Seminar on Combinatorial Computing
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Monotone graph properties

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Abstract

A graph property P is called *monotone* if, whenever a graph has property P, so does each of its subgraphs. For example, the property that a graph is H-*free*, i.e., it contains no subgraph isomorphic to a fixed so-called forbidden subgraph H, is monotone. We measure the *size* of a property P by counting the number of graphs with property P on n labeled vertices.

Turán's theorem provides an upper bound of roughly $(1-1/p)\binom{n}{2}$ on the number of edges of a K_{p+1}-free graph with n vertices. Clearly, every subgraph of a K_{p+1}-free graph is K_{p+1}-free, which gives a lower bound on the size of this property. Erdős, Frankl, Rödl, Prömel, Steger, and others proved that this bound is not far from being tight, and generalized their result in various directions.

In a recent paper, joint with Bollobás and Simonovits, I managed to establish, among other results, the following theorem: for every monotone property P, there is an integer k and a positive constant c such that the size of P is between $2^{(1-1/k)\binom{n}{2}}$ and $2^{(1-1/k)\binom{n}{2}+n^{2-c}}$. We applied probabilistic arguments and Szemerédi's Regularity Lemma. We also present many open problems.

For further information contact Janos Pach at pachcims.nyu.edu, or visit our website: http://cs.gc.cuny.edu/ pach/