Seminar on Combinatorial Computing
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Room 6417, Graduate Center
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Old and new directions in Ramsey theory on integers

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Abstract

A famous result of van der Waerden states that for every pair of positive integers k and r, there exists a positive integer $W := W(k, r)$, such that every r-coloring of integers in $[W] = \{1, 2, \ldots, W\}$ contains a *monochromatic* $AP(k)$ (k-term arithmetic progression). This theorem was generalized in numerous ways, one being the following "density"-type theorem of Szemerédi: for every $k \in \mathbf{N}$ and a real number $\delta > 0$, there exists a positive integer N, such that every $S \subseteq [N]$, with $|S| \geq \delta N$, contains an $AP(k)$. In this talk, I will survey several other generalizations of van der Waerden's theorem, that have received considerable interest in the last decade.

1. (Rainbow analogues) Every 3-coloring of $[n]$, such that all the color classes have the same cardinality, contains a *rainbow* $AP(3)$ (joint work with Fox, Jungić, Mahdian, and Nešetřil). However, for infinitely many natural numbers n, we can construct 4-colorings of $[n]$, with equinumerous color classes, that contain no rainbow $AP(4)$ (joint work with Conlon and Jungić).
2. (Sub-Ramsey analogues; joint work with Fox and Jungić) For a given positive integer k, Alon, Caro, and Tuza in 1989 defined $sr(m, k)$ to be the minimal positive integer such that every coloring of $[n]$, $n \geq sr(m, k)$, *that uses each color at most k times*, yields a rainbow $AP(m)$. We can show that $\frac{17}{8}k + O(1) \leq sr(3, k) \leq \frac{15}{7}k + O(1)$ and $sr(m, k) = \Omega(m^2 k)$, improving the previous bounds.
3. (generalized van der Waerden triples; joint work with Fox) Let a and b be positive integers such that $a \leq b$ and $(a, b) \neq (1, 1)$ (this case being equivalent to van der Waerden's theorem). We prove that there exists a 6-coloring of $\mathbf{N}$ that does not contain a monochromatic (a, b)-triple, that is, a triple (x, y, z) of positive integers such that $y = ax + d$ and $z = bx + 2d$ for some positive integer d. This confirms a conjecture of Landman and Robertson.

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or visit our website
http://www.math.nyu.edu/~pach/public_html/combinatorics_seminar.html