Seminar on Combinatorial Computing
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Room 6417, Graduate Center
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Sylvester-Gallai type theorems for pseudoparabolas

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Abstract

The Gallai-Sylvester Theorem states the following: In any arrangement of n lines in the plane, not all of them through one point, there is a point through which precisely two lines pass. Such a point is called an *ordinary point*. In their paper from 1993, Csima and Sawyer showed that in such arrangement ($n > 7$) of lines there are at least $\frac{6n}{13}$ ordinary points. Their proof holds also for arrangements of pseudolines (this is an arrangement of simple curves, every two of which intersect precisely once).

We are interested in that property in arrangements of pairwise intersecting pseudoparabolas, which is a set of x-monotone bi-infinite curves, every two of which intersect precisely twice. The area bounded by two curves is called a *lens*. If the lens is not intersected by any other curve it is said to be an *empty lens*. There is an arrangement of four pseudoparabolas with no ordinary point. We present a new arrangement of seven pseudoparabolas with no ordinary point and show that in any arrangement of n pairwise intersecting pseudoparabolas the number or ordinary points plus the number of empty lenses is at least $\frac{n}{7}$. We also present some new observations about arrangements of pseudoparabolas and arrangements of circles.

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or visit our website
http://www.math.nyu.edu/p̃ach/public_html/combinatorics_seminar.html