Seminar on Combinatorial Computing
November 30, Wednesday, 6:30 p.m.
Room 6417, Graduate Center
365 Fifth Avenue, New York

Ramsey-Type Results for the Hypercube

Jan Vondrák
Microsoft Research

Abstract

We consider the question of existence of monochromatic cycles for edge colorings of the hypercube, raised by Fan Chung in 1992. She proved that for any fixed $k \geq 2$ and even $\ell \geq 4$, for a sufficiently large hypercube, any k-coloring of the edges contains a monochromatic cycle of length 2ℓ. On the other hand, there is a 2-coloring of any hypercube which avoids monochromatic cycles of length 4, and a 3-coloring which avoids monochromatic cycles of length 6. Fan Chung asked what happens for odd $\ell \geq 5$, i.e. whether it is possible to avoid monochromatic cycles of length $10, 14, 18, \ldots$.

We answer this question by proving that for any fixed $k \geq 2$ and $\ell \geq 5$, any k-coloring of a sufficiently large hypercube contains a monochromatic cycle of length 2ℓ. More generally, we provide a characterization of all subgraphs of the hypercube with this Ramsey property. In addition, we show the existence of subgraphs H_k such that for a sufficiently large hypercube, any k-coloring of the edges contains a monochromatic copy of H_k but this is not the case for all $(k+1)$-colorings.

For further information contact Janos Pach at pach@cims.nyu.edu,
or visit our website
http://www.math.nyu.edu/p̃ach/public_html/combinatorics_seminar.html