Seminar on Combinatorial Computing
October 18, Wednesday, 6:30 p.m.
Room 6417, Graduate Center
365 Fifth Avenue, New York

Necklaces, convolutions, and $X + Y$

John Iacono
Polytechnic University, Brooklyn

Abstract

We give subquadratic algorithms that, given two necklaces, each with n beads at arbitrary positions, compute the optimal rotation of the necklaces to best align the beads. Here alignment is measured according to the ℓ_p norm of the vector of distances between pairs of beads from opposite necklaces in the best perfect matching. We show surprisingly different results for $p = 1, p = 2$, and $p = \infty$. For $p = 2$, we reduce the problem to standard convolution, while for $p = \infty$ and $p = 1$, we reduce the problem to $(\min, +)$ convolution and $(\text{median}, +)$ convolution. Then we solve the latter two convolution problems in subquadratic time, which are interesting results in their own right. These results shed some light on the classic sorting $X + Y$ problem, because the convolutions can be viewed as computing order statistics on the antidiagonals of the $X + Y$ matrix. All of our algorithms run in $o(n^2)$ time, whereas the obvious algorithms for these problems run in $\Theta(n^2)$ time.

This is joint work with David Bremner, Timothy M. Chan, Erik D. Demaine, Jeff Erickson, Ferran Hurtado, Stefan Langerman, and Perouz Taslakian.

For further information contact Janos Pach at pach@cims.nyu.edu,
or visit our website
http://www.math.nyu.edu/p̃ach/public_html/combinatorics_seminar.html