Seminar on Combinatorial Computing
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Counting and enumerating pseudo-triangulations

Hervé Brönnimann
Polytechnic University, Brooklyn

Abstract

Decompositions called pseudo-triangulations or geodesic triangulations have been studied for convex sets and for simple polygons in the plane because of their applications to visibility and ray shooting. They have been used in a number of kinetic data structures (KDSs) for collision detection among moving objects in the plane because they can be maintained by edge swaps as points move and can form a partition of the free space whose size is related to minimum link separators of the objects. Streinu used them in an elegant proof that it is always possible to unfold a chain in the plane without self-intersection.

In this talk, I will introduce pseudo-triangulations (PST) for a given point set in the plane; PSTs have many properties of triangulations, but have more freedom: a PST consists of a decomposition into pseudo-triangles, which are polygons with more than three vertices but exactly three have angles less than π. We will be interested only in minimal such decompositions (i.e. removing an edge violates the PST property). Such minimal PSTs have exactly $2n-3$ edges.

Our work started with the following conjecture: "For any set S of points in general position in the plane, there are at least as many (minimal) pseudo-triangulations of S as of triangulations of S, with equality iff the points are in convex position."

Though the conjecture remains unproved, we have taken the experimental route and implemented an algorithm that counts PSTs of a given point set. We will show that PSTs, like triangulations, have an edge-flip operation with efficient implementations. We will then use edge-flips to enumerate all PSTs of a given point set, based on the greedy flip algorithm of Pocchiola and Vegter for computing the visibility complex of a scene of n convex objects in the plane. We will present some experimental support of the conjecture for small values of n.

For further information contact Janos Pach at pach@cims.nyu.edu, or visit our website: http://www.math.nyu.edu/ pach/combinatorics_seminar.html