Seminar on Combinatorial Computing
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Room 6417, Graduate Center
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Bijective decomposition of tree-rooted maps

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Abstract

A map is an embedding of a planar graph in the sphere considered up to continuous deformation. In this talk, we consider tree-rooted maps, that is, maps with a distinguished spanning tree. Tree-rooted maps are well behaved objects that can be encoded by shuffles of two parenthesis systems. This encoding makes their enumeration easy and it was noticed by Mullin (1967), that the number of tree-rooted maps with n edges is $C_n C_{n+1}$, where $C_n = \frac{1}{n+1}\binom{2n}{n}$ is the n^{th} Catalan number.

Catalan numbers appear in the enumeration of many combinatorial structures. For instance, the parenthesis systems of length $2n$, the plane trees with n edges and the non-crossing partitions of size n are all counted by the n^{th} Catalan number. Therefore, Mullin's result makes it appealing to look for a bijection between tree-rooted maps with n edges and pairs of Catalan objects of size n and $n+1$.

We will present a bijection between tree-rooted maps and pairs made of a plane tree and a non-crossing partition. This bijection allows the random generation of tree-rooted maps in linear time. It also involves a general correspondence between spanning trees and a class of orientations which appears to be central in many other bijective results on maps.

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or visit our website
http://www.math.nyu.edu/p̃ach/public_html/combinatorics_seminar.html