Seminar on Combinatorial Computing
December 6, Wednesday, 6:30 p.m.
Room 8404, Graduate Center
365 Fifth Avenue, New York

Graph coloring via the probabilistic method

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Abstract

We discuss a simple but surprisingly powerful graph colouring technique. As we will see, this technique is the main idea behind many of the strongest results in graph colouring over the past decade or so.

The idea is to generate a random partial colouring of a graph in perhaps the simplest way possible. Assign to each vertex a colour chosen uniformly at random. Of course, with very high probability this will not be a partial colouring, so we fix it by uncolouring any vertex which receives the same colour as one of its neighbours. What remains must be a proper partial colouring of the graph. We then extend this partial colouring (perhaps greedily) to obtain a colouring of G. A straightforward probabilistic analysis of this simple procedure allows us to obtain good bounds on the chromatic number of some graphs.

This is the general approach we discuss. Of course, if $N(v)$ is a clique then there will be no repeated colours in $N(v)$ under any partial colouring. Thus, we will need to modify the procedure to obtain results for graphs which have small dense clique-like subgraphs. To do so, we need to introduce a structural decomposition theory.

We will present some results on obtaining optimal or near optimal colourings of various graphs using this mixture of probabilistic and structural techniques. The talk is directed at a general audience and no background in probability is required.

For further information contact Janos Pach at pach@cims.nyu.edu, or visit our website: http://www.math.nyu.edu/ pach/combinatorics_seminar.html