Seminar on Combinatorial Computing
March 2, Wednesday, 6:30 p.m.
Room 6417, Graduate Center
365 Fifth Avenue, New York

Independence in Euclidean Ramsey Theory

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Abstract

In this talk, I will present several remarkable new developments on independence in Euclidean Ramsey theory. S. Shelah and A. Soifer recently constructed a graph on the real line with chromatic number 2 in the Zermelo-Fraenkel-Choice (ZFC) system of axioms, but with uncountable chromatic number (if it exists) in a consistent system of axioms with limited choice, studied by R. Solovay in 1970. Motivated by these recent results, R. Radoičić and I discovered that the statement "every 3-coloring of the non-zero real numbers contains a monochromatic solution to the equation $x_1 + 2x_2 - 4x_3 = 0$" is independent of the Zermelo-Fraenkel axioms for set theory. A system $L : A\mathbf{x} = \mathbf{0}$ of linear homogeneous equations is called a-regular over $\mathbf{R}$ if every a-coloring of the real numbers contains a monochromatic solution to L in distinct variables. In 1943, R. Rado classified those L that are a-regular over $\mathbf{R}$ for all finite a. In ZFC, if a is an infinite cardinal, we classify those L that are a-regular. This classification depends on the cardinality of the continuum. In the Solovay model, we classify those L that are $\aleph_0$-regular over $\mathbf{R}$. We also leave several problems concerning the chromatic number of graphs on Euclidean space.

For further information contact Janos Pach at pach@cims.nyu.edu, or visit our website
http://www.math.nyu.edu/~pach/public_html/combinatorics_seminar.html