Seminar on Combinatorial Computing
April 5, Wednesday, 6:20 p.m.
Room 3212, Graduate Center
365 Fifth Avenue, New York

On the Structure of Point Sets Containing Many Similar Copies of a Given Pattern

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Abstract

Consider a k-element subset P of the plane. Earlier results from Elekes and Erdős, and by Laczkovich and Ruzsa indicate that the maximum number of sets similar to P that can be found among n points in the plane is $\Theta(n^2)$ if and only if the cross ratio of any quadruplet of points in P is algebraic.

In this talk we will study the structure of the extremal sets A which contain cn^2 subsets similar to P. As our main result we prove that if an n-element set A contains cn^2 subsets similar to P then A must contain large lattice-like structures. In particular we prove that, for n large enough, A must contain m points in a line forming an arithmetic progression, or $m \times m$ lattices, when P is not cocyclic or collinear. On the other hand we show that for cocyclic or collinear sets P, there are n-element sets A with $c_P n^2$ copies of P and without $k \times k$ lattice subsets. If time permits we will show some constructions of sets in general position having a large number of equilateral triangles. Most of this material is joint work together with György Elekes.

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or visit our website: http://math.gc.cuny.edu/Seminars/Combinatorics.html