Seminar on Combinatorial Computing
March 7, 2007, Wednesday, 6:30 p.m.
Room 6417, Graduate Center
365 Fifth Avenue, New York

Adaptive annealing: A near-optimal connection between sampling and counting

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Abstract

We present a near-optimal reduction from approximately counting the cardinality of a discrete set to approximately sampling elements of the set. Our result is set in the framework of approximating the partition function Z of a discrete system, such as the Ising model, the matchings, or colorings of a graph. The typical approach to estimating the partition function $Z(\beta)$ at some desired (inverse) temperature β^* is to define a sequence, which we call a *cooling schedule*, $\beta_0 = 0 < \beta_1 < \ldots < \beta_\ell = \beta^*$ where $Z(0)$ is trivial to compute and the ratios $Z(\beta_{i+1})/Z(\beta_i)$ are easy to estimate by sampling from the distribution corresponding to $Z(\beta_i)$. Previous approaches required a cooling schedule of length $O^*(\ln A)$ where $A = Z(0)$, thereby ensuring that each ratio $Z(\beta_{i+1})/Z(\beta_i)$ is bounded. We present a cooling schedule of length $O^*(\sqrt{\ln A})$. For well-studied problems such as estimating the partition function of the Ising model, or approximating the number of k-colorings or matchings of a graph, our cooling schedule is of length $O^*(\sqrt{n})$, which yields an overall savings of $O^*(n)$ in the running time of the approximate counting algorithm compared to previous results.

Joint work with Santosh Vempala and Eric Vigoda.

For further information contact Janos Pach at pach@cims.nyu.edu, or visit our website
http://www.math.nyu.edu/p̃ach/public_html/combinatorics_seminar.html