Seminar on Combinatorial Computing
March 9, Wednesday, 6:30 p.m.
Room 6417, Graduate Center
365 Fifth Avenue, New York

Entropy and graph homomorphisms

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Abstract

A homomorphism from a graph G to a graph H is a function $f : V(G) \to V(H)$ which preserves adjacency (if $x \equiv y$ in G, then $f(x) \equiv f(y)$ in H). Questions about independent (stable) sets in G and proper q-colourings of G, among many others, can be framed in terms of graph homomorphisms. Write $\hom(G, H)$ for the number of homomorphisms from G to H.

Using entropy methods, we show that for any H and any N-vertex, d-regular, bipartite G, $\hom(G, H)$ is at most $(\hom(K(d, d), H))^{N/2d}$ where $K(d, d)$ is the complete bipartite graph with d vertices in each partition class. This generalizes a result of J. Kahn on independent sets in regular bipartite graphs.

We also give a weighted version of this result which can be interpreted as a statement about the partition function of a statistical physics "spin-system".

This represents joint work with Prasad Tetali, Georgia Tech.

For further information contact Janos Pach at pach@cims.nyu.edu,
or visit our website
http://www.math.nyu.edu/p̃ach/public_html/combinatorics_seminar.html