CSC 220 Algorithms

Midterm Test B, November 4, 2002

1. Suppose $f(1) = 0$ and for all $n \geq 2$,

$$f(n) = 3\sum_{i=1}^{n-1} f(i) + 10.$$

What is $f(n)$?

2. Prove that if there in an algorithm for finding the *second smallest* element in a given set of distinct numbers using at most 100 comparisons, then one can also find in the same set simultaneously the *smallest* and the *second smallest* elements using at most 100 comparisons.

3. We have 10 coins that look the same. 8 of them have weight 1, one has weight .99 and one has weight 0.98.

a. Is it always possible to determine the weights of the coins by a *digital scale* using fewer than 4 measurements?

b. Design an algorithm for identifying the 8 good (heavy) coins by a two-pan *balance*, using at most 5 measurements.

4. Using the method of *MERGESORT*, put in increasing order the sequence

$$35, 90, 88, 6, 49, 34, 79, 2.$$

What is the total number of comparisons you made?

5. a. Let $f(n), g(n), h(n)$ and $j(n)$ be positive valued functions defined on the set of positive integers, and assume that $f(n) = o(g(n))$ and $h(n) = O(j(n))$. Prove or disprove: $f(n)h^2(n) = O(g(n)j^2(n))$.

b. Let $f(n) = n! + 1 + 2 + \ldots + 2^n$ and $g(n) = 1 + 2 + \ldots + 10^n$ for all $n \geq 0$. Prove or disprove: $f(n) = O(g(n))$.

6. Design an algorithm for finding simultaneously the smallest *and* the second largest elements among 64 distinct numbers using at most 100 comparisons.

Please explain all of your answers! Good luck! - J.P.