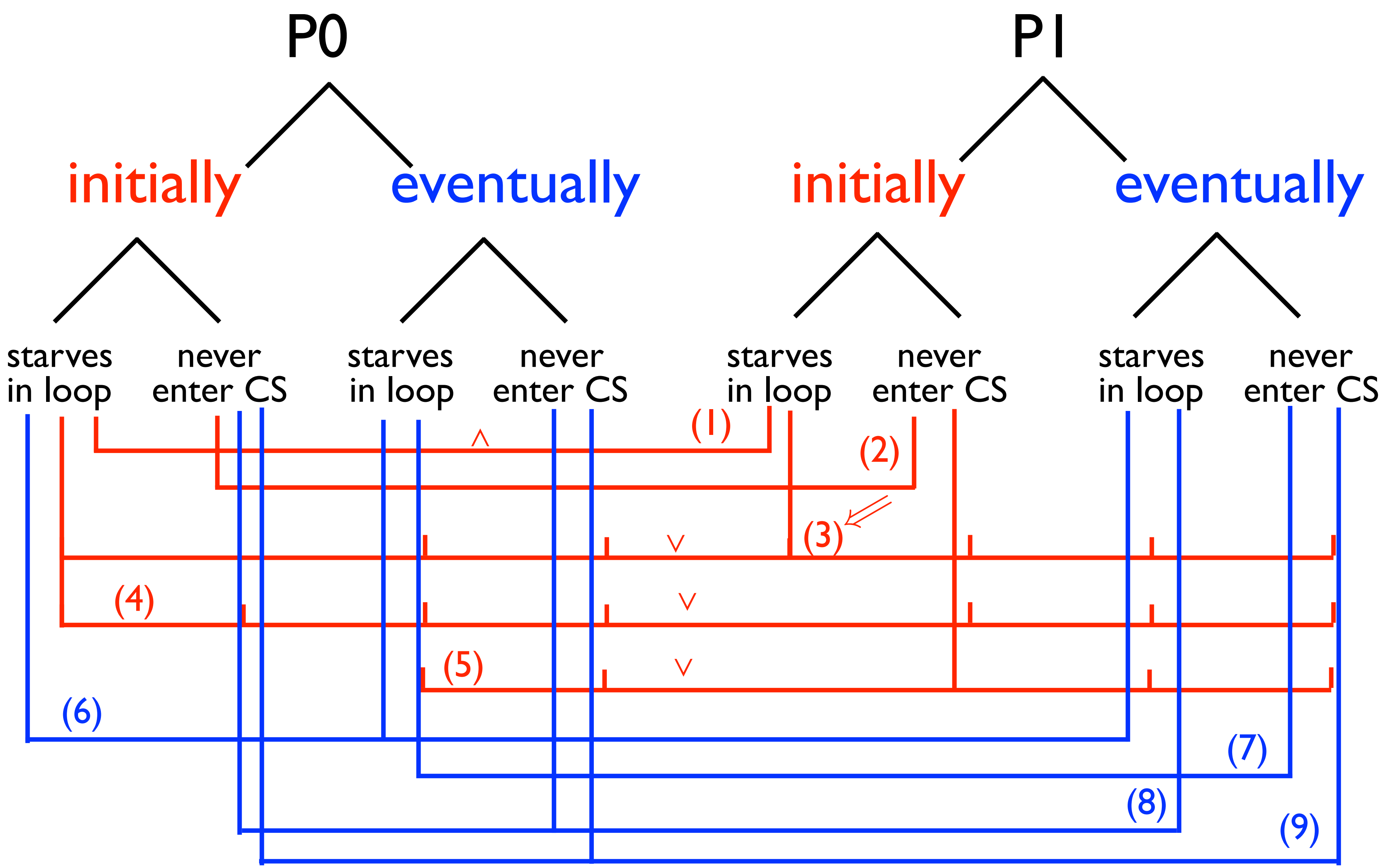
$\Longrightarrow$ entangled with the verification conditions

$\Longrightarrow$ impossible to reason on one execution trace only

Reasoning on only one execution

- An execution is entirely determined by its read-from relation rf
- The verification conditions depend on a set Γ of verification conditions
- By choosing Γ = {rf}, we can reason on this execution
- This execution satisfies the inductive invariant $S_{ind}(\{\mathrm{rf}\})$
- To prove that this execution is impossible it is sufficient to prove that $S_{ind}(\{\mathrm{rf}\})$ cannot hold (according to the verification conditions)
- Since the method is sound, if the verification conditions are not satisfied, the execution is excluded by the semantics

9 cases of starvation



(1) Both processes starve in spin loops

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\texttt{true}\}$
do $\{i\}$
2: $\{\texttt{true}\}$
do $\{j_i\}$
3: $\{\texttt{true}\}$
`r[] R10 latch0` $\{\rightsquigarrow L0^i_{j_i}\}$
4: $\{\texttt{R10} = \texttt{L0}^{\texttt{i}}_{\texttt{j}_\texttt{i}} \wedge$
$(\mathrm{r0R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}])\}$
`while (R10=0)` $\{k_i\}$
5: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
`w[] latch0 0`
6: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
`r[] Rf0 flag0` $\{\rightsquigarrow F0^i\}$
7: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \texttt{Rf0} = \texttt{F0}^{\texttt{i}} \wedge$
$(\mathrm{r0Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}])\}$
`if (Rf0`$\neq$`0) then`
8: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
`(* critical section *)`
false
`w[] flag0 0`
9: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
`w[] flag1 1`
10: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
`w[] latch1 1`
11: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
`fi`
12: $\{\texttt{true}\}$
`while true`
13: $\{\texttt{false}\}$

||

21: $\{\texttt{true}\}$
do $\{\ell\}$
22: $\{\texttt{true}\}$
do $\{m_\ell\}$
23: $\{\texttt{true}\}$
`r[] R11 latch1` $\{\rightsquigarrow L1^\ell_{m_\ell}\}$
24: $\{\texttt{R11} = \texttt{L1}^{\ell}_{\texttt{m}_\ell} \wedge$
$(\mathrm{r0R11}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1R11}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}])\}$
`while (R11=0)` $\{n_\ell\}$
25: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
`w[] latch1 0`
26: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
`r[] Rf1 flag1` $\{\rightsquigarrow F1^\ell\}$
27: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \texttt{Rf1} = \texttt{F1}^{\ell} \wedge$
$(\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}])\}$
`if (Rf1`$\neq$`0) then`
28: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
`(* critical section *)`
false
`w[] flag1 0`
29: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
`w[] flag0 1`
30: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
`w[] latch0 1`
31: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
`fi`
32: $\{\texttt{true}\}$
`while true`
33: $\{\texttt{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- invariant false after both spin loops
- so `latch1` in 23: can only be read from initialization
- so `latch1` is 1 not 0, a contradiction

(2) Both processes never enter their critical section

```
{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }
1:  {true}
    do {i}
2:    {true}
      do {j_i}
3:      {true}
        r[] Rl0 latch0 {⇝ L0^i_{j_i}}
4:      {Rl0 = L0^i_{j_i} ∧
          (r0Rl0^i_{j_i}[Γ_rf] ∨ r1Rl0^i_{j_i}[Γ_rf])}
      while (Rl0=0) {k_i}
5:    {r1Rl0^i_{k_i}[Γ_rf]}
      w[] latch0 0
6:    {r1Rl0^i_{k_i}[Γ_rf]}
      r[] Rf0 flag0 {⇝ F0^i}
7:    {r1Rl0^i_{k_i}[Γ_rf] ∧ Rf0 = F0^i ∧
        (r0Rf0^i[Γ_rf] ∨ r1Rf0^i[Γ_rf])}
      if (Rf0≠0) then
8:      {r1Rl0^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
        (* critical section *)
        w[] flag0 0
9:      {r1Rl0^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
        w[] flag1 1
10:     {r1Rl0^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
        w[] latch1 1
11:     {r1Rl0^i_{k_i}[Γ_rf] ∧ r1Rf0^i[Γ_rf]}
      fi
12:   {true}
    while true
13: {false}
```

```
21: {true}
    do {ℓ}
22:   {true}
      do {m_ℓ}
23:     {true}
        r[] Rl1 latch1 {⇝ L1^ℓ_{m_ℓ}}
24:     {Rl1 = L1^ℓ_{m_ℓ} ∧
          (r0Rl1^ℓ_{m_ℓ}[Γ_rf] ∨ r1Rl1^ℓ_{m_ℓ}[Γ_rf])}
      while (Rl1=0) {n_ℓ}
25:   {r1Rl1^ℓ_{n_ℓ}[Γ_rf]}
      w[] latch1 0
26:   {r1Rl1^ℓ_{n_ℓ}[Γ_rf]}
      r[] Rf1 flag1 {⇝ F1^ℓ}
27:   {r1Rl1^ℓ_{n_ℓ}[Γ_rf] ∧ Rf1 = F1^ℓ ∧
        (r0Rf1^ℓ[Γ_rf] ∨ r1Rf1^ℓ[Γ_rf])}
      if (Rf1≠0) then
28:     {r1Rl1^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
        (* critical section *)
        w[] flag1 0
29:     {r1Rl1^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
        w[] flag0 1
30:     {r1Rl1^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
        w[] latch0 1
31:     {r1Rl1^ℓ_{n_ℓ}[Γ_rf] ∧ r1Rf1^ℓ[Γ_rf]}
      fi
32:   {true}
    while true
33: {false}
```

- let rf be the communication for such a trace (encoded in Γ_{rf})

(2) Both processes never enter their critical section

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: $\{\text{true}\}$
do $\{i\}$
2: $\{\text{true}\}$
do $\{j_i\}$
3: $\{\text{true}\}$
r[] R10 latch0 $\{\rightsquigarrow L0^{i}_{j_i}\}$
4: $\{\mathtt{R10} = \mathtt{L0}^{\mathrm{i}}_{\mathrm{j_i}} \wedge (\mathrm{r0R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathrm{rf}}] \vee \mathrm{r1R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathrm{rf}}])\}$
while (R10=0) $\{k_i\}$
5: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}]\}$
w[] latch0 0
6: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}]\}$
r[] Rf0 flag0 $\{\rightsquigarrow F0^{i}\}$
7: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{\mathrm{i}} \wedge (\mathrm{r0Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}] \vee \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}])\}$
if (Rf0$\neq$0) then
8: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}]\}$
(* critical section *)
w[] flag0 0
9: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}]\}$
false
w[] flag1 1
10: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}]\}$
w[] latch1 1
11: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}]\}$
fi
12: $\{\text{true}\}$
while true
13: $\{\text{false}\}$

||

21: $\{\text{true}\}$
do $\{\ell\}$
22: $\{\text{true}\}$
do $\{m_\ell\}$
23: $\{\text{true}\}$
r[] R11 latch1 $\{\rightsquigarrow L1^{\ell}_{m_\ell}\}$
24: $\{\mathtt{R11} = \mathtt{L1}^{\ell}_{\mathrm{m}_\ell} \wedge (\mathrm{r0R11}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathrm{rf}}] \vee \mathrm{r1R11}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathrm{rf}}])\}$
while (R11=0) $\{n_\ell\}$
25: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}]\}$
w[] latch1 0
26: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}]\}$
r[] Rf1 flag1 $\{\rightsquigarrow F1^{\ell}\}$
27: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathrm{rf}}] \vee \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}])\}$
if (Rf1$\neq$0) then
28: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]\}$
(* critical section *)
w[] flag1 0
29: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]\}$
w[] flag0 1
false
30: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]\}$
w[] latch0 1
31: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]\}$
fi
32: $\{\text{true}\}$
while true
33: $\{\text{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant inside critical sections must be false

(2) Both processes never enter their critical section

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathsf{true}\}$
 `do` $\{i\}$
2: $\{\mathsf{true}\}$
 `do` $\{j_i\}$
3: $\{\mathsf{true}\}$
 `r[] Rl0 latch0` $\{\rightsquigarrow L0^{i}_{j_i}\}$
4: $\{\mathtt{Rl0} = \mathtt{L0}^{\mathrm{i}}_{\mathrm{j_i}} \wedge (\mathrm{r0Rl0}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}])\}$
 `while (Rl0=0)` $\{k_i\}$
5: $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
 `w[] latch0 0`
6: $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
 `r[] Rf0 flag0` $\{\rightsquigarrow F0^{i}\}$
7: $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{\mathrm{i}} \wedge (\mathrm{r0Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]})\}$
 `if (Rf0≠0) then`
8: $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
 `(* critical section *)`
 `w[] flag0 0`
9: $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
false
 `w[] flag1 1`
10: $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
 `w[] latch1 1`
11: $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
 `fi`
12: $\{\mathsf{true}\}$
 `while true`
13: $\{\mathsf{false}\}$

||

21: $\{\mathsf{true}\}$
 `do` $\{\ell\}$
22: $\{\mathsf{true}\}$
 `do` $\{m_\ell\}$
23: $\{\mathsf{true}\}$
 `r[] Rl1 latch1` $\{\rightsquigarrow L1^{\ell}_{m_\ell}\}$
24: $\{\mathtt{Rl1} = \mathtt{L1}^{\ell}_{\mathrm{m}_\ell} \wedge (\mathrm{r0Rl1}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rl1}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}])\}$
 `while (Rl1=0)` $\{n_\ell\}$
25: $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
 `w[] latch1 0`
26: $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
 `r[] Rf1 flag1` $\{\rightsquigarrow F1^{\ell}\}$
27: $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]})\}$
 `if (Rf1≠0) then`
28: $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
 `(* critical section *)`
 `w[] flag1 0`
29: $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
 `w[] flag0 1`
false
30: $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
 `w[] latch0 1`
31: $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
 `fi`
32: $\{\mathsf{true}\}$
 `while true`
33: $\{\mathsf{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant inside critical sections must be false
- tests (`Rf0≠0`) and (`Rf1≠0`) must be false (written ~~xxx~~)

(2) Both processes never enter their critical section

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathsf{true}\}$
  `do` $\{i\}$
2: $\{\mathsf{true}\}$
  `do` $\{j_i\}$
3: $\{\mathsf{true}\}$
  `r[] R10 latch0` $\{\leadsto L0^i_{j_i}\}$
4: $\{\mathtt{R10} = \mathtt{L0}^{\mathtt{i}}_{\mathtt{j_i}} \wedge (\mathrm{r0R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}])\}$
  `while (R10=0)` $\{k_i\}$
5: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
  `w[] latch0 0`
6: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
  `r[] Rf0 flag0` $\{\leadsto F0^i\}$
7: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{\mathtt{i}} \wedge (\mathrm{r0Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]})\}$
  `if (Rf0≠0) then`
8: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
  `(* critical section *)`
  `w[] flag0 0`
9: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
false
  `w[] flag1 1`
10: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
  `w[] latch1 1`
11: $\{\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
  `fi`
12: $\{\mathsf{true}\}$
  `while true`
13: $\{\mathsf{false}\}$

rf

21: $\{\mathsf{true}\}$
  `do` $\{\ell\}$
22: $\{\mathsf{true}\}$
  `do` $\{m_\ell\}$
23: $\{\mathsf{true}\}$
  `r[] R11 latch1` $\{\leadsto L1^\ell_{m_\ell}\}$
24: $\{\mathtt{R11} = \mathtt{L1}^{\ell}_{\mathtt{m}_\ell} \wedge (\mathrm{r0R11}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1R11}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}])\}$
  `while (R11=0)` $\{n_\ell\}$
25: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
  `w[] latch1 0`
26: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
  `r[] Rf1 flag1` $\{\leadsto F1^\ell\}$
27: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]})\}$
  `if (Rf1≠0) then`
28: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
  `(* critical section *)`
rf
  `w[] flag1 0`
29: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
  `w[] flag0 1`
false
30: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
  `w[] latch0 1`
31: $\{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
  `fi`
32: $\{\mathsf{true}\}$
  `while true`
33: $\{\mathsf{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant inside critical sections must be false
- tests (`Rf0`≠0) and (`Rf1`≠0) must be false (written ~~xxx~~)
- so read of `Rf0` and `Rf1` is 0 from a reachable write

(2) Both processes never enter their critical section

{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }

1: {true}
do {i}
2: {true}
do {j_i}
3: {true}
r[] R10 latch0 {$\leadsto L0^{i}_{j_i}$}
4: {$\mathtt{R10} = \mathtt{L0}^{\mathtt{i}}_{\mathtt{j_i}} \wedge$
$(\mathrm{r0R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathrm{rf}}] \vee \mathrm{r1R10}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathrm{rf}}])$}
while (R10=0) {k_i}
5: {$\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}]$}
w[] latch0 0
6: {$\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}]$}
r[] Rf0 flag0 {$\leadsto F0^{i}$}
7: {$\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{\mathtt{i}} \wedge$
$(\mathrm{r0Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}] \vee \cancel{\mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}]})$}
if (Rf0$\neq$0) then
8: {$\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}]$}
(* critical section *)
w[] flag0 0
9: {$\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}]$}
w[] flag1 1
false
10: {$\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}]$}
w[] latch1 1
11: {$\mathrm{r1R10}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathrm{rf}}]$}
fi
12: {true}
while true
13: {false}

rf

21: {true}
do {ℓ}
22: {true}
do {m_ℓ}
23: {true}
r[] R11 latch1 {$\leadsto L1^{\ell}_{m_\ell}$}
24: {$\mathtt{R11} = \mathtt{L1}^{\ell}_{\mathtt{m}_\ell} \wedge$
$(\mathrm{r0R11}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathrm{rf}}] \vee \cancel{\mathrm{r1R11}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathrm{rf}}]})$}
while (R11=0) {n_ℓ}
25: {$\cancel{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}]}$}
w[] latch1 0
26: {$\cancel{\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}]}$}
r[] Rf1 flag1 {$\leadsto F1^{\ell}$}
27: {$\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge$
$(\cancel{\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]} \vee \cancel{\mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]})$}
if (Rf1$\neq$0) then
28: {$\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]$}
(* critical section *)
rf
w[] flag1 0
29: {$\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]$}
w[] flag0 1
false
30: {$\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]$}
w[] latch0 1
31: {$\mathrm{r1R11}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathrm{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathrm{rf}}]$}
fi
32: {true}
while true
33: {false}

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant inside critical sections must be false
- tests (Rf0≠0) and (Rf1≠0) must be false (written ~~xxx~~)
- so read of Rf0 and Rf1 is 0 from a reachable write
- impossible for Rf1 so loop 23—24 is never exited

$\Longrightarrow$ we are in case (3), P1 stuck in spin loop

(3) Process `P1` stuck in spin loop (no hypothesis on `P0`)

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\mathsf{true}\}$
`do` $\{i\}$
2: $\{\mathsf{true}\}$
`do` $\{j_i\}$
3: $\{\mathsf{true}\}$
`r[] Rl0 latch0` $\{\leadsto L0^{i}_{j_i}\}$
4: $\{\mathtt{Rl0} = \mathtt{L0}^{i}_{j_i} \wedge (\mathrm{r0Rl0}^{i}_{j_i}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rl0}^{i}_{j_i}[\Gamma_{\mathsf{rf}}])\}$
`while (Rl0=0)` $\{k_i\}$
5: $\{\mathrm{r1Rl0}^{i}_{k_i}[\Gamma_{\mathsf{rf}}]\}$
`w[] latch0 0`
6: $\{\mathrm{r1Rl0}^{i}_{k_i}[\Gamma_{\mathsf{rf}}]\}$
`r[] Rf0 flag0` $\{\leadsto F0^{i}\}$
7: $\{\mathrm{r1Rl0}^{i}_{k_i}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{i} \wedge (\mathrm{r0Rf0}^{i}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf0}^{i}[\Gamma_{\mathsf{rf}}])\}$
`if (Rf0`$\neq$`0) then`
8: $\{\mathrm{r1Rl0}^{i}_{k_i}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathsf{rf}}]\}$
`(* critical section *)`
`w[] flag0 0`
9: $\{\mathrm{r1Rl0}^{i}_{k_i}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathsf{rf}}]\}$
false
`w[] flag1 1`
10: $\{\mathrm{r1Rl0}^{i}_{k_i}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathsf{rf}}]\}$
`w[] latch1 1`
11: $\{\mathrm{r1Rl0}^{i}_{k_i}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{i}[\Gamma_{\mathsf{rf}}]\}$
`fi`
12: $\{\mathsf{true}\}$
`while true`
13: $\{\mathsf{false}\}$

21: $\{\mathsf{true}\}$
`do` $\{\ell\}$
22: $\{\mathsf{true}\}$
`do` $\{m_\ell\}$
23: $\{\mathsf{true}\}$
`r[] Rl1 latch1` $\{\leadsto L1^{\ell}_{m_\ell}\}$
24: $\{\mathtt{Rl1} = \mathtt{L1}^{\ell}_{m_\ell} \wedge (\mathrm{r0Rl1}^{\ell}_{m_\ell}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathrm{r1Rl1}^{\ell}_{m_\ell}[\Gamma_{\mathsf{rf}}]})\}$
`while (Rl1=0)` $\{n_\ell\}$
25: $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}]\}$
`w[] latch1 0`
26: $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}]\}$
`r[] Rf1 flag1` $\{\leadsto F1^{\ell}\}$
27: $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge (\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}])\}$
`if (Rf1`$\neq$`0) then`
28: $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
`(* critical section *)`
`w[] flag1 0`
29: $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
false
`w[] flag0 1`
30: $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
`w[] latch0 1`
31: $\{\mathrm{r1Rl1}^{\ell}_{n_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
`fi`
32: $\{\mathsf{true}\}$
`while true`
33: $\{\mathsf{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant after 25: must be false
- read of `latch1` in 23: must be a 0
- only possibility if from 25:
- A contradiction since 25: is unreachable

(4) Process `P0` starves in spin loop, no hypothesis on `P1`

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

P0:

1: $\{\texttt{true}\}$
  do $\{i\}$
2:   $\{\texttt{true}\}$
    do $\{j_i\}$
3:     $\{\texttt{true}\}$
      `r[] R10 latch0` $\{\leadsto L0^{i}_{j_i}\}$
4:     $\{\texttt{R10} = \texttt{L0}^{\mathrm{i}}_{\mathrm{j_i}} \wedge$
       $(\mathrm{r0Rl0}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}] \vee \cancel{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}]})\}$
    `while (R10=0)` $\{k_i\}$
5:   $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
    `w[] latch0 0`
6:   $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
    `r[] Rf0 flag0` $\{\leadsto F0^{i}\}$
7:   $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \texttt{Rf0} = \texttt{F0}^{\mathrm{i}} \wedge$
     $(\mathrm{r0Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}])\}$
    `if (Rf0`$\neq$`0) then`
8:     $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
false
      `(* critical section *)`
      `w[] flag0 0`
9:     $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
      `w[] flag1 1`
10:     $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
      `w[] latch1 1`
11:     $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
    `fi`
12:   $\{\texttt{true}\}$
  `while true`
13: $\{\texttt{false}\}$

co

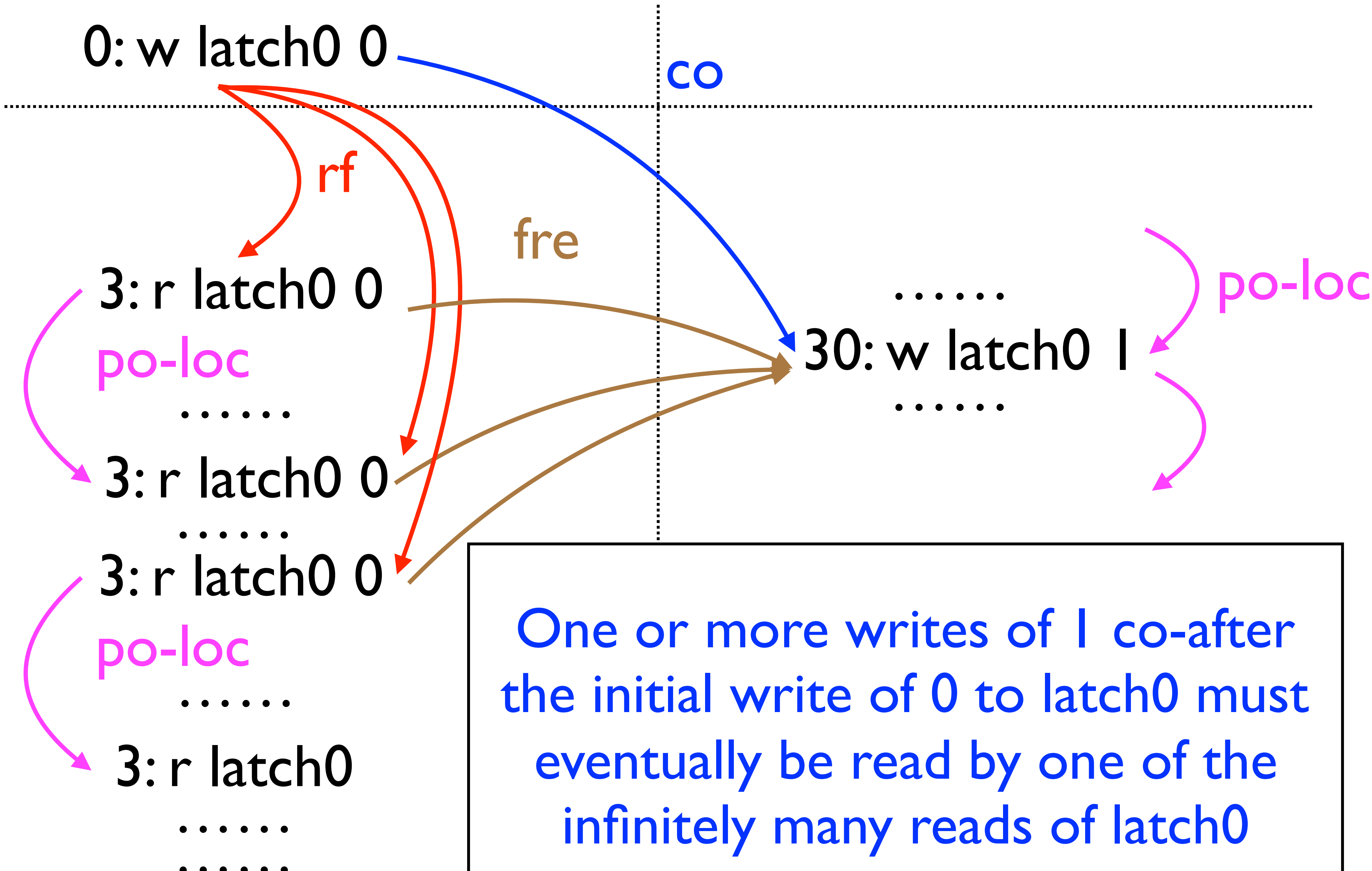
fr

P1:

21: $\{\texttt{true}\}$
  do $\{\ell\}$
22:   $\{\texttt{true}\}$
    do $\{m_\ell\}$
23:     $\{\texttt{true}\}$
      `r[] R11 latch1` $\{\leadsto L1^{\ell}_{m_\ell}\}$
24:     $\{\texttt{R11} = \texttt{L1}^{\ell}_{\mathrm{m}_\ell} \wedge$
       $(\mathrm{r0Rl1}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rl1}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}])\}$
    `while (R11=0)` $\{n_\ell\}$
25:   $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
    `w[] latch1 0`
26:   $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
    `r[] Rf1 flag1` $\{\leadsto F1^{\ell}\}$
27:   $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \texttt{Rf1} = \texttt{F1}^{\ell} \wedge$
     $(\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}])\}$
    `if (Rf1`$\neq$`0) then`
28:     $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
      `(* critical section *)`
      `w[] flag1 0`
29:     $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
      `w[] flag0 1`
30:     $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
      `w[] latch0 1`
31:     $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
    `fi`
32:   $\{\texttt{true}\}$
  `while true`
33: $\{\texttt{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- the invariant after 5: must be false so P0 never enters its critical section
- read of `latch0` in 3: must be a 0, with 2 possibilities
- cannot be from write at 5: which is unreachable
- so is from initial write 0:
- but P1 enters its critical section (otherwise see case 1)
- so `w[] latch0 1` will be executed later in co order
- so all `3:r[] R10 latch0` are fr to all 30: w[] latch0 1
- by fairness of communications, this write of 1 to latch0 will eventually be read at 3:
- in contradiction with always reading 0

(4) Process `P0` starves in spin loop, `P1` does not



One or more writes of 1 co-after the initial write of 0 to latch0 must eventually be read by one of the infinitely many reads of latch0

Communication fairness hypothesis(*)

- All writes eventually hit the memory:
  - If, at a cut of the execution, all the processes infinitely often write the same value v to a shared variable `x` and only that value v
  - and from a later cut point of that execution, a process infinitely often repeats reads to that variable `x`
  - then the reads will end up reading that value v

(*) The SPARC Architecture Manual, Version 8, Section K2, p. 283: ``if one processor does an *S* , and another processor repeatedly does *L* 's to the same location, then there is an *L* that will be after the *S*".

(5) Process `P1` never enters its CS

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

Process P0

1: $\{\mathsf{true}\}$
  `do` $\{i\}$
2:   $\{\mathsf{true}\}$
    `do` $\{j_i\}$
3:     $\{\mathsf{true}\}$
      `r[] Rl0 latch0` $\{\rightsquigarrow\ L0^{i}_{j_i}\}$
4:     $\{\mathtt{Rl0} = \mathtt{L0}^{\mathrm{i}}_{\mathrm{j_i}}\ \wedge$
      $(\mathrm{r0Rl0}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{j_i}}[\Gamma_{\mathsf{rf}}])\}$
    `while (Rl0=0)` $\{k_i\}$
5:   $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
    `w[] latch0 0`
6:   $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}]\}$
    `r[] Rf0 flag0` $\{\rightsquigarrow\ F0^{i}\}$
7:   $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{\mathrm{i}}\ \wedge$
    $(\mathrm{r0Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}])\}$
    `if (Rf0`$\neq$`0) then`
8:     $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
      `(* critical section *)`
      `w[] flag0 0`
9:     $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
      `w[] flag1 1`
10:    $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
      `w[] latch1 1`
11:    $\{\mathrm{r1Rl0}^{\mathrm{i}}_{\mathrm{k_i}}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf0}^{\mathrm{i}}[\Gamma_{\mathsf{rf}}]\}$
    `fi`
12: $\{\mathsf{true}\}$
  `while true`
13: $\{\mathsf{false}\}$

||

Process P1

21: $\{\mathsf{true}\}$
  `do` $\{\ell\}$
22:   $\{\mathsf{true}\}$
    `do` $\{m_\ell\}$
23:     $\{\mathsf{true}\}$
      `r[] Rl1 latch1` $\{\rightsquigarrow\ L1^{\ell}_{m_\ell}\}$
24:     $\{\mathtt{Rl1} = \mathtt{L1}^{\ell}_{\mathrm{m}_\ell}\ \wedge$
      $(\mathrm{r0Rl1}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rl1}^{\ell}_{\mathrm{m}_\ell}[\Gamma_{\mathsf{rf}}])\}$
    `while (Rl1=0)` $\{n_\ell\}$
25:   $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
    `w[] latch1 0`
26:   $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}]\}$
    `r[] Rf1 flag1` $\{\rightsquigarrow\ F1^{\ell}\}$
27:   $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell}\ \wedge$
    $(\mathrm{r0Rf1}^{\ell}[\Gamma_{\mathsf{rf}}] \vee \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}])\}$
    `if (Rf1`$\neq$`0) then`
28:     $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
      `(* critical section *)`
      `w[] flag1 0` rf
29:     $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$ false
      `w[] flag0 1`
30:     $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
      `w[] latch0 1`
31:     $\{\mathrm{r1Rl1}^{\ell}_{\mathrm{n}_\ell}[\Gamma_{\mathsf{rf}}] \wedge \mathrm{r1Rf1}^{\ell}[\Gamma_{\mathsf{rf}}]\}$
    `fi`
32: $\{\mathsf{true}\}$
  `while true`
33: $\{\mathsf{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- `P1` exits loop 23:–24: (else see cases (1) or (3))
- must read `Rl1` = 1 from 0: or 10:
- read of `Rf1` at 26: must be 0
- only possibility is from 28:
- impossible from unreachable code

(5) Process P0 leaves spin loop but always fails entering its CS

`{0: latch0 = 0; flag0 = 0; latch1 = 1; flag1 = 1; }`

1: $\{\text{true}\}$
  do $\{i\}$
2: $\{\Gamma_{\text{rf}}\}$
  do $\{j_i\}$
3: $\{\text{true}\}$
  `r[] Rl0 latch0` $\{\rightsquigarrow L0^i_{j_i}\}$
fences
4: $\{\mathtt{Rl0} = \mathtt{L0}^{\text{i}}_{\text{j}_\text{i}} \wedge (\text{r0Rl0}^{\text{i}}_{\text{j}_\text{i}}[\Gamma_{\text{rf}}] \vee \text{r1Rl0}^{\text{i}}_{\text{j}_\text{i}}[\Gamma_{\text{rf}}])\}$
  `while (Rl0=0)` $\{k_i\}$
5: $\{\text{r1Rl0}^{\text{i}}_{\text{k}_\text{i}}[\Gamma_{\text{rf}}]\}$
  `w[] latch0 0`
  $\{\text{r1Rl0}^{\text{i}}_{\text{k}_\text{i}}[\Gamma_{\text{rf}}]\}$
  `f[fdep] {3} {6}`
6: $\{\text{r1Rl0}^{\text{i}}_{\text{k}_\text{i}}[\Gamma_{\text{rf}}]\}$
  `r[] Rf0 flag0` $\{\rightsquigarrow F0^i\}$
7: $\{\text{r1Rl0}^{\text{i}}_{\text{k}_\text{i}}[\Gamma_{\text{rf}}] \wedge \mathtt{Rf0} = \mathtt{F0}^{\text{i}} \wedge (\text{r0Rf0}^{\text{i}}[\Gamma_{\text{rf}}] \vee \text{r1Rf0}^{\text{i}}[\Gamma_{\text{rf}}])\}$
  `if (Rf0≠0) then`
8: $\{\text{r1Rl0}^{\text{i}}_{\text{k}_\text{i}}[\Gamma_{\text{rf}}] \wedge \text{r1Rf0}^{\text{i}}[\Gamma_{\text{rf}}]\}$
  `(* critical section *)`
  `w[] flag0 0`
9: $\{\text{r1Rl0}^{\text{i}}_{\text{k}_\text{i}}[\Gamma_{\text{rf}}] \wedge \text{r1Rf0}^{\text{i}}[\Gamma_{\text{rf}}]\}$
  `w[] flag1 1`
10: $\{\text{r1Rl0}^{\text{i}}_{\text{k}_\text{i}}[\Gamma_{\text{rf}}] \wedge \text{r1Rf0}^{\text{i}}[\Gamma_{\text{rf}}]\}$
false
  `w[] latch1 1`
11: $\{\text{r1Rl0}^{\text{i}}_{\text{k}_\text{i}}[\Gamma_{\text{rf}}] \wedge \text{r1Rf0}^{\text{i}}[\Gamma_{\text{rf}}]\}$
  `fi`
12: $\{\text{true}\}$
  `while true`
13: $\{\text{false}\}$

co

rf

fre

21: $\{\text{true}\}$
  do $\{\ell\}$
22: $\{\text{true}\}$
  do $\{m_\ell\}$
23: $\{\text{true}\}$
  `r[] Rl1 latch1` $\{\rightsquigarrow L1^\ell_{m_\ell}\}$
24: $\{\mathtt{Rl1} = \mathtt{L1}^{\ell}_{\text{m}_\ell} \wedge (\text{r0Rl1}^{\ell}_{\text{m}_\ell}[\Gamma_{\text{rf}}] \vee \text{r1Rl1}^{\ell}_{\text{m}_\ell}[\Gamma_{\text{rf}}])\}$
  `while (Rl1=0)` $\{n_\ell\}$
25: $\{\text{r1Rl1}^{\ell}_{\text{n}_\ell}[\Gamma_{\text{rf}}]\}$
  `w[] latch1 0`
26: $\{\text{r1Rl1}^{\ell}_{\text{n}_\ell}[\Gamma_{\text{rf}}]\}$
  `r[] Rf1 flag1` $\{\rightsquigarrow F1^\ell\}$
27: $\{\text{r1Rl1}^{\ell}_{\text{n}_\ell}[\Gamma_{\text{rf}}] \wedge \mathtt{Rf1} = \mathtt{F1}^{\ell} \wedge (\text{r0Rf1}^{\ell}[\Gamma_{\text{rf}}] \vee \text{r1Rf1}^{\ell}[\Gamma_{\text{rf}}])\}$
  `if (Rf1≠0) then`
28: $\{\text{r1Rl1}^{\ell}_{\text{n}_\ell}[\Gamma_{\text{rf}}] \wedge \text{r1Rf1}^{\ell}[\Gamma_{\text{rf}}]\}$
  `(* critical section *)`
  `w[] flag1 0`
29: $\{\text{r1Rl1}^{\ell}_{\text{n}_\ell}[\Gamma_{\text{rf}}] \wedge \text{r1Rf1}^{\ell}[\Gamma_{\text{rf}}]\}$
  `w[] flag0 1`
  $\{\text{r1Rl1}^{\ell}_{\text{n}_\ell}[\Gamma_{\text{rf}}] \wedge \text{r1Rf1}^{\ell}[\Gamma_{\text{rf}}]\}$
  `f[flw] {29} {30}`
30: $\{\text{r1Rl1}^{\ell}_{\text{n}_\ell}[\Gamma_{\text{rf}}] \wedge \text{r1Rf1}^{\ell}[\Gamma_{\text{rf}}]\}$
  `w[] latch0 1`
fences
31: $\{\text{r1Rl1}^{\ell}_{\text{n}_\ell}[\Gamma_{\text{rf}}] \wedge \text{r1Rf1}^{\ell}[\Gamma_{\text{rf}}]\}$
  `fi`
32: $\{\text{true}\}$
  `while true`
33: $\{\text{false}\}$

- let rf be the communication for such a trace (encoded in Γ_{rf})
- loop 2:–4: exited
- read of `Rl0` = 1 at 3: is from 30:
- invariant false in critical section 8:–11:
- read of `Rf0` = 0 at 6: is from 0: (8: not reachable)

```
withco
let l-fencerel(S) =
        ((po&(_*S));po)&fromto(S)
let Fdep = F & tag2events('fdep)
let deps = l-fencerel(Fdep) & (R*_)
let Flw = F & tag2events('flw)
let flw = l-fencerel(Flw)
let fences = deps | flw
let free = (rf^-1;co) & ext
irreflexive fre;fences;rfe;fences
```

In TSO there is no need for a fence since it is MP. For weaker than PSO, a fence is needed.

(6) Both processes eventually starve in spin loop

```
{0:  w latch0 0;                    w latch1 1;
     w flag0 0;                     w flag1 1;}
                               ||
... ...                        ||  ... ...
3:   r Rl0 latch0 1            ||  23: r Rl1 latch1 1
5:   w latch0 0                ||  25: w latch1 0
6:   r Rf0 flag0 1             ||  26: r Rf1 flag1 1
8:   (* critical section *)    ||  28: (* critical section *)
     w flag0 0                 ||      w flag1 0
                               ||      f[bar] {25:} {29:}
9:   w flag1 1                 ||  29: w flag0 1
     f[bar] {5:} {10:}         ||
10:  w latch1 1                ||  30: w latch0 1
... ...                        ||  ... ...
3:   r Rl0 latch0 1            ||  23: r Rl1 latch1 1
5:   w latch0 0                ||  25: w latch1 0
6:   r Rf0 flag0 1             ||  26: r Rf1 flag1 1
8:   (* critical section *)    ||  28: (* critical section *)
     w flag0 0                 ||      w flag1 0
                               ||      f[bar] {25:} {29:}
9:   w flag1 1                 ||  29: w flag0 1
     f[bar] {5:} {10:}         ||
10:  w latch1 1                ||  30: w latch0 1
... ...                        ||  ... ...
3:   r Rl0 latch0 0            ||  23: r Rl1 latch1 0
3:   r Rl0 latch0 0            ||  23: r Rl1 latch1 0
... ...                        ||  ... ...
```

bar co bar

- let rf be the communication for such a trace (encoded in Γ_{rf})
- so `latch0` is always 0 and `latch1` is always 0
- so `latch0` in 23 is always read from 25:
- so `10: w latch1 1` was co-before (since otherwise by the communication hypothesis it would be eventually read)
- and `3: Rl0 latch0 0` is from 0: or 5:
- so `30: w latch0 1` is co-before them (since otherwise by the communication hypothesis it would be eventually read)
- impossible by fences
- irreflexive co; bar; co; bar

(7) Eventually, P0 starves in spin loop, P1 never enters its CS

Process P0 enters & exits CS multiple times

then, never exits the waiting loop

```
{0:  w latch0 0;
     w flag0 0;
...  ...
3:   r Rl0 latch0 1
5:   w latch0 0
6:   r Rf0 flag0 1
8:   (* critical section *)
     w flag0 0
9:   w flag1 1
10:  w latch1 1
...  ...
3:   r Rl0 latch0 1
5:   w latch0 0
6:   r Rf0 flag0 1
8:   (* critical section *)
     w flag0 0
9:   w flag1 1
10:  w latch1 1
...  ...
3:   r Rl0 latch0 0
3:   r Rl0 latch0 0
3:   r Rl0 latch0 0
...  ...
```

```
     w latch1 1;
     w flag1 1;}
...  ...
23:  r Rl1 latch1 1
25:  w latch1 0
26:  r Rf1 flag1 1
28:  (* critical section *)
     w[] flag1 0
29:  w[] flag0 1
30   w[] latch0 1
...  ...
23:  r Rl1 latch1 1
25:  w latch1 0
26:  r Rf1 flag1 0
...  ...
23:  r Rl1 latch1 1
25:  w latch1 0
26:  r Rf1 flag1 0

...  ...

...  ...
```

last CS entrance

- P1 does not eventually starves in spin loop (otherwise case 6)
- case P1 eventually never starves and never enters its critical section
- P1 then does a last write of 1 to latch0
- P0 eventually makes infinitely many reads of latch0
- A contradiction (since otherwise by the communication hypothesis, this 1 would be eventually read)

(8) Eventually, `P1` starves in spin loop, `P0` never enters its CS

symmetric of (7)

(9) P0 and P1 always leave spin loop and never enter their CS

```
{0:  w[] latch0 0;          w[] latch1 1;
     w[] flag0 0;           w[] flag1 1;}
```

```
... ...
3:  r[] Rl0 latch0 1
5:  w[] latch0 0
6:  r[] Rf0 flag0 1
8:  (* critical section *)
    w[] flag0 0
9:  w[] flag1 1
10: w[] latch1 1
... ...
3:  r Rl0 latch0 1
5:  w[] latch0 0
6:  r[] Rf0 flag0 1
8:  (* critical section *)
    w[] flag0 0
9:  w[] flag1 1
10: w[] latch1 1
... ...
3:  r[] Rl0 latch0 1
5:  w[] latch0 0
6:  r[] Rf0 flag0 0
... ...
3:  r[] Rl0 latch0 1
5:  w[] latch0 0
6:  r[] Rf0 flag0 0
... ...
3:  r[] Rl0 latch0 1
5:  w[] latch0 0
6:  r[] Rf0 flag0 0
... ...
```

```
... ...
23: r[] Rl1 latch1 1
25: w[] latch1 0
26: r[] Rf1 flag1 1
28: (* critical section *)
    w[] flag1 0
29: w[] flag0 1
30  w[] latch0 1
... ...
23: r[] Rl1 latch1 1
25: w[] latch1 0
26: r[] Rf1 flag1 0
28: (* critical section *)
23: w[] flag1 0
29: w[] flag0 1
30: w[] latch0 1
... ...
23: r[] Rl1 latch1 1
25: w[] latch1 0
26: r[] Rf1 flag1 0
... ...
23: r[] Rl1 latch1 1
25: w[] latch1 0
26: r[] Rf1 flag1 0
... ...
23: r[] Rl1 latch1 1
25: w[] latch1 0
26: r[] Rf1 flag1 0
... ...
```

rf

...

- P0 and P1 eventually never starve and never enter their critical sections
- They both have a last entrance in their critical sections
- This last write of 1 to the latches will, by communication fairness, eventually reach the memory
- Then we only have infinitely many writes of 0 to the latches
- So the read of the latches in the spin loops will eventually always read 0
- So from then on, by communication fairness, all the reads will be from 0, in reads of the latch will be zero
- In contradiction with the fact that the spin loop is always exited
- The barrier prevents infinitely postponing the write 0 actions

Conclusion

Conclusion

- The proof method is parameterized by consistency hypotheses, expressed in
  - Invariance form: S_{com}
  - Consistency form: H_{com} (e.g. in `cat`)
- Program not logic/architecture/consistency model dependent (hence the proof is portable)
- Can reason on *arbitrary* subsets of anarchic executions (hence flexible e.g. non-starvation)

Proposed design methodology

1. Design the algorithm A and its specification S_{inv} (e.g. in the sequential consistency model of parallelism)
2. Consider the anarchic semantics of algorithm A
3. Add communication specifications S_{com} to restrict anarchic communications and ensure the correctness of A with respect to specification S_{inv}
4. Do the invariance proof under WCM with S_{com}
5. Infer H_{com} (in `cat`) from invariant S_{com}
6. Prove that the machine memory model M in `cat` implies H_{cm}

Challenges

- Modern machines have complex memory models

  ⇒ portability has a price (refencing)

  ⇒ debugging is very hard/quasi-impossible

  ⇒ proofs are much harder than with sequential consistency (but still feasible?, mechanically?)

  ⇒ static analysis parameterized by a WCM will be a challenge

  ⇒ but we can start with S_{com}

Thanks

- *Patrick Cousot thanks Luc Maranget for his precious help at Dagstuhl on the non-starvation part.*

The End, Thank You