Seminar on Combinatorial Computing
April 15, Tuesday (!), 6:30 p.m.
Room 6417, Graduate Center
365 Fifth Avenue, New York

Whitehead's graph: Automorphic conjugacy in the free group of rank two

Bilal Khan
CUNY Graduate Center

Abstract

In the study of the automorphism group of a free group $F = F(X)$ on a set X, J. H. C. Whitehead introduced a graph whose vertices are elements of F, where two vertices are connected if and only if the corresponding elements of F are related by the action of one of a specially chosen set of generators of $Aut(F)$. In this introductory talk, I will outline recent results on the structure of Whitehead's graph in the case when F is the free group of rank 2.

Whitehead's graph is first partitioned into levels, and a combinatorial/algebraic framework is established to quantify the structure within and between levels. *Intra-level structure:* I show that a particular set of graphs are forbidden within any level, and that this fact implies that any "sufficiently large" connected level subgraph must have a very particular and simple form. *Inter-level structure:* I show that except for bounded defective regions in each automorphic orbit, the graph has a predictable and regular structure.

These results are harnessed to devise the first polynomial-time algorithm for testing automorphic conjugacy in F_2, surpassing Whitehead's exponential algorithm, which was the best known, to date. The intuition about the structure of Whitehead's graph was developed using computational experiments, which will also be described.

For further information contact Janos Pach at (212) 650-6123, or visit our website: http://www.math.nyu.edu/~pach/combinatorics_seminar.html