Seminar on Combinatorial Computing
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Room 6421, Graduate Center
365 Fifth Avenue, New York

Graphs, free groups, and the Hanna Neumann Conjecture

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Abstract

Let H, K be non-trivial finitely generated subgroups of a free group $F = F(\{a, b\})$, with ranks m and n respectively. In 1958, Hanna Neumann proved the following inequality on the rank of the intersection of two groups:

$$rank(H \cap K) - 1 \leq 2(m-1)(n-1)$$

and then asked whether the factor of 2 in this inequality could be dropped. The tighter form of the inequality (without the 2) has since come to be known as the Hanna Neumann Conjecture.

Since 1992, Gabor Tardos and Warren Dicks have succeeded in proving the conjecture for several cases, e.g. when $m, n \leq 2$. Their proofs have involved graph-theoretic and combinatorial arguments.

This introductory talk is intended for combinatorics enthusiasts who would like to see what the above group-theoretic inequalities have to do with graphs. Specifically, we will describe the mapping between the class of finitely generated subgroups of F and a certain class of directed edge-labelled graphs. Then, in the context of graph theory, we will prove above inequality and derive the combinatorial equivalent of the Hanna Neumann Conjecture.

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or visit our website: http://math.gc.cuny.edu/Seminars/Combinatorics.html