Progress on Abstract Interpretation Based Formal Methods and Future Challenges

Patrick Cousot

D partement d'informatique, cole normale sup rieure,
45 rue d'Ulm, 75230 Paris cedex 05, France
Patrick.Cousot@ens.fr http://www.di.ens.fr/~cousot/

Abstract. In order to contribute to the software reliability problem, tools have been designed in order to analyze statically the run-time behavior of programs. Because the correctness problem is undecidable, some form of approximation is needed. The whole purpose of abstract interpretation is to formalize this idea of approximation. We illustrate informally the application of abstraction to the semantics of programming languages as well as to program static analysis. The main point is that in order to reason or compute about a complex system, some information must be lost, that is the observation of executions must be either partial or at a high level of abstraction.
In the second part of the paper, we compare program static analysis with deductive methods, model-checking and type inference. Their foundational ideas are shortly reviewed, and the shortcomings of these four tools are discussed, including when they are combined. Alternatively, since program debugging is still the main program verification method used in the software industry, we suggest to combine formal with informal methods.
Finally, the grand challenge for all formal methods and tools is to solve the software reliability, trustworthiness or robustness problems. Few challenges more specific to program analysis by abstract interpretation are shortly discussed.

1 Introductive Motivations

The evolution of hardware by a factor of 10^6 over the past 25 years has lead to the explosion of the size of programs in similar proportions. The scope of application of very large programs (from 1 to 40 millions of lines) is likely to widen rapidly in the next decade. These big programs will have to be designed at a reasonable cost and then modified and maintained during their lifetime (which is often over 20 years). The size and efficiency of the programming and maintenance teams in charge of their design and follow-up cannot grow up in similar proportions. At a not so uncommon (and often optimistic) rate of one bug per thousand lines such huge programs might rapidly become hardly manageable in particular for safety critical systems (128). Therefore in the next 10 years, the *software reliability problem* is likely to become a major concern and challenge to modern highly computer-dependent societies.

In the past decade a lot of progress has been done both on *thinking/methodological tools* (to enhance the human intellectual ability) to cope with complex software systems and *mecanical tools* (using the computer) to help the programmer to reason on programs.

The mecanical tools for computer aided program verification started empirically by executing or simulating the program in enough representative possible environments. However debugging of the compiled code or simulation of a model of the source program hardly scale up and often offer a low coverage of the program dynamic behavior.

Formal program verification methods attempt to mecanically prove that program execution is correct in all specified environments. This includes *deductive methods*, *model checking*, *program typing* and *program analysis*.

Since program verification is undecidable, computer aided program verification methods are all partial or incomplete. The undecidability or complexity is always solved by using some form of *approximation*. This means that the mechanical tool will sometimes suffer from practical time and space complexity limitations, rely on finiteness hypotheses or provide only semi-algorithms, require user interaction or be able to consider restricted forms of specifications or programs only. The mechanical program verification tools are all quite similar and essentially differ in their choices regarding the approximations which have to be done in order to cope with undecidability or complexity. The ambition of *abstract interpretation* is to formalize this notion of approximation in a unified framework (45; 48).

2 Abstract Interpretation

Since program verification deals with properties, that is sets (of objects with these properties), abstract interpretation can be formulated in an application independent setting, as a theory for approximating sets and set operations as considered in set (or category) theory. A more restricted understanding of abstract interpretation is to view it as a theory of approximation of the behavior of dynamic discrete systems (such as the formal semantics of programs or a communication protocole specification). Since such behaviors can be characterized by fixpoints (e.g. corresponding to iteration), an essential part of the theory provides constructive and effective methods for fixpoint approximation and checking by abstraction (52).

2.1 Fixpoint Semantics

The *semantics of a programming language* defines the semantics of any program written in this language. The *semantics of a program* provides a formal mathematical model of all possible behaviors of a computer system executing this program in interaction with any possible environment. In the following we will try to explain informally why the semantic of a program can be defined as the solution of a fixpoint equation. Then, in order to compare semantics, we will

show that all semantics of a program can be organized in a hierarchy by abstraction. By observing computations at different levels of abstraction, one can approximate fixpoints hence organize the semantics of a program in a lattice (43).

2.2 Trace Semantics

Our finer grain of observation of program execution, that is the most precise of the semantics that we will consider, is that of *trace semantics*, a model also frequently used in temporal logic (139). An execution of a program for a given specific interaction with its environment is a sequence of states, observed at discrete intervals of time, starting from an initial state, then moving from one state to the next one by executing an atomic program step or transitions and either ending in a final regular or erroneous state or non terminating, in which case the trace is infinite (see Fig. 1).

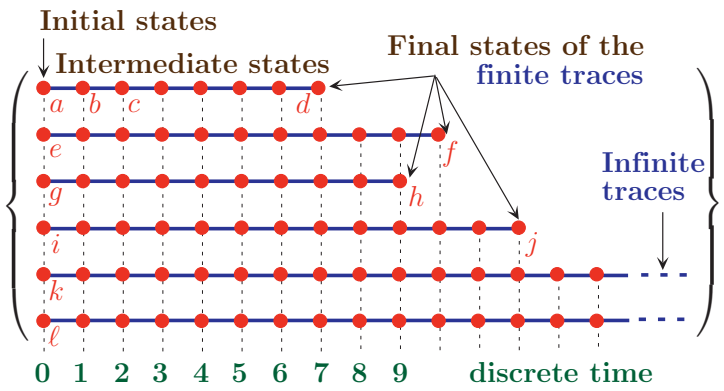


Fig. 1. Examples of Computation Traces

2.3 Least Fixpoint Trace Semantics

The trace semantics can be defined in fixpoint form (43), that is as a solution of an equation of the form $X = F(X)$ where X ranges over sets of finite and infinite traces.

More precisely, let **Behaviors** be the set of execution traces of a program, possibly starting in any state. We denote by $\mathbf{Behaviors}^+$ the subset of finite traces and by $\mathbf{Behaviors}^\infty$ the subset of infinite traces.

A finite trace $\overset{a}{\bullet}\!\!-\!\!\cdots\!\!-\!\!\overset{z}{\bullet}$ in $\mathbf{Behaviors}^+$ is either reduced to a final state (in which case there is no possible transition from state $\overset{a}{\bullet} = \overset{z}{\bullet}$) or the initial state $\overset{a}{\bullet}$ is not final and the trace consists of a first computation step $\overset{a}{\bullet}\!\!-\!\!\overset{b}{\bullet}$ after which, from the intermediate state $\overset{b}{\bullet}$, the execution goes on with the shorter finite trace

ending in the final state . The finite traces are therefore all well defined by induction on their length.

An infinite trace in $\mathbf{Behaviors}^\infty$ starts with a first computation step after which, from the intermediate state , the execution goes on with an infinite trace starting from the intermediate state . These remarks lead to the following fixpoint equation:

$$
\begin{aligned}
\mathbf{Behaviors} = \; & \{ \overset{a}{\bullet} \mid \overset{a}{\bullet} \text{ is a final state} \} \\
& \cup \{ \overset{a}{\bullet}\!\!-\!\!\overset{b}{\bullet}\!\!-\!\!\cdots\!\!-\!\!\overset{z}{\bullet} \mid \overset{a}{\bullet}\!\!-\!\!\overset{b}{\bullet} \text{ is an elementary step } \& \\
& \qquad \overset{b}{\bullet}\!\!-\!\!\cdots\!\!-\!\!\overset{z}{\bullet} \in \mathbf{Behaviors}^+ \} \\
& \cup \{ \overset{a}{\bullet}\!\!-\!\!\overset{b}{\bullet}\!\!-\!\!\cdots\!\!-\!\!\cdots \mid \overset{a}{\bullet}\!\!-\!\!\overset{b}{\bullet} \text{ is an elementary step } \& \\
& \qquad \overset{b}{\bullet}\!\!-\!\!\cdots\!\!-\!\!\cdots \in \mathbf{Behaviors}^\infty \}
\end{aligned}
$$

In general, the equation has multiple solutions. For example if there is only state , it is not final and the only possible elementary step is then the equation is $\mathbf{Behaviors} = \{ \overset{a}{\bullet}\!\!-\!\!\overset{a}{\bullet}\!\!-\!\!\cdots\!\!-\!\!\cdots \mid \overset{a}{\bullet}\!\!-\!\!\cdots\!\!-\!\!\cdots \in \mathbf{Behaviors} \}$. One solution is $\{ \overset{a}{\bullet}\!\!-\!\!\overset{a}{\bullet}\!\!-\!\!\overset{a}{\bullet}\!\!-\!\!\overset{a}{\bullet}\!\!-\!\!\cdots\!\!-\!\!\cdots \}$ but another one is the empty set $\emptyset$. Therefore, we choose the least solution for the *computational partial ordering*:

« More finite traces & less infinite traces » .

2.4 Abstractions

A programming language semantics is more or less precise according to the considered observation level of the program executions (105). This intuitive idea can be formalized by *Abstract interpretation* (43) and applied to different languages (13; 79), including for proof methods (39).

The *abstract interpretation theory* formalizes this notion of approximation/abstraction in a mathematical setting which is independent of particular applications. In particular, abstractions must be provided for all mathematical constructions used in programming and specification languages semantic definitions (48; 52).

If the approximation is rough enough, the abstraction of a concrete semantics can lead to an abstract semantics which is less precise but is *effectively computable* by a computer. By effective computation of the abstract semantics, the computer is able to analyze the behavior of programs and of software before and without executing them (44). *Abstract interpretation algorithmics* provide approximate methods for computing this abstract semantics. The main abstract interpretation algorithmics provide effective methods for the exact or approximate iterative resolution of fixpoint equations (45).

We will first illustrate formal and effective abstractions for sets. Then we will show that such abstractions can be lifted to functions and finally to fixpoints.

The abstraction idea and its formalization are equally applicable in other areas of computer science such as artificial intelligence (87) e.g. for intelligent

planning (21), proof checking (85), automated deduction and theorem proving (86), etc.

2.5 Hierarchy of Abstractions

As shown in Fig. 2 (from (43), where **Behaviors**, denoted $\tau^{\vec{\infty}}$ for short, is the lattice infimum), all abstractions of a semantics can be organized in a lattice (which is part of the lattice of abstract interpretations introduced in (48)). The *approximation partial ordering* of this lattice formally corresponds to logic implication, intuitively to the idea that a semantics is more precise than another.

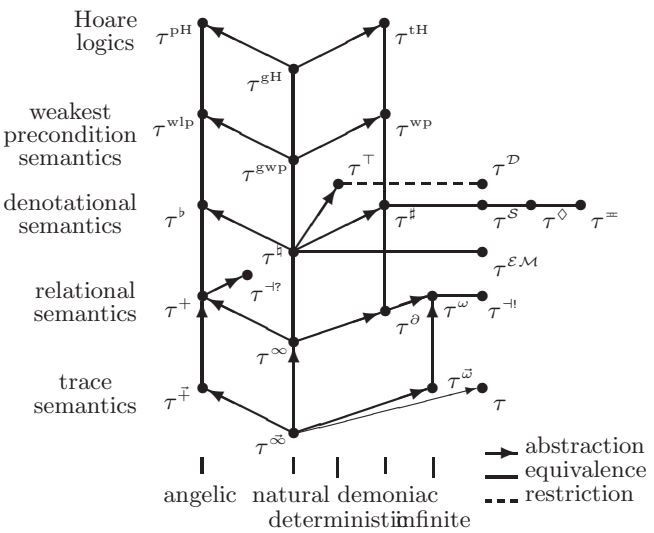


Fig. 2. The Hierarchy of Semantics

Fig. 3 illustrates the derivation of a *relational semantics* (105) (denoted τ^{∞} in Fig. 2) from a trace semantics (denoted $\tau^{\vec{\infty}}$ in Fig. 2). The abstraction α from trace to relational semantics consists in replacing the finite traces a ... z by the pair $\langle a, z\rangle$ of the initial and final states. The infinite traces a b ... ... are replaced by the pair $\langle a, \bot\rangle$ where the symbol $\bot$ denotes non-termination. Therefore the abstraction is:

$$\alpha(X) = \{\langle a, z\rangle \mid \text{a} \text{---} \ldots \text{---} \text{z} \in X\} \cup \{\langle a, \bot\rangle \mid \text{a} \text{---} \text{b} \text{---} \ldots \text{---} \ldots \in X\} .$$

The *denotational semantics* (denoted $\tau^{\natural}$ in Fig. 2) is the isomorphic representation of a relation by its right-image:

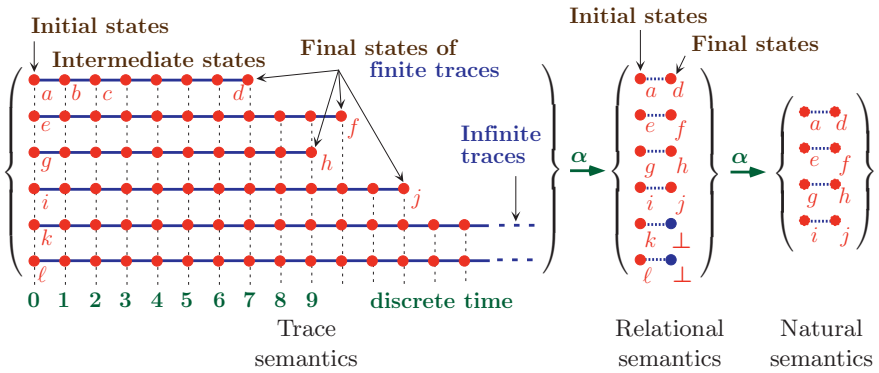


Fig. 3. Abstraction from Trace to Relational and Natural Semantics

$$\alpha(R) = \boldsymbol{\lambda}\, a \cdot \{x \mid \langle a, x\rangle \in R\}.$$

The abstraction from relational to *big-step operational or natural semantics* (denoted τ^+ in Fig. 2) simply consists in forgetting everything about non-termination, so $\alpha(R) = \{\langle a, x\rangle \in R \mid x \neq \bot\}$, as illustrated in Fig. 3.

A non comparable abstraction consists in collecting the set of initial and final states as well as all transitions $\langle x, y\rangle$ appearing along some finite or infinite trace of the trace semantics. One gets the *small-step operational or transition semantics* (138) (denoted τ in Fig. 2 and also called Kripke structure in modal logic) as illustrated in Fig. 4.

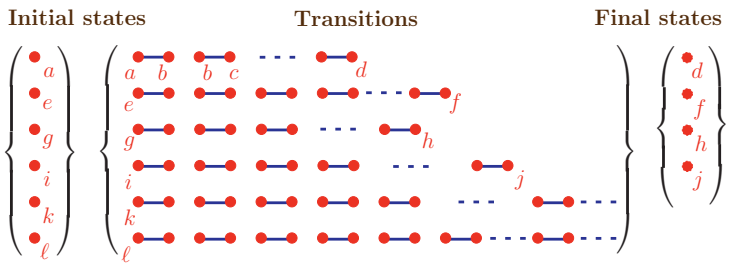


Fig. 4. Transition Semantics

A further abstraction consists in collecting all states appearing along some finite or infinite trace as illustrated in Fig. 5. This is the *partial correctness semantics* (39) or the *collecting semantics* (45) for proving invariance properties of programs.

All abstractions considered in this paper are "from above" so that the abstract semantics describes a superset or logical consequence of the concrete

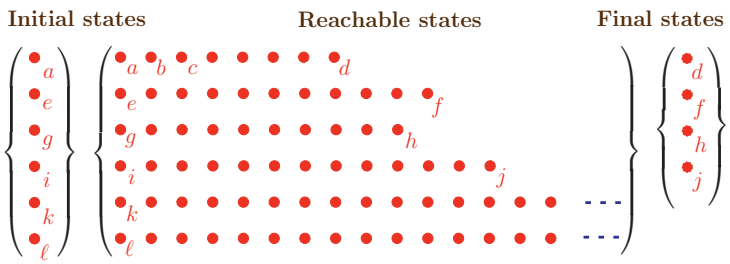


Fig. 5. Collecting / Partial Correctness Semantics

semantics. Abstractions "from below" are dual and consider a subset of the concrete semantics. An exemple of approximation "from below" is provided by debugging techniques which consider a subset of the possible program executions or by existential checking where one wants to prove the existence of an execution trace fullfilling some given specification. In order to avoid repeating two times dual concepts, we only consider approximations "from above", knowing that approximations "from below" can be easily derived by applying the duality principle (as found e.g. in lattice theory).

2.6 Effective Abstractions

Numerical Abstractions Assume that a program has two integer variables `X` and `Y`. The trace semantics of the program (Fig. 1) can be abstracted in the collecting semantics (Fig. 5). A further abstraction consists in forgetting in a state all but the values x and y of variables `X` and `Y`. In this way the trace semantics is abstracted to a set of points, as illustrated in Fig. 6(a).

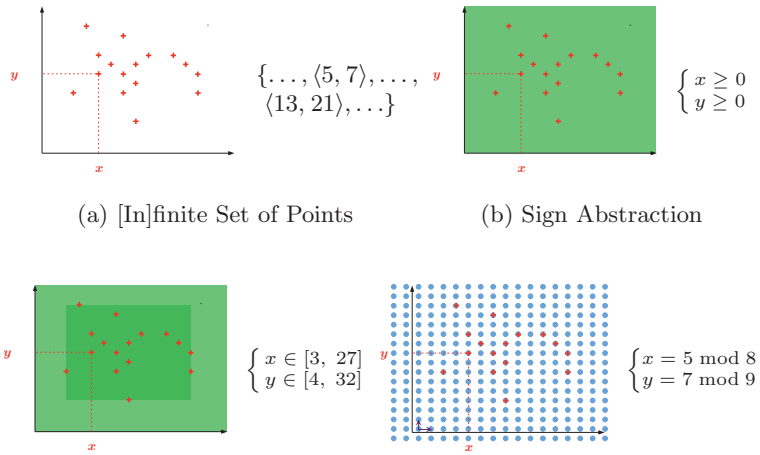
We now illustrate informally a number of effective abstractions of an [in]finite set of points.

Non-relational Abstractions The nonn-relational, attibute independent or cartesian abstractions (48, example 6.2.0.2) consists in ignoring the possible relationships between the values of the `X` and `Y` variables. So a set of pairs is approximated through projection by a pair of sets. Each such set may still be infinite and in general not exactly computer representable. Further abstractions are therefore needed.

The *sign abstraction* (48) illustrated in Fig. 6(b) consists in replacing integers by their sign thus ignoring their absolute value.

The *interval abstraction* (44) illustrated in Fig. 6(c) is more precise since it approximates a set of integers by it minimal and maximal values (including $-\infty$ and $+\infty$ as well as the empty set if necessary).

The *congruence abstraction* (92) illustrated in Fig. 6(d) is not comparable.



(a) [In]finite Set of Points

(b) Sign Abstraction

(c) Interval Abstraction

(d) Simple Congruence Abstraction

Fig. 6. Non-relational Abstractions

Relational Abstractions Relational abstractions are more precise than non relational ones (112) in that some of the relationships between values of the program states are preserved by the abstraction.

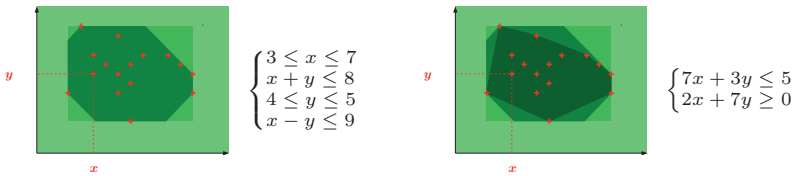
For example the *polyhedral abstraction* (61) illustrated in Fig. 7(b) approximates a set of integers by its convex hull. Only non-linear relationships between the values of the program variables are forgotten.

The use of an *octogonal abstraction* (2) illustrated in Fig. 7(a) is less precise since only some shapes of polyhedra are retained or equivalently only linear relations between any two variables are considered with +1 or -1 coefficients (of the form $\pm x \pm y \leq c$ where c is an integer constant).

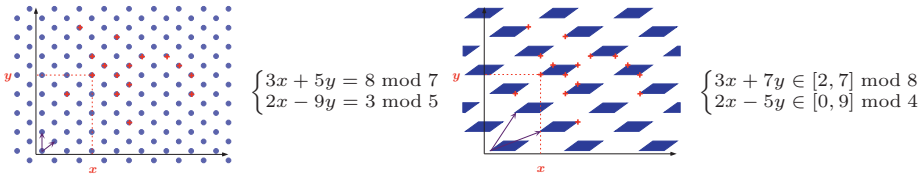
A non comparable relational abstraction is the *linear congruence abstraction* (93) illustrated in Fig. 7(c).

A combination of non-relational dense approximations (like intervals) and relational sparse approximations (like congruences) is the *trapezoidal linear congruence abstraction* of (122) as illustrated in Fig. 7(d).

Symbolic Abstractions Most structures manipulated by programs are *symbolic structures* such as control structures (call graphs), data structures (search trees), communication structures (distributed & mobile programs), etc. It is very difficult to find compact and expressive abstractions of such sets of objects (sets of languages, sets of automata, sets of trees or graphs, etc.). For example Büchi automata or automata on trees are very expressive but algorithmically expensive.



(a) Octagonal Abstraction (b) Polyhedral Abstraction



(c) Relational Congruence Abstraction (d) Trapezoidal Congruence Abstraction

Fig. 7. Relational Abstractions

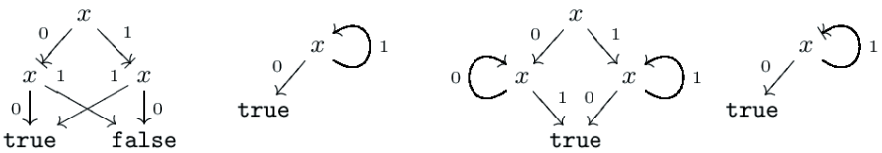


Fig. 8. Binary Decision Graphs

A compromise between semantic expressivity and algorithmic efficiency was recently introduced by (124; 123) using *Binary Decision Graphs* and *Tree Schemata* to abstract infinite sets of infinite trees is illustrated in Fig. 8 & 9.

2.7 Information Loss

Any abstraction introduces some loss of information. For example the abstraction of the trace semantics into relational or denotational semantics loses all information on the computation cost since all intermediate steps in the execution are removed.

All answers given by the abstract semantics are always correct with respect to the concrete semantics. For example if termination is proved using the relational semantics then there no execution abstracted to $\langle a, \bot \rangle$ so no infinite trace a —— b —— . . . —— . . . in the trace semantics whence non termination is impossible when starting execution in initial state a.

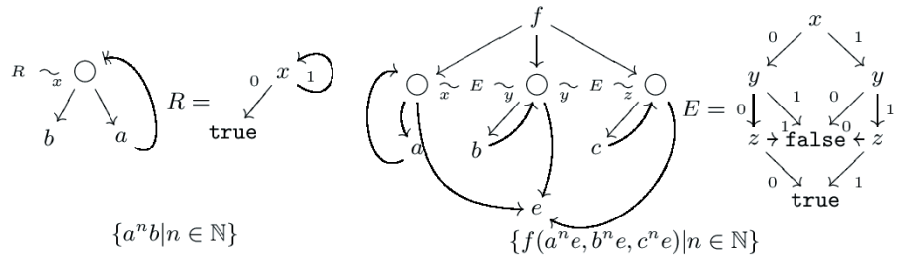
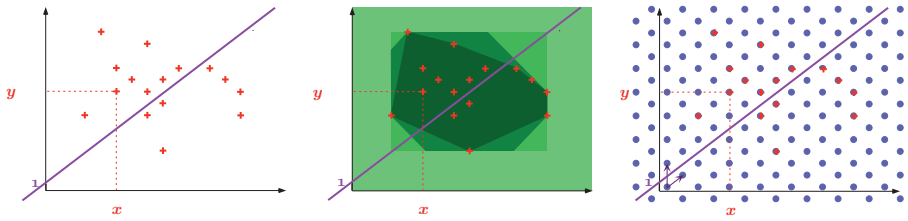


Fig. 9. Tree Schemata



(a) yes (b) unkown (c) yes

Fig. 10. Is `1/(X+1-Y)` well-defined?

However, because of the information loss, not all questions can be definitely answered with the abstract semantics. For example the natural semantics cannot answer questions about termination as can be done with the relational or denotational semantics. These semantics cannot answer questions about concrete computation costs

The more concrete semantics can answer more questions. The more abstract semantics are simpler. Non comparable abstract semantics (such as intervals and congruences) can neither answer more nor less questions.

To illustrate the loss of information, let us consider the problem of deciding whether the operation `1/(X+1-Y)` appearing in a program is always well defined at run-time. The answer can certainly be given by the concrete semantics since it has no point on the line $x + 1 - y = 0$, as shown in Fig. 10(a).

In practice the concrete abstraction is not computable so it is hardly usable in a useful effective tool. The dense abstractions that we have considered are too approximate as illustrated in Fig. 10(b).

However the answer is positive when using the relational congruence abstraction, as shown in Fig. 10(c).

Abstract interpretation theory has mainly been concerned with the *soundness* of the abstract semantics/interpreter, relative to which questions can be

answered corectly despite the loss of information, which is essential in practice and leads to a formal design method.

However *completeness*, relative to the formalization of the loss of information in a controlled way so as to answer a given set of questions, has also been studied (48; 82), including in the context of model checking (42). In practice complete abstractions, including a most abstract one do exist, but most often are not computable and even hard to design manually since the design of a complete abstraction is logically equivalent to a formal correctness proof (42).

A more limited but certainly feasible objective towards expressive analyses is by combination of abstract domains (such as the reduced product (48), disjunctive completion (48; 76), complementation (36)) and their refinement (81), which can be implemented in static analyser generators (e.g. (117)).

2.8 Function Abstraction

We now show how the abstraction of complex mathematical objects used in the semantics of programming or specification languages can be defined by composing abstractions of simpler mathematical structures.

For example knowing abstractions of the parameter and result of a monotonic function on sets, a function F can be abstracted into an abstract function $F^\sharp$ as illustrated in Fig. 11 (48). Mathematically, $F^\sharp$ takes its parameter x in the

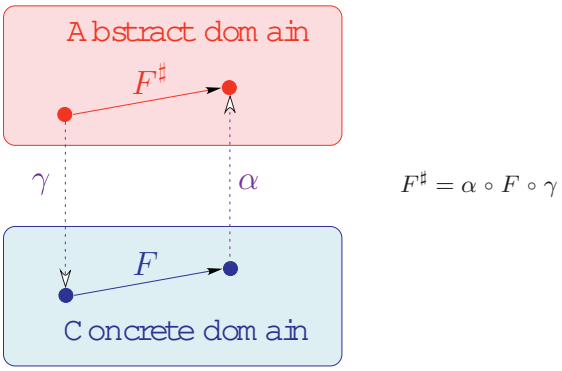


Fig. 11. Function Abstraction

abstract domain. Let $\gamma(x)$ be the corresponding concrete set (γ is the adjoined, intuitively the inverse of the abstraction function α). The function F can be applied to get the concrete result $\circ\ F\ \circ\ \gamma(x)$. The abstraction function α can then be applied to approximate the result $F^\sharp(x) = \alpha \circ F \circ \gamma(x)$.

In general neither F nor α and γ is computable even though the abstraction α may be effective. So we have got a formal specification of the abstract function $F^\sharp$ and an algorithm has to be found for an effective implementation.

2.9 Fixpoint Abstraction

A fixpoint of a function F can often be obtained as the limit of the iterations of F from a given initial value $\bot$. In this case the abstraction of the fixpoint can often be obtained as the abstract limit of the iteration of the abstraction $F^\sharp$ of F starting from the abstraction $\alpha(\bot)$ of the initial value $\bot$. The basic result is that the concretization of the abstract fixpoint is related to the concrete fixpoint by the approximation relation expressing the soundness of the abstraction (48). This is illusrated in Fig. 12.

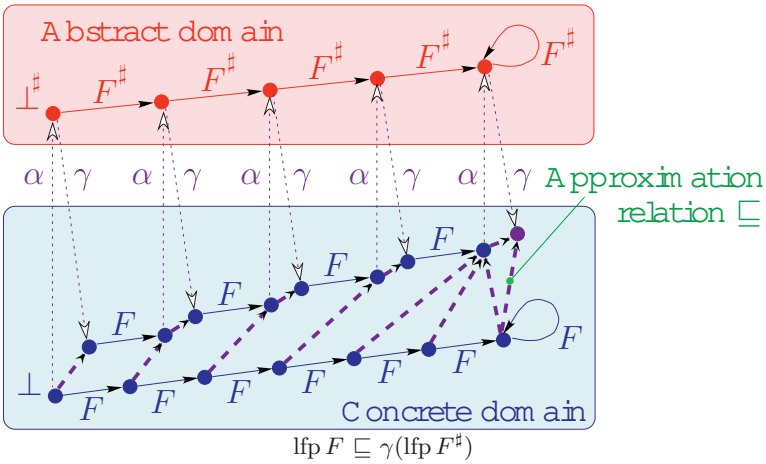


Fig. 12. Fixpoint Abstraction

Often states have some finite component (e.g. a program counter) which can be used to get a fixpoint system of equations by projection along that component. Then *chaotic* and *asynchronous iteration stategies* can be used to solve the equations iteratively (46). Various efficient iteration strategies have been studied, including ones taking particular properties of abstractions into account and others to speed up the convergence of the iterates (45; 53).

2.10 Composing Abstractions

Abstractions hence abstract interpreters for program analysis can be designed compositionally by stepwise abstraction, combination or refinement (48; 80).

An example of stepwise abstraction is the functional abstraction of Sec. 2.8. The abstraction of a function is parameterized by abstractions for the function parameters and the function result which can be chosen later in the modular design of the abstract interpreter. An example of abstraction combination is the *reduced product* of two abstractions (48) which is the most abstract abstraction more precises than these two abstractions. An example of refinement is the

power operation (48) which complete an abstract domain by adding missing disjunctions and the complement (36) adding missing negations.

It is always possible to refine an abstraction so as to check a given specification for a given program (42; 82). Nevertheless this approach has severe practical limitations since, in general, the design of this abstraction is logically equivalent to the design of an inductive argument for the formal proof that the given program satisfies the given specification while the soundness proof of this abstraction logically amounts to checking the inductive verification conditions of this formal proof (42). Such proofs can hardly be fully automated hence human interaction is unavoidable. Moreover the whole process has to be repeated each time the program or specification is modified.

Instead of considering such strong specifications for a given specific program, the objective of program analysis is to consider (often predefined) specifications and all possible programs. The practical problem in program analysis is therefore to design useful abstractions which are computable for all programs and expressive enough to yield interesting information for most programs.

3 Program Analysis

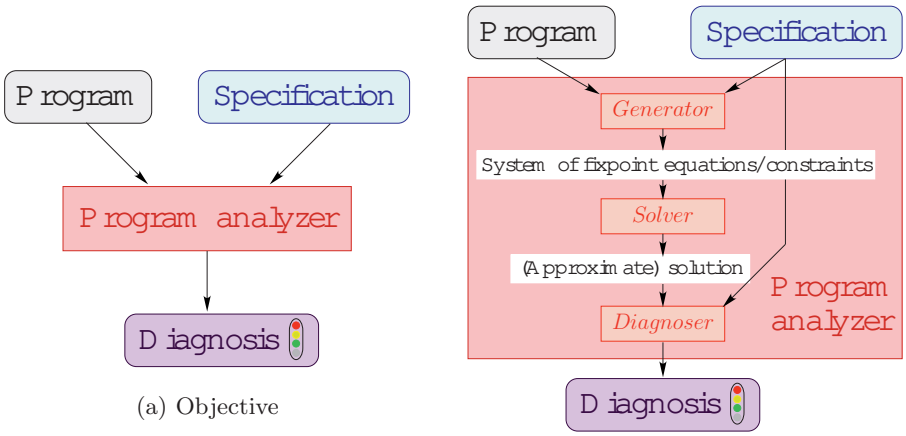
Program analysis is the automatic static determination of dynamic run-time properties of programs.

3.1 Foundational Ideas of Program Analysis

Given a program and a specification, a program analyzer will check if the program semantics satisfies the specification (Fig. 13(a)). In case of failure,the analyser will provide hints to understand the origin of errors (e.g. by providing necessary conditions to be satisfied by counter-examples).

The principle of the analysis is to compute an approximate semantics of the program in order to check a given specification. Abstract interpretation is used to derive, from a standard semantics, the approximate and computable abstract semantics. The derivation can often be done by composing standard abstractions to fit a particular kind of information which has to be discovered about program execution. This derivation is itself not (fully) mechanizable but *static analyser generators* such as PAG (71) and others can provide generic abstractions to be composed with problem specific ones.

In practice, the program analyser contains a *generator* reading the program text and producing equations or constraints which solution is a computer representation of the program abstract semantics. A *solver* is then used to solve these abstract equations/constraints. A popular resolution method is to use iteration. In this case the convergence may have to be accelerated using *widening* to over estimate the solution followed by a *narrowing* to improve it (45). The approximation of the abstract semantics is then used by a *diagnoser* to check the specification. Because of the loss of information, the diagnosis is always of the form "yes", "no", "unknown" or "irrelevant" (e.g. a safety specification for



(a) Objective

(b) Principle

Fig. 13. Program Analysis

unreachable code). The general structure of program analysers is illustrated in Fig. 13(b).

3.2 Shortcomings of Program Analysis

Static program analysis can be used for large programs (220 000 lines of C) without user interaction. The abstractions are chosen to be of wide scope without specialization to a particular program. Abstract algebras can be designed and implemented into libraries which are resuable for different programming languages. The objective is to discover invariants that are likely to appear in many programs so that the abstraction must be widely reusable for the program analyzer to be of economic interest.

The drawback of this general scope is that the considered abstract specifications and properties are often simple, mainly concerning elementary safety properties. For example non-linear abstractions of sets of points are very difficult et very few mathematical results are of practical interest and directly applicable to program analysis (4). Checking termination and similar liveness properties is trivial with finite state systems, at least from a theoretical if not algorithmical point of view (e.g. finding loops in finite graphs). The same problem for infinite state systems with potentially infinite data structures (as considered e.g. in partial evaluation) requires the discovery of *variant functions* which is also very difficult in full generality and even more for fair concurrent systems (123).

Even when considering restricted simple abstract properties, the semantics of real-life programming languages is very complex (recursion, concurrency, modularity, etc.) whence so is the corresponding abstract interpreter. The abstraction

of this semantics hence the design of the analyzer is mostly manual (and beyond the hability of casual programmers or theorem provers) whence costly. The considered abstractions must have a large scope of application and must be easily reusable to be of economic interest.

From a user point of view, the results of the analysis have to be presented in a simple way (for example by pointing at errors only or by providing abstract counter-examples, or less frequently concrete ones). Experience shows that the cases of uncertainty represent 5 to 10 % of the possible cases. They must be handle with other empirical or formal methods (including more refined abstract interpretations).

3.3 Applications of Program Analysis

Among the numerous applications of program analysis, let us cite *Data flow analysis* (60); program optimization and transformation (including partial evaluation and program specialization (110) and data dependence analysis for the parallelisation of sequential languages); *set-based analysis* (56); *Type inference* (41) (including undecidable systems and soft typing); Verification of *reactive* (109), *real-time* (98) and *(linear) hybrid systems* (97) including state space reduction (57); cryptographic protocol analysis(129); *Abstract model-checking* of infinite systems (58; 60); *Abstract debugging*, testing and verification (38; 17); *Cache and pipeline behavior prediction* (72); *Probabilistic analysis* (130); *Communication topology analysis* for mobile/distributed code (74); *Automatic differentiation* of numerical programs (152); *Abstract simulation* of temporal specifications (26); Semantic tattooing/*watermarking* of software; etc.

Program static analysis has been intensively studied for grammars and polynomial systems (40), term graph rewriting (89), typesetting languages (107), procedural languages (16; 47) (for alias analysis (157), pointer analysis (68; 69), parameter boxing/unboxing (90), copy elimination (146), dependence analysis (121), exception analysis (145), constant propagation (115), (linear) equality or inequality relationskips analysis (61) etc.), parallel procedural languages (62; 88), functional languages (for binding time analysis (156), strictness analysis (23; 54; 132), inverse image analysis (70), projection analysis (22), comportment analysis (55), dependency analysis (12), path/trace analysis (34), closure analysis (135), control flow analysis (149), value flow analysis (14), compile-time garbage collection (108), stackability and escape analysis (10), data structures and abstract data type analysis (119), heap shape analysis (111; 151), exception analysis (158), polymorphic function analysis (3), kind/sort analysis (94), typing (41), effect systems (113), termination analysis (136), time complexity analysis (143), parallelization (154), etc.), functional parallel languages (63), data parallel languages (27), logic languages including Prolog (51; 66) (for mode (125) and type analysis (106) and their combination (18), finiteness analysis (8), relational argument size analysis (147), dependency analysis (131), detecting determinate/functional computations (83), mutually exclusive rules detection (140), occur check reduction (150), WAM code optimization (64), copy avoidance (78), groundness analysis (37), sharing analysis (35), freeness analysis

(32) and their combinations (33), termination analysis (114), time complexity and cost analysis (67), parallelisation (19), etc.) including its search rule and the cut (77) and database programming languages (1), concurrent logic languages (24), functional logic languages (99), constraint logic languages (9), concurrent constraint logic languages (159), specification languages (84), synchronous languages (96), concurrent/parallel languages (50), communicating and distributed languages (49; 126) and more recently object-oriented languages (11).

Abstract interpretation has been used (including interval analysis) for the static analysis of the embedded ADA software of the Ariane 5 launcher[1] (116). The static program analysis aims at the automatic detection of the *definiteness*, *potentiality*, *impossibility* or *inaccessibility* of run-time errors such as scalar and floating-point overflows, array index errors, divisions by zero and related arithmetic exceptions, uninitialized variables, data races on shared data structures, etc. The analyser was able to automatically discover the Ariane 501 flight error. This was a success for the later 502 and 503 flights and the ARD[2] (116) and in the verification of avionic software (142).

3.4 Industrialization of Static Analysis by Abstract Interpretation

The impressive results obtained by the static analysis of real-life embedded critical software (116; 142) is quite promising for the industrialization of abstract interpretation.

This is the explicit objective of AbsInt Angewandte Informatik GmbH created in Germany by R. Wilhelm and C. Ferdinand in 1998 commercializing the program analyser generator PAG and an application to determine the worst-case execution time for modern computer architectures with memory caches pipelines, etc (73).

Polyspace Technologies was created in France by A. Deutsch and D. Pilaud in 1999 to develop and commercialize ADA and C program analyzers.

Other companies like Connected Components Corporation created in the U.S.A. by W.L. Harrison in 1993 use abstract interpretation internally e.g. for compiler design (103).

4 Abstract Formal Methods

No automatic formal method can ultimately find all errors in a software system and so for their combinations. We will shortly review the automatic formal methods for computer-aided program verification, briefly discussing their principles, advantages and shortcomings. Since program analysis has already been discussed, we now consider typing, model-checking, deductive methods and their combination.

[1] Flight software (60,000 lines of Ada code) and Inertial Measurement Unit (30,000 lines of Ada code).

[2] Atmospheric Reentry Demonstrator: module coming back to earth.

4.1 Typing

Polymorphic typing and type inference (127) was a definite step in the design of programming languages and compilers (102). The question for the next decade seems to be to scale to more expressive properties.

Foundational Ideas of Typing Typing is based on decidable program analyses. This approach is always possible by restricting both on specifications (allowed types) and on programs, as shown when considering types as abstract interpretations (41). In theory, type systems have a clean presentation of the type analysis (inference algorithm (127)) through an equivalent logical formal system (type verification (65). Monomorphic typing (104) was extended to polymorphism (127), complex data structures, references (100), exceptions and separate modules (101) in a way that scales up for very large programs. It is nicely integrated in the compiler and the certification can go down to the generated code (proof-carrying code (133), certified compiler (153)).

Shortcomings of Typing Type systems (e.g. with subtle subtyping) can be very complex to understand for the casual user. One difficulty is that typing is compositional but not fully abstract (e.g. the same polymorphic code can type differently in different utilization contexts). The interaction with the user is often crude (no hint is given to understand why wrong programs do not type well). It is hardly possible for the user to provide hints to help the typing process. The logical specification of the type system is often inexistent in the reference manual, not equivalent to the type inference algorithm or so inextricable that it is useless both to the programmer and compiler designer. The programs considered in type theory are both complex (higher-order modules) and too restricted (mainly functional languages). The most severe restrictions are on the considered properties (arithmetic, out of range, null pointer dereferencing, ... errors are checked at run-time, all liveness properties are ignored). These restrictions and the difficulty to generalize to more expressive properties mainly follow from the encoding of types as terms/formulæ and from the one iterate fixpoint approximation.

4.2 Deductive Methods

Foundational Ideas of Deductive Methods Deductive methods use a (manually designed abstraction of) the program semantics to obtain minimal verification conditions to prove program correctness. These verification conditions can be derived from the program trace semantics by abstract interpretation (43). Then a theorem prover (134) or a proof assistant (137) is used to check the verification conditions.

Shortcomings of Deductive Methods Deductive methods use the schema of Fig. 13(b) but for the fact that the solver is replaced by a verifier or checker thus

avoiding fixpoint computations. So the constraints or equations corresponding to the verification conditions are not solved. This means that an inductive argument (e.g. invariant, variant function) has to be provided, generally by the user. Since the implication involved in the verification condition is itself undecidable, the proof verification can only be partially automatized, even though the solution to the equations/constraints is provided. Therefore interaction of the programmer with the prover is ultimately needed. This (wo)man/prover interaction is hard if not despairing, in particular because the size of the proof is often exponential in the program size. Therefore debugging an unsuccessful proof (because of a program error or a prover weakness) can be as complex as (if not much more complex than) debugging the program itself.

An alternative (118) consists in restricting the form of predicates considered by the prover, (which is an abstract interpretation (48, Sec. 5)). This can go up to unsound verification condition simplifications, essentially to make verifier simpler (e.g. modular arithmetic).

Because theorem provers are driven by unformalized heuristics, and these heuristics and their interactions are changed over time for improving proof strategies, theorem provers are often unstable over time (e.g. proof strategies get changed so that old proofs no longer work). Another weakness which makes interaction with other formal methods somewhat difficult is the uniform encoding of properties as syntactical terms/formulæ (so that e.g. BBDs are hardly efficiently encodable). It follows that the theorem prover has ultimately to be extented with program analysers, model checkers, typing, among others (148), often without supporting theory, in particular for mechanizing and combining abstractions.

4.3 Model Checking

Model checking (28; 141) has been very successful for the verification of hardware (7), communication protocoles (31), cryptographic protocoles (5), and real-time (25) or probabilistic (155) processes. As far as software systems are concerned, the question for the next decade is whether model checking can be extended to the verification of very large real-life programs.

Foundational Ideas of Model Checking First a model of the program (i.e. manually designed abstraction of the program semantics) must be designed (in the form of a transition system similar to a small step operational semantics). Then a specification of the program must be provided by the user in a very expressive temporal logic (139). A model checker can then check the specification by exhaustive search/symbolic exploration of the state space.

The spectacular success of model checking followed from the clever design of data structures (e.g. BDDs) and algorithms (e.g. minimal state graph generation (15), fixpoint computation (120) or SAT (6)) for representing very large sets of booleans and their transformations.

The approximation is that the model must be finite-state or some form of abstract interpretation must be used (30; 91) to reduce the verification problem

to finite state, including symmetries (29), etc. Also clever semantics of concurrent systems have been considered, e.g. to avoid the combinatorial explosion of interleaving (31).

Another trend in infinite-state model checking consider safety properties only and polyhedral abstractions, with variants (e.g. Presburger arithmetic (20)). This is a direct application of polyhedral program analysis (61), including the use of widenings. This allows e.g. for the analysis of reactive (75), real-time (95) and hybrid systems (96).

Shortcomings of Model Checking Although model checking gained a factor of 100 in 10 years, it is very difficult to scale up because of the state explosion problem. So, the necessary restriction to available computer resources often reduces the model checker from formal verification to debugging on part of the state space. Since the model must ultimately be finite (to allow for exhaustive search/symbolic exploration), abstraction is mandatory, which is a very difficult task to do manually and/or is left informal. Moreover, some forms of abstractions (such as interval (45) or polyhedral (61) abstractions) do not abstract concrete transition systems into abstract transition systems so that the model checker may not be reusable in the abstract. One can use abstraction for model checking which are complete in that there always exists a program specific abstraction into a finite model to prove a given specification correct (see (42) for safety properties) but none will be complete for all programs, even for simple properties as considered in program analysis (53). It follows that complete abstractions are difficult and not reusable hence not cost effective.

5 Combining Program Verification Methods

Since no single formal method can ultimately solve the verification problem, a current trend is to combine formal methods.

For example, one can rely on a user designed abstraction and derive a a program finite abtract model by abstract interpretation, prove the correctness of the abstraction by deductive methods and later verify the abstract model by model-checking (144).

A fundamental limitation (42) is that the abstraction discovery and the abstract semantics derivation are respectively logically equivalent hence practically as difficult as invariant discovery and invariant verification in a formal proof. So we have the feeling that combination of tools might simplify formal proofs but still will ultimately not solve the program verification problem.

6 Combining Empirical and Formal Methods

Formal methods have made a lot of progress in the last decade. Nevertheless there are few automatic light weight tools to apply them in practice. Integration of such tools is difficult and cannot ultimately solve all verification problems.

It follows that the only mechanical tool for verifying programs, which defaults and incompleteness are well known, is still *debugging*. There again progress was slow, in particular because theory never took debugging seriously. The main advantage of debugging is that a debugger is a light weight tool which is very easily understood by all programmers. Because of its well-known incredible cost for weak results, debugging may not scale up in the next decade for very large software.

An alternative which still remains to be investigated is the combination of informal methodslike debugging with verification tools. Let us consider for example *abstract testing* (59).

The classical debugging methodology consists in running the program on test data, checking if the execution satisfies informal specifications. This process is repeated by providing more tests until reaching a satisfactory coverage.

By an easily understandable analogy, the abstract testing methodology (59) consists in computing the abstract semantics for a finitary or infinitary abstraction chosen by the programmer among a predefined palette (not user defined, which would be too difficult). The abstract semantics is then checked against user-provided abstract assertions or the abstraction of a formal specification. This process is repeated with more refined abstractions until enough assertions are proved or no predefined abstraction can do.

Observe that one can prove the absence of (some categories of) bugs, not only their presence. Moreoever abstract evaluation can range from an analogy with program execution to the application of proof methods (using e.g. forward as well as backward reasonings providing abstract counter-examples) without attempting to make a one-shot complete formal proof of the specification.

7 Conclusions on the Past Decade

Full program verification by formal methods (e.g. model checking/deductive methods), which requires user interaction (for discovering an abstraction or inductive argument) is very costly in human resources hence is not likely to scale up for very large software. Abstraction is mandatory for program verification but difficult, hardly automatizable and beyond the common capabilities of most programmers.

Partial program verification by static analysis (with typing being considered as a particular and successfull case) is *cost-effective*[3] because no user intervention is mandatory for performing the analysis and universal abstractions are reusable hence commercializable.

For large and complex programs, complete verification by formal methods is not likely to be viable at low cost. Program debugging is still and will probably remain for some time the prominent industrial program "verification" method.

In this context, abstract interpretation based program static analysis can be extended to *abstract program testing*. Abstract interpretation based methods

[3] e.g. less than 0.25$ per program line costing 50 to 80$.

offer powerful techniques which, in the presence of approximation, can be viable alternatives or complements both to the exhaustive search of model-checking and to the partial exploration methods of classical debugging.

8 Grand Challenge for the Next Decade

We believe that in the next decade the software industry will certainly have to face its responsability imposed by a computer-dependent society. Consequently, SOFTWARE RELIABILITY[4] will be a grand challenge for computer science and practice.

The grand challenge for formal methods, in particular abstract interpretation based formal tools, is both the large scale industrialization and the intensification of the fundamental research effort.

General-purpose, expressive and cost-effective abstractions have to be developped e.g. to handle floating point numbers, data dependences (e.g. for parallelization), liveness properties with fairness (to extend finite-state model-checking to software), probabilistic properties, etc. Present-day tools will have to be enhanced to handle higher-order compositional modular analyses and to new programming paradigms (such as threads, mobile/network programming, etc.), to automatically combine and refine abstracts, to interact nicely with users and other formal or informal methods.

The most challenging objective might be to integrate formal analysis by abstract interpretation in the full software development process.

[4] other suggestions were “trustworthiness” (C. Jones) and “robustness” (R. Leino).

References

[1] G. Amato, F. Giannotti, and G. Mainetto. Data sharing analysis for a database programming language via abstract interpretation. In R. Agrawal, S. Baker, and D.A. Bell, editors, *Proc. 19th Int. Conf. VLDB '93*, Dublin, IE, pages 405–415. Morgan Kaufmann Pub., 24–27 Aug. 1993.

[2] V. Balasundaram and K. Kennedy. A technique for summarizing data access and its use in parallelism enhancing transformations. In *Proc. ACM SIGPLAN '89 Conf. PLDI. ACM SIGPLAN Not. 24(7)*, pages 41–53, Portland, OR, US, 21–23 June 1989.

[3] G. Baraki and R.J.M. Hughes. Abstract interpretation of polymorphic functions. In K. Davis and J. Hughes, editors, *Functional Programming, Glasgow 1989*, Proc. 1989 Glasgow Workshop, Fraserburgh, UK. Springer-Verlag and BCS, 31–40 Aug. 1989.

[4] S. Bensalem, M. Bozga, J.-C. Fernandez, L. Ghirvu, and L. Lakhnech. A transformational approach for generating non-linear invariants. In J. Palsberg, editor, *Proc. 7th Int. Symp. SAS '2000*, Santa Barbara, CA, US, LNCS 1824, pages 58–74. Springer-Verlag, 29 June – 1 Jul. 2000.

[5] A. Biere. μcke - efficient μ-calculus model checking. In O. Grumberg, editor, *Proc. 9th Int. Conf. CAV '97*, Haifa, IL,LNCS 1254, pages 468–471. Springer-Verlag, 22–25 Jul. 1997.

[6] A. Biere, A. Cimatti, E.M. Clarke, M. Fujita, and Y. Zhu. Symbolic model checking using SAT procedures instead of BDDs. In *Proc. 36th Conf. DAC '99*, New Orleans, LA, US, pages 317–320. ACM Press, 21–25 June 1999.

[7] A. Biere, E.M. Clarke, R. Raimi, and Y. Zhu. Properties of a power PC microprocessor using symbolic model checking without BDDs. In N. Halbwachs and D. Peled, editors, *Proc. 11th Int. Conf. CAV '99*, Trento, IT, LNCS 1633, pages 60–71. Springer-Verlag, 6–10 Jul. 1999.

[8] P.A. Bigot, S.K. Debray, and K. Marriott. Understanding finiteness analysis using abstract interpretation. In K.R. Apt, editor, *Proc. JICSLP '92*, Washington, DC, US, pages 735–749. MIT Press, Nov. 1992.

[9] S. Bistarelli, P. Codognet, and F. Rossi. An abstraction framework for soft constraints and its relationship with constraint propagation. In B.Y. Choueiry and T. Walsh, editors, *Proc. 4th Int. Symp. SARA '2000*, Horseshoe Bay, TX, US, LNAI 1864, pages 71–86. Springer-Verlag, 26–29 Jul. 2000.

[10] B. Blanchet. Escape analysis: Correctness proof, implementation and experimental results. In *25th POPL*, pages 25–37, San Diego, CA, US, 19–21 Jan. 1998. ACM Press.

[11] B. Blanchet. Escape analysis for object-oriented languages: Application to java. In *Proc. ACM SIGPLAN Conf. OOPSLA '99. ACM SIGPLAN Not. 34(10)*, pages 20–34, Denver, CO, US, 1–5 Nov. 1999.

[12] M. Blume. Dependency analysis for Standard ML. *TOPLAS*, 21(4):790–812, Jul. 1999.

[13] C. Bodei, P. Degano, and C. Priami. Constructing specific SOS semantics for concurrency via abstract interpretation. In G. Levi, editor, *Proc. 5th Int. Symp. SAS '98*, Pisa, IT, 14–16 Sep. 1998, LNCS 1503, pages 168–183. Springer-Verlag, 1998.

[14] R. Bod k and S. Anik. Path-sensitive value-flow analysis. In *25th POPL*, pages 237–251, San Diego, CA, US, 19–21 Jan. 1998. ACM Press.

[15] A. Bouajjani, J.-C. Fernandez, N. Halbwachs, P. Raymond, and C. Ratel. Minimal state graph generation. *Sci. Comput. Programming*, 18:247–269, 1992.

[16] F. Bourdoncle. Interprocedural abstract interpretation of block structured languages with nested procedures, aliasing and recursivity. In P. Deransart and J. Małuszyński, editors, *Proc. Int. Work. PLILP '90*, Linköping, SE, LNCS 456, pages 307–323. Springer-Verlag, 20–22 Aug. 1990.

[17] F. Bourdoncle. Abstract debugging of higher-order imperative languages. In *Proc. ACM SIGPLAN '93 Conf. PLDI. ACM SIGPLAN Not. 28(6)*, pages 46–55, Albuquerque, NM, US, 23–25 June 1993. ACM Press.

[18] M. Bruynooghe and G. Janssens. An instance of abstract interpretation integrating type and mode inferencing (extended abstract). In R. Kowalski and K. Bowen, editors, *Proc. 5th Int. Conf. & Symp. on Logic Programming, Volume 1*, Seattle, WA, US, pages 669–683. MIT Press, 15–19 Aug. 1988.

[19] F. Bueno, M.J. Garc a de la Banda, and M.V. Hermenegildo. Effectiveness of abstract interpretation in automatic parallelization: A case study in logic programming. *TOPLAS*, 21(2):189–239, Mar. 1999.

[20] T. Bultan, R. Gerber, and W. Pugh. Symbolic model checking of infinite state systems using presburger arithmetic. In O. Grumberg, editor, *Proc. 9th Int. Conf. CAV '97*, Haifa, IL,LNCS 1254, pages 400–411. Springer-Verlag, 22–25 Jul. 1997.

[21] A. Bundy, F. Giunchiglia, R. Sebastiani, and T. Walsh. Computing abstraction hierarchies by numerical simulation. In *Proc. 30th Nat. Conf. AAAI '96*, pages 523–529, Portland, OR, US, 4–8 Aug. 1996. AAAI Press / MIT Press. ftp://ftp.mrg.dist.unige.it/pub/mrg-ftp/9604-01.ps.gz, http://aaai.org/Press/Proceedings/AAAI/1996/aaai96-contents.html.

[22] G.L. Burn. A relationship between abstract interpretation and projection analysis (extended abstract). In *17th POPL*, pages 151–156, San Francisco, CA, 1990. ACM Press.

[23] G.L. Burn, C.L. Hankin, and S. Abramsky. Strictness analysis of higher-order functions. *Sci. Comput. Programming*, 7:249–278, Nov. 1986.

[24] M.-M. Corsini C. Codognet, P. Codognet. Abstract interpretation for concurrent logic languages. In S.K. Debray and M.V. Hermenegildo, editors, *NACLP 1997*, Austin, TX, US, pages 215–232. MIT Press, 29 Oct. – 1 Nov. 1990.

[25] S.V.A. Campos, E.M. Clarke, and M. Minea. The Verus tool: A quantitative approach to the formal verification of real-time systems. In O. Grumberg, editor, *Proc. 9th Int. Conf. CAV '97*, Haifa, IL,LNCS 1254, pages 452–455. Springer-Verlag, 22–25 Jul. 1997.

[26] D. Cansell and D. Méry. Abstract animator for temporal specifications: Application to TLA. In A. Cortesi and G. Filé, editors, *Proc. 6th Int. Symp. SAS '99*, Venice, IT, 22–24 Sep. 1999, LNCS 1694, pages 284–299. Springer-Verlag, 1999.

[27] S. Chatterjee, B.E. Blelloch, and A.L. Fisher. Size and access inference for data-parallel programs. In *Proc. ACM SIGPLAN '91 Conf. PLDI. ACM SIGPLAN Not. 26(6)*, pages 130–144, Toronto, Ontario, CA, 26–28 June 1991.

[28] E.M. Clarke and E.A. Emerson. Synthesis of synchronization skeletons for branching time temporal logic. In *IBM Workshop on Logics of Programs*, Yorktown Heights, NY, US, LNCS 131. Springer-Verlag, May 1981.

[29] E.M. Clarke, E.A. Emerson, S. Jha, and A.P. Sistla. Symmetry reductions in model checking. In A.J. Hu and M.Y. Vardi, editors, *Proc. 10th Int. Conf. CAV '98*, Vancouver, BC, CA,LNCS 1427, pages 147–158. Springer-Verlag, 28 June – 2 Jul. 1998.

[30] E.M. Clarke, O. Grumberg, and D.E. Long. Model checking and abstraction. In *19th POPL*, pages 343–354, Albuquerque, NM, 1992. ACM Press.

[31] E.M. Clarke, S. Jha, and W.R. Marrero. Partial order reductions for security protocol verification. In S. Graf and M.I. Schwartzbach, editors, *Proc. 6th Int. Conf. TACAS '2000*, Berlin, DE, 25 Mar. – 2 Apr. 2000, LNCS 1785, pages 503–518. Springer-Verlag, 2000.

[32] M. Codish, D. Dams, G. Fil , and M. Bruynooghe. Freeness analysis for logic programs – and correctness? In D.S. Warren, editor, *Proc. 10th ICLP '93*, Budapest, HU, pages 116–131. MITpress, 21–25 June 1993.

[33] M. Codish, H. Søndergaard, and P.J. Stuckey. Sharing and groundness dependencies in logic programs. *TOPLAS*, 21(5):948–976, Sep. 1999.

[34] C. Colby and P. Lee. Trace-based program analysis. In *23rd POPL*, pages 195–207, St. Petersburg Beach, FL, 1996. ACM Press.

[35] A. Cortesi and G. Filé. Sharing is optimal. *J. Logic Programming*, 38(3):371–386, 1999.

[36] A. Cortesi, G. Filé, R. Giacobazzi, C. Palamidessi, and F. Ranzato. Complementation in abstract interpretation. *TOPLAS*, 19(1):7–47, Jan. 1997.

[37] A. Cortesi, G. Filé, and W.H. Winsborough. Optimal groundness analysis using propositional logic. *J. Logic Programming*, 27(2):137–167, 1996.

[38] P. Cousot. Semantic foundations of program analysis. In S.S. Muchnick and N.D. Jones, editors, *Program Flow Analysis: Theory and Applications*, chapter 10, pages 303–342. Prentice-Hall, 1981.

[39] P. Cousot. Methods and logics for proving programs. In J. van Leeuwen, editor, *Formal Models and Semantics*, volume B of *Handbook of Theoretical Computer Science*, chapter 15, pages 843–993. Elsevier, 1990.

[40] P. Cousot. Constructive design of a hierarchy of semantics of a transition system by abstract interpretation. *ENTCS*, 6, 1997. URL: http://www.elsevier.nl/locate/entcs/volume6.html, 25 pages.

[41] P. Cousot. Types as abstract interpretations, invited paper. In *24th POPL*, pages 316–331, Paris, FR, Jan. 1997. ACM Press.

[42] P. Cousot. Partial completeness of abstract fixpoint checking, invited paper. In B.Y. Choueiry and T. Walsh, editors, *Proc. 4th Int. Symp. SARA '2000*, Horseshoe Bay, TX, US, LNAI 1864, pages 1–25. Springer-Verlag, 26–29 Jul. 2000.

[43] P. Cousot. Constructive design of a hierarchy of semantics of a transition system by abstract interpretation. *Theoret. Comput. Sci.*, To appear (Preliminary version in (40)).

[44] P. Cousot and R. Cousot. Static determination of dynamic properties of programs. In *Proc. 2nd Int. Symp. on Programming*, pages 106–130. Dunod, 1976.

[45] P. Cousot and R. Cousot. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In *4th POPL*, pages 238–252, Los Angeles, CA, 1977. ACM Press.

[46] P. Cousot and R. Cousot. Automatic synthesis of optimal invariant assertions: mathematical foundations. In *ACM Symposium on Artificial Intelligence & Programming Languages*, Rochester, NY, ACM SIGPLAN Not. 12(8):1–12, 1977.

[47] P. Cousot and R. Cousot. Static determination of dynamic properties of recursive procedures. In E.J. Neuhold, editor, *IFIP Conf. on Formal Description of Programming Concepts, St-Andrews, N.B., CA*, pages 237–277. North-Holland, 1977.

[48] P. Cousot and R. Cousot. Systematic design of program analysis frameworks. In *6th POPL*, pages 269–282, San Antonio, TX, 1979. ACM Press.

[49] P. Cousot and R. Cousot. Semantic analysis of communicating sequential processes. In J.W. de Bakker and J. van Leeuwen, editors, *7th ICALP*, LNCS 85, pages 119–133. Springer-Verlag, Jul. 1980.

[50] P. Cousot and R. Cousot. Invariance proof methods and analysis techniques for parallel programs. In A.W. Biermann, G. Guiho, and Y. Kodratoff, editors, *Automatic Program Construction Techniques*, chapter 12, pages 243–271. Macmillan, 1984.

[51] P. Cousot and R. Cousot. Abstract interpretation and application to logic programs. *J. Logic Programming*, 13(2–3):103–179, 1992. (The editor of J. Logic Programming has mistakenly published the unreadable galley proof. For a correct version of this paper, see http://www.di.ens.fr/~cousot.).

[52] P. Cousot and R. Cousot. Abstract interpretation frameworks. *J. Logic and Comp.*, 2(4):511–547, Aug. 1992.

[53] P. Cousot and R. Cousot. Comparing the Galois connection and widening/narrowing approaches to abstract interpretation, invited paper. In M. Bruynooghe and M. Wirsing, editors, *Proc. 4th Int. Symp. PLILP '92*, Leuven, BE, 26–28 Aug. 1992, LNCS 631, pages 269–295. Springer-Verlag, 1992.

[54] P. Cousot and R. Cousot. Galois connection based abstract interpretations for strictness analysis, invited paper. In D. Bjørner, M. Broy, and I.V. Pottosin, editors, *Proc. FMPA*, Akademgorodok, Novosibirsk, RU, LNCS 735, pages 98–127. Springer-Verlag, 28 June – 2 Jul. 1993.

[55] P. Cousot and R. Cousot. Higher-order abstract interpretation (and application to comportment analysis generalizing strictness, termination, projection and PER analysis of functional languages), invited paper. In *Proc. 1994 ICCL*, pages 95–112, Toulouse, FR, 16–19 May 1994. IEEE Comp. Soc. Press.

[56] P. Cousot and R. Cousot. Formal language, grammar and set-constraint-based program analysis by abstract interpretation. In *Proc. 7th FPCA*, pages 170–181, La Jolla, CA, 25–28 June 1995. ACM Press.

[57] P. Cousot and R. Cousot. Parallel combination of abstract interpretation and model-based automatic analysis of software. In R. Cleaveland and D. Jackson, editors, *Proc. 1st ACM SIGPLAN Workshop on Automatic Analysis of Software, AAS '97*, pages 91–98, Paris, FR, Jan. 1997. ACM Press.

[58] P. Cousot and R. Cousot. Refining model checking by abstract interpretation. *Aut . Soft . Eng.*, 6:69–95, 1999.

[59] P. Cousot and R. Cousot. Abstract interpretation based program testing. In *Proc. SSGRR 2000 Computer & eBusiness International Conference*, Compact disk paper 248, L'Aquila, Italy, 31 Jul. – 6 Aug. 2000. Scuola Superiore G. Reiss Romoli.

[60] P. Cousot and R. Cousot. Temporal abstract interpretation. In *27th POPL*, pages 12–25, Boston, MA, Jan. 2000. ACM Press.

[61] P. Cousot and N. Halbwachs. Automatic discovery of linear restraints among variables of a program. In *5th POPL*, pages 84–97, Tucson, AZ, 1978. ACM Press.

[62] R. Cridlig. Semantic analysis of shared-memory concurrent languages using abstract model-checking. In *Proc. PEPM '95*, La Jolla, CA, 21–23 June 1995. ACM Press.

[63] R. Cridlig and É. Goubault. Semantics and analysis of Linda-based languages. In P. Cousot, M. Falaschi, G. Filé, and A. Rauzy, editors, *Proc. 3rd Int. Work. WSA '93*, Padova, IT, LNCS 724, pages 72–86. Springer-Verlag, 22–24 Sep. 1993.

[64] G. Filé D. Baldan. Abstract interpretation from improving WAM code. In P. Van Hentenryck, editor, *Proc. 4th Int. Symp. SAS '97*, Paris, FR, 8–10 Sep. 1997, LNCS 1302, page 364. Springer-Verlag, 1997.

[65] L. Damas and R. Milner. Principal type-schemes for functional programs. In *9th POPL*, pages 207–212, Albuquerque, NM, Jan. 1982. ACM Press.

[66] S.K. Debray. Formal bases for dataflow analysis of logic programs. In G. Levi, editor, *Advances in Logic Programming Theory*, Int. Schools for Computer Scientists, section 3, pages 115–182. Clarendon Press, 1994.

[67] S.K. Debray, P. López-Garc a, M.V. Hermenegildo, and N.-W. Lin. Lower bound cost estimation for logic programs. In J. Małuszyński, editor, *Proc. Int. Symp. ILPS '1997*, Port Jefferson, Long Island, NY, US, pages 291–305. MIT Press, 13–16 Oct. 1997.

[68] A. Deutsch. Semantic models and abstract interpretation techniques for inductive data structures and pointers, invited paper. In *Proc. PEPM '95*, pages 226–229, La Jolla, CA, 21–23 June 1995. ACM Press.

[69] N. Dor, M. Rodeh, and M. Sagiv. Checking cleanness in linked lists. In J. Palsberg, editor, *Proc. 7th Int. Symp. SAS '2000*, Santa Barbara, CA, US, LNCS 1824, pages 115–134. Springer-Verlag, 29 June – 1 Jul. 2000.

[70] P. Dybjer. Inverse image analysis generalises strictness analysis. *Inform. and Comput.*, 90:194–216, 1991.

[71] C. Ferdinand. *Generating Program Analyzers*. Verfasser – Pirrot Verlag, Saarbrücken, DE, 1999.

[72] C. Ferdinand, F. Martin, R. Wilhelm, and M. Alt. Cache behavior prediction by abstract interpretation. *Sci. Comput. Programming*, 35(1):163–189, 1999.

[73] C. Ferdinand, F. Martin, R. Wilhelm, and M. Alt. Cache behavior prediction by abstract interpretation. *Sci. Comput. Programming, Special Issue on SAS'96*, 35(1):163–189, September 1999.

[74] J. Feret. Confidentiality analysis of mobile systems. In J. Palsberg, editor, *Proc. 7th Int. Symp. SAS '2000*, Santa Barbara, CA, US, LNCS 1824, pages 135–154. SPRINGER, 29 June – 1 Jul. 2000.

[75] J.-C. Fernandez. Abstract interpretation and verification of reactive systems. In P. Cousot, P. Falaschi, G. Filé, and A. Rauzy, editors, *Proc. 3rd Int. Work. WSA '93*, Padova, IT, LNCS 724, pages 60–71. Springer-Verlag, 22–24 Sep. 1993.

[76] G. Filé and . Ranzato. The powerset operator on abstract interpretations. *Theoret. Comput. Sci.*, 222(1-2):77–111, Jul. 1999.

[77] G. Filé and S. Rossi. Static analysis of Prolog with cut. In A. Voronkov, editor, *Proc. 4th Int. Conf. LPAR '93*, pages 134–145, St. Petersburg, RU, LNCS 698, 13–20 Jul. 1993. Springer-Verlag.

[78] I.T. Foster and W.H. Winsborough. Copy avoidance through compile-time analysis and local reuse. In K. Ueda V.A. Saraswat, editor, *Proc. 1991 Int. Symp. ISLP '91*, San Diego, CA, US, pages 455–469. MIT Press, 28 Oct. – 1 Nov. 1997.

[79] R. Giacobazzi. "optimal" collecting semantics for analysis in a hierarchy of logic program semantics. In C. Puech and R. Reischuk, editors, *Proc. Annual Symp. STACS '96*, LNCS 1046, pages 503–514. Springer-Verlag, 1996.

[80] R. Giacobazzi. A tutorial on domain theory in abstract interpretation. In G. Levi, editor, *Proc. 5th Int. Symp. SAS '98*, Pisa, IT, 14–16 Sep. 1998, LNCS 1503, pages 349–350. Springer-Verlag, 1998.

[81] R. Giacobazzi and F. Ranzato. Refining and compressing abstract domains. In P. Degano, R. Gorrieri, and A. Marchetti-Spaccamela, editors, *Proc. 24th Int. Coll. ICALP '97*, volume 1256 of *LNCS*, pages 771–781. Springer-Verlag, 1997.

[82] R. Giacobazzi, F. Ranzato, and F. Scozzari. Making abstract intrepretations complete. *J. ACM*, 2000. To appear.

[83] R. Giacobazzi and L. Ricci. Detecting determinate computations by bottom-up abstract interpretation. In B. Krieg-Brückner, editor, *Proc. 4th ESOP '92*, Rennes, FR, LNCS 582, pages 167–181. Springer-Verlag, 26–28 Feb. 1992.

[84] F. Giannotti and D. Latella. Gate splitting in LOTOS specifications using abstract interpretation. *Sci. Comput. Programming*, 23((2-3)):127–149, 1994.

[85] F. Giunchiglia and A. Villafiorita. ABSFOL: a proof checker with abstraction. In M.A. McRobbie and J.K. Slaney, editors, *Proc. 30th Int. Conf. CADE '96*, volume 1104 of *New Brunswick, NJ, US, LNAI*, pages 136–140. Springer-Verlag, Jul. 30–Aug. 3 1996. ftp://ftp.mrg.dist.unige.it/pub/mrg-ftp/9602-11.ps.gz.

[86] F. Giunchiglia and T. Walsh. Abstract theorem proving. In N.S. Sridharan, editor, *Proc. 11th IJCAI '89*, pages 372–377, Detroit, MI, US, Aug. 1989. Morgan Kaufmann Pub. ftp://ftp.mrg.dist.unige.it/pub/mrg-ftp/8902-03.ps.gz.

[87] F. Giunchiglia and T. Walsh. A theory of abstraction. *Art. Int.*, 56(2–3):323–390, Oct. 1992. ftp://ftp.mrg.dist.unige.it/pub/mrg-ftp/9001-14.ps.gz.

[88] É. Goubault. Schedulers as abstract interpretations of higher-dimensional automata. In *Proc. PEPM '95*, La Jolla, CA, pages 134–145. ACM Press, 21–23 June 1995.

[89] É. Goubault and C. Hankin. A lattice for the abstract interpretation of term graph rewriting systems. In R. Sleep, R. Plasmeijer, and M. van

Eekelen, editors, *Term Graph Rewriting: Theory and Practice*, chapter 10, pages 131–140. Wiley & S., 1993.

[90] J. Goubault. Generalized boxings, congruences and partial inlining. In B. Le Charlier, editor, *Proc. 1st Int. Symp. SAS '94*, Namur, BE, 20–22 Sep. 1994, LNCS 864, pages 147–161. Springer-Verlag, 1994.

[91] S. Graf and C. Loiseaux. A tool for symbolic program verification and abstraction. In C. Courcoubetis, editor, *Proc. 5th Int. Conf. CAV '93*, Elounda, GR, LNCS 697, pages 71–84. Springer-Verlag, 28 June –1 Jul. 1993.

[92] P. Granger. Static analysis of arithmetical congruences. *Int. J. Comput. Math.*, 30:165–190, 1989.

[93] P. Granger. Static analysis of linear congruence equalities among variables of a program. In S. Abramsky and T.S.E. Maibaum, editors, *Proc. Int. J. Conf. TAPSOFT '91, Volume 1 (CAAP '91)*, Brighton, GB, LNCS 493, pages 169–192. Springer-Verlag, 1991.

[94] C.A. Gunter, E.L. Gunter, and D.B. MacQueen. Computing ML equality kinds using abstract interpretation. *Inform. and Comput.*, 107(2):303–323, Dec. 1993.

[95] N. Halbwachs. Delays analysis in synchronous programs. In C. Courcoubatis, editor, *Proc. 5th Int. Conf. CAV '93*, Elounda, GR, LNCS 697, pages 333–346. Springer-Verlag, 28 June –1 Jul. 1993.

[96] N. Halbwachs. About synchronous programming and abstract interpretation. *Sci. Comput. Programming*, 31(1):75–89, May 1998.

[97] N. Halbwachs, J.-É. Proy, and P. Raymond. Verification of linear hybrid systems by means of convex approximations. In B. Le Charlier, editor, *Proc. 1st Int. Symp. SAS '94*, Namur, BE, 20–22 Sep. 1994, LNCS 864, pages 223–237. Springer-Verlag, 1994.

[98] N. Halbwachs, Y.E. Proy, and P. Roumanoff. Verification of real-time systems using linear relation analysis. *Formal Methods in System Design*, 11(2):157–185, Aug. 1997.

[99] M. Hanus. Towards the global optimization of functional logic programs. In P.A. Fritzson, editor, *Proc. 5th Int. Conf. CC '94*, Edinburg, UK, LNCS 786, pages 68–82. Springer-Verlag, Apr. 1994.

[100] R. Harper. A simplified account of polymorphic references. *Inf. Process. Lett.*, 54(4):201–206, 1994.

[101] R. Harper, R. Milner, and M. Tofte. A type discipline for program modules. In H. Ehrig, R. Kowalski, G. Levi, and U. Montanari, editors, *Proc. Int. J. Conf. TAPSOFT '87, Volume 2 (AFISD/CFLP)*, Pisa, IT, LNCS 250, pages 308–319. Springer-Verlag, 23–27 Mar. 1987.

[102] R. Harper and J.C. Mitchell. On the type structure of Standard ML. *TOPLAS*, 15(2):211–252, 1993.

[103] W.L. Harrison. Can abstract interpretation become a main stream compiler technology? (abstract). In P. Van Hentenryck, editor, *Proc. 4th Int. Symp. SAS '97*, Paris, FR, 8–10 Sep. 1997, LNCS 1302, page 395. Springer-Verlag, 1997.

[104] R. Hindley. The principal type-scheme of an object in combinatory logic. *Trans. Amer. Math. Soc.*, 146:29–60, 1969.

[105] C.A.R. Hoare and P.E. Lauer. Consistent and complementary formal theories of the semantics of programming languages. *Acta Informat.*, 3(2):135–153, 1974.

[106] K. Horiuchi and T. Kanamori. Polymorphic type inference in Prolog by abstract interpretation. In K. Furukawa, H. Tanaka, and T. Fujisaki, editors, *Proc. 6th Conf. on Logic Programming '87*, Tokyo, JP, LNCS 315, pages 195–214. Springer-Verlag, June 1987.

[107] N.R. Horspool and J. Vitek. Static analysis of PostScript code. *Comput. Lang.*, 19(2):65–78, 1993.

[108] S. Hughes. Compile-time garbage collection for higher-order functional languages. *J. Logic and Comp.*, 2(4):483–464, Aug. 1992.

[109] B. Jeannet, N. Halbwachs, and P. Raymond. Dynamic partitioning in analyses of numerical properties. In A. Cortesi and G. Filé, editors, *Proc. 6th Int. Symp. SAS '99*, Venice, IT, 22–24 Sep. 1999, LNCS 1694, pages 18–38. Springer-Verlag, 1999.

[110] N.D. Jones. Combining abstract interpretation and partial evaluation (brief overview). In P. Van Hentenryck, editor, *Proc. 4th Int. Symp. SAS '97*, Paris, FR, 8–10 Sep. 1997, LNCS 1302, pages 396–405. Springer-Verlag, 1997.

[111] N.D. Jones and S.S. Muchnich. Flow analysis and optimization of LISP-like structures. In *6th POPL*, pages 244–256, San Antonio, TX, 1979. ACM Press.

[112] N.D. Jones and S.S. Muchnick. Complexity of flow analysis, inductive assertion synthesis and a language due to Dijkstra. In S.S. Muchnick and N.D. Jones, editors, *Program Flow Analysis: Theory and Applications*, chapter 12, pages 380–393. Prentice-Hall, 1981.

[113] P. Jouvelot and D.K. Gifford. Algebraic reconstruction of types and effects. In *18th POPL*, pages 303–310, Orlando, FL, 1991. ACM Press.

[114] T. Kanamori, K. Horiuchi, and T. Kawamura. Detecting termination of logic programs based on abstract hybrid interpretation. Tech. rep. 398, ICOT, Tokyo, JP, 1987.

[115] G. Kildall. A unified approach to global program optimization. In *1st POPL*, pages 194–206, Boston, MA, Oct. 1973. ACMpress.

[116] P. Lacan, J.N. Monfort, L.V.Q. Ribal, A. Deutsch, and G. Gonthier. The software reliability verification process: The Ariane 5 example. In *Proceedings DASIA 98 – DAta Systems In Aerospace*, Athens, GR. ESA Publications, SP-422, 25–28 May 1998.

[117] B. Le Charlier and P. Van Hentenryck. Experimental evaluation of a generic abstract interpretation algorithm for Prolog. In *Proc. 1992 ICCL*, Oakland, CA, pages 137–146. IEEE Comp. Soc. Press, 20–23 Apr. 1992.

[118] K.R.M. Leino and G. Nelson. An extended static checker for Modula-3. In K. Koskimies, editor, *Proc. 7th Int. Conf. CC '98*, Lisbon, PT, LNCS 1383, pages 302–305. Springer-Verlag, 28 Mar. – 4 Apr. 1998.

[119] Y.A. Liu and S.D. Stroller. Eliminating dead code on recursive data. In A. Cortesi and G. Filé, editors, *Proc. 6th Int. Symp. SAS '99*, Venice, IT, 22–24 Sep. 1999, LNCS 1694, pages 179–193. Springer-Verlag, 1999.

[120] D.E. Long, A. Browne, E.M. Clarke, S. Jha, and W.R. Marrero. An improved algorithm for the evaluation of fixpoint expressions. *Theoret. Comput. Sci.*, 178(1-2):237–255, 1997.

[121] F. Masdupuy. Using abstract interpretation to detect array data dependencies. In *Proc. Int. Symp. on Supercomputing*, pages 19–27, Fukuoka, JP, Nov. 1991. Kyushu U. Press.

[122] F. Masdupuy. Semantic analysis of interval congruences. In D. Bjørner, M. Broy, and I.V. Pottosin, editors, *Proc. FMPA*, Akademgorodok, Novosibirsk, RU, LNCS 735, pages 142–155. Springer-Verlag, 28 June – 2 Jul. 1993.

[123] L. Mauborgne. Tree schemata and fair termination. In J. Palsberg, editor, *Proc. 7th Int. Symp. SAS '2000*, Santa Barbara, CA, US, LNCS 1824, pages 302–321. Springer-Verlag, 29 June – 1 Jul. 2000.

[124] L. Mauborgne. Improving the representation of infinite trees to deal with sets of trees. In G. Smolka, editor, *Programming Languages and Systems, Proc. 9th ESOP '2000*, Berlin, DE, LNCS 1782, pages 275–289. Springer-Verlag, Mar. – Apr. 2000.

[125] C.S. Mellish. Abstract interpretation of Prolog programs. In E. Shapiro, editor, *3rd ICLP '86*, London, GB, LNCS 225, pages 463–474. Springer-Verlag, 14–18 Jul. 1986.

[126] N. Mercouroff. An algorithm for analyzing communicating processes. In S. Brookes, M. Main, A. Melton, M. Mislove, and D. Schmidt, editors, *Proc. 7th Int. Conf. on Mathematical Foundations of Programming Semantics*, Pittsburgh, PA, pages 312–325. Springer-Verlag, 25–28 Mar. 1991.

[127] R. Milner. A theory of polymorphism in programming. *J. Comput. System Sci.*, 17(3):348–375, Dec. 1978.

[128] M. Minasi. *The Software Conspiracy: What You Don't Know About the Software Industry and How It's Taking Control of Your Life*. McGraw-Hill, 1999.

[129] D. Monniaux. Abstracting cryptographic protocols with tree automata. In A. Cortesi and G. Filé, editors, *Proc. 6th Int. Symp. SAS '99*, Venice, IT, 22–24 Sep. 1999, LNCS 1694, pages 149–163. Springer-Verlag, 1999.

[130] D. Monniaux. Abstract interpretation of probabilistic semantics. In J. Palsberg, editor, *Proc. 7th Int. Symp. SAS '2000*, Santa Barbara, CA, US, LNCS 1824, pages 322–339. Springer-Verlag, 29 June – 1 Jul. 2000.

[131] K. Muthukumar and M. Hermenegildo. Compile-time derivation of variable dependency using abstract interpretation. *J. Logic Programming*, 13(2–3):315–347, Jul. 1992.

[132] A. Mycroft. *Abstract Interpretation and Optimising Transformations for Applicative Programs*. Ph.D. Dissertation, CST-15-81, Department of Computer Science, University of Edinburgh, Edinburg, UK, Dec. 1981.

[133] G.C. Necula. Proof-carrying code. In *24th POPL*, pages 106–119, Paris, FR, Jan. 1997. ACM Press.

[134] S. Owre, N. Shankar, and D.W.J. Stringer-Calvert. PVS: An experience report. In D. Hutter, W. Stephan, P. Traverso, and M. Ullmann, editors, *PROC Applied Formal Methods - FM-Trends'98, International Workshop on Current Trends in Applied Formal Method*, Boppard, DE, LNCS 1641, pages 338–345. Springer-Verlag, 7–9 Oct. 1999.

[135] J. Palsberg. Closure analysis in constraint form. *TOPLAS*, 17(1):47–62, Jan. 1995.

[136] S.E. Panitz and M. Schmidt-Schauß. TEA: Automatically proving termination of programs in a non-strict higher-order functional language. In P. Van Hentenryck, editor, *Proc. 4th Int. Symp. SAS '97*, Paris, FR, 8–10 Sep. 1997, LNCS 1302, pages 345–360. Springer-Verlag, 1997.

[137] C. Paulin-Mohring and B. Werner. Synthesis of ML programs in the system Coq. *J. Symbolic Logic*, 15(5/6):607–640, 1993.

[138] G.D. Plotkin. A structural approach to operational semantics. Technical Report DAIMI FN-19, Aarhus University, DK, Sep. 1981.

[139] A. Pnueli. The temporal logic of programs. In *Proc. 18th FOCS*, pages 46–57, Providence, RI, Nov. 1977.

[140] K. Post. Mutually exclusive rules in logic programming. In M. Bruynooghe, editor, *Proc. Int. Symp. ILPS '1994*, Ithaca, NY, US, pages 472–486. MIT Press, 13–17 Nov. 1994.

[141] J.-P. Queille and J. Sifakis. Verification of concurrent systems in Cesar. In *Proc. Int. Symp. on Programming*, LNCS 137, pages 337–351. Springer-Verlag, 1982.

[142] F. Randimbivololona, J. Souyris, and A. Deutsch. Improving avionics software verification cost-effectiveness Abstract interpretation based technology contribution. In *Proceedings DASIA 2000 – DAta Systems In Aerospace*, Montreal, CA. ESA Publications, 22–26 May 2000.

[143] B. Reistad and D.K. Gifford. Static dependent costs for estimating execution time. In *Proc. ACM Conf. Lisp & Func. Prog.*, Orlando, FL, US, pages 65–78. ACM Press, 27–29 June 1994.

[144] S. Sa di. Model checking guided abstraction and analysis. In J. Palsberg, editor, *Proc. 7th Int. Symp. SAS '2000*, Santa Barbara, CA, US, LNCS 1824, pages 377–396. Springer-Verlag, 29 June – 1 Jul. 2000.

[145] C.F. Schaefer and G.N. Bundy. Static analysis of exception handling in Ada. *Soft.–Pract. & Exp.*, 23(10):1157–1174, Oct. 1993.

[146] P. Schnorf, M. Ganapathi, and J.L. Hennessy. Compile-time copy elimination. *Soft.–Pract. & Exp.*, 23(11):1175–1200, Nov. 1993.

[147] D. De Schreye and K. Verschaetse. Deriving linear size relations for logic programs by abstract interpretation. *New Gen. Comp.*, 13(2):117–154, 1995.

[148] N. Shankar. Unifying verification paradigms. In *FTRTFT'96*, 1996.

[149] O. Shivers. The semantics of scheme control-flow analysis. In P. Hudak and N.D. Jones, editors, *Proc. PEPM '91*, Yale U., New Haven, CT, US, 17–19 June 1991, ACM SIGPLAN Not. 26(9), pages 190–198. ACM Press, Sep. 1991.

[150] H. Søndergaard. An application of abstract interpretation of logic programs: Occur check reduction. In B. Robinet and R. Wilhelm, editors, *Proc. ESOP '86*, Saarbrücken, DE, 17-19 Mar. 1986, LNCS 213, pages 327–338. Springer-Verlag, 1986.

[151] J. Stransky. A lattice for abstract interpretation of dynamic (LISP-like) structures. *Inform. and Comput.*, 101(1):70–102, Nov. 1992.

[152] M. Tadjouddine, F. Eyssette, and C. Faure. Sparse jacobian computation in automatic differentiation by static program analysis. In G. Levi, editor, *Proc. 5th Int. Symp. SAS '98*, Pisa, IT, 14–16 Sep. 1998, LNCS 1503, pages 311–326. Springer-Verlag, 1998.

[153] D. Tarditi, J.G. Morrisett, P. Cheng, R. Harper, and P. Lee. TIL: A type-directed optimizing compiler for ML. In *Proc. ACM SIGPLAN '96 Conf. PLDI. ACM SIGPLAN Not. 31(5)*, pages 181–192, Philadephia, PA, US, 21–24 May 1996.

[154] K.R. Traub, D.E. Culler, and K.E. Schauser. Global analysis for partitioning non-strict programs into sequential threads. *LISP Pointers*, 5(1):324–334, Jan. – Mar. 1992.

[155] M.Y. Vardi. Probabilistic linear-time model checking: An overview of the automata-theoretic approach. In J.-P. Katoen, editor, *Formal Methods for Real-Time and Probabilistic Systems, 5th Int. Symp. AMAST Workshop, ARTS '99*, Bamberg, DE, 26–28 May 1999, LNCS 1601, pages 265–276. Springer-Verlag, 1993.

[156] F. Védrine. Binding-time analysis and strictness analysis by abstract interpretation. In A. Mycroft, editor, *Proc. 2nd Int. Symp. SAS '95*, Glasgow, UK, 25–27 Sep. 1995, LNCS 983, pages 400–417. Springer-Verlag, 1995.

[157] A. Venet. Automatic analysis of pointer aliasing for untyped programs. *Sci. Comput. Programming, Special Issue on SAS'96*, 35(1):223–248, September 1999.

[158] Kwangkeun Yi. An abstract interpretation for estimating uncaught exceptions in standard ML programs. *Sci. Comput. Programming*, 31(1):147–173, May 1998.

[159] E. Zaffanella, R. Giacobazzi, and G. Levi. Abstracting synchronization in concurrent constraint programming. In M.V. Hermenegildo and J. Penjam, editors, *Proc. 6th Int. Symp. PLILP '94*, Madrid, ES, 14–16 Sep. 1994, LNCS 844, pages 57–72. Springer-Verlag, 1994.